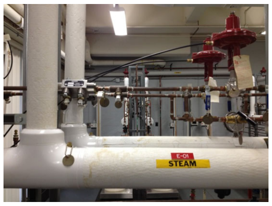
CM3110
Transport/Unit Ops I
Part II: Heat Transfer



Complex Heat Transfer – Dimensional Analysis

(Forced convection heat transfer)

Professor Faith Morrison

Department of Chemical Engineering
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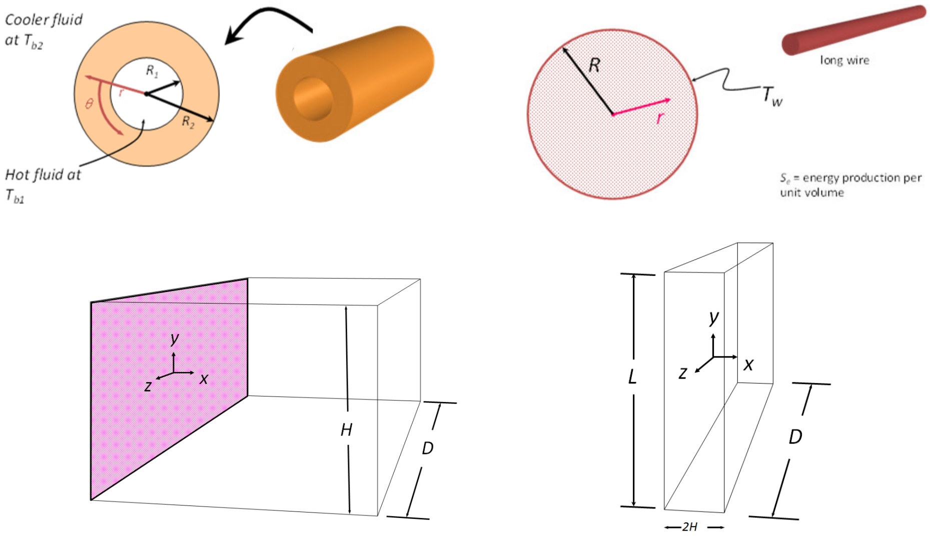
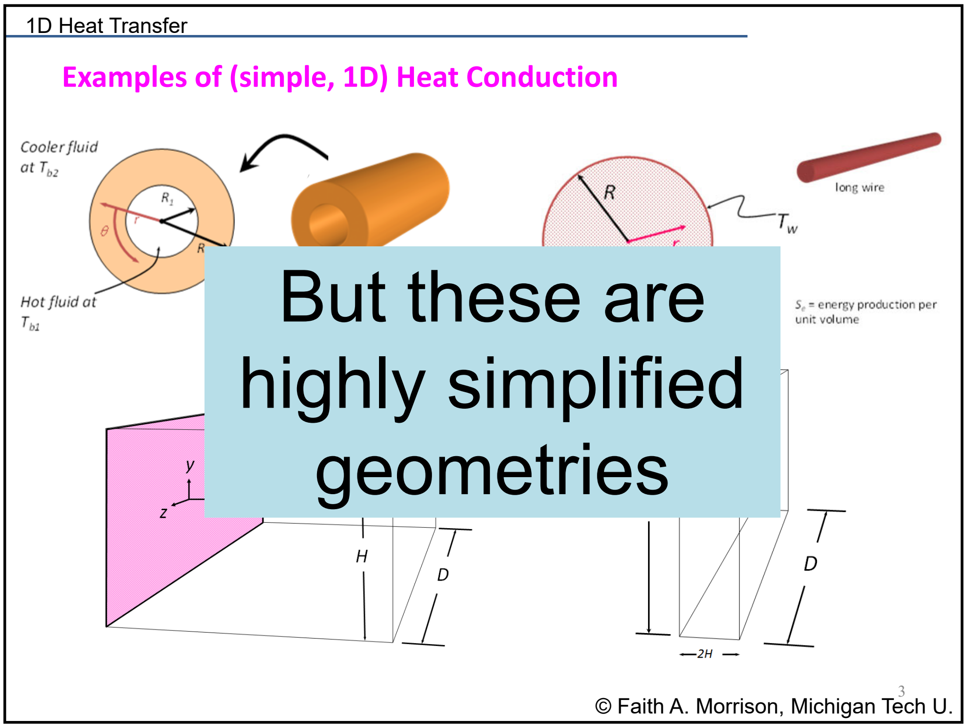
1D Heat Transfer

(what have we been up to?)

Examples of (simple, 1D) Heat Conduction





1D Heat Transfer

Examples of (simple, 1D) Heat Conduction

But these are highly simplified geometries

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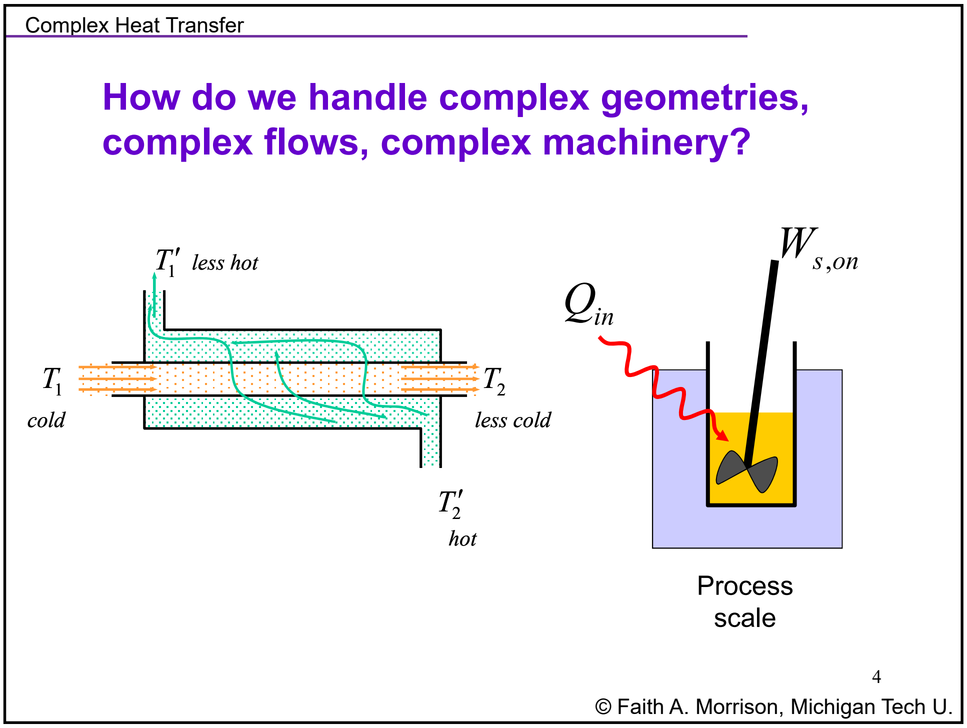

Complex Heat Transfer

How do we handle complex geometries, complex flows, complex machinery?

Process scale

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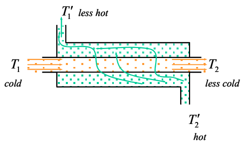
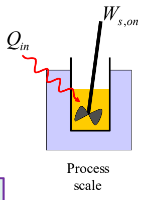
Complex Heat Transfer – Dimensional Analysis

*(**Answer**: Use the same techniques we have been using in fluid mechanics)*





Engineering Modeling

- Choose an idealized problem and solve it
- From insight obtained from **ideal** problem, identify governing equations of **real** problem
- Nondimensionalize the governing equations; deduce dimensionless scale factors (e.g. Re, Fr for fluids)
- Design experiments to test modeling thus far
- Revise modeling (structure of dimensional analysis, identity of scale factors, e.g. add roughness lengthscale)
- Design additional experiments
- Iterate until useful correlations result

Complex Heat Transfer – Dimensional Analysis

Experience with Dimensional Analysis thus far:

- Flow in pipes at all flow rates (laminar and turbulent)

  Solution: Navier-Stokes, Re, Fr, L/D, dimensionless wall force $= f$; $f = f(\mathrm{Re}, L/D)$

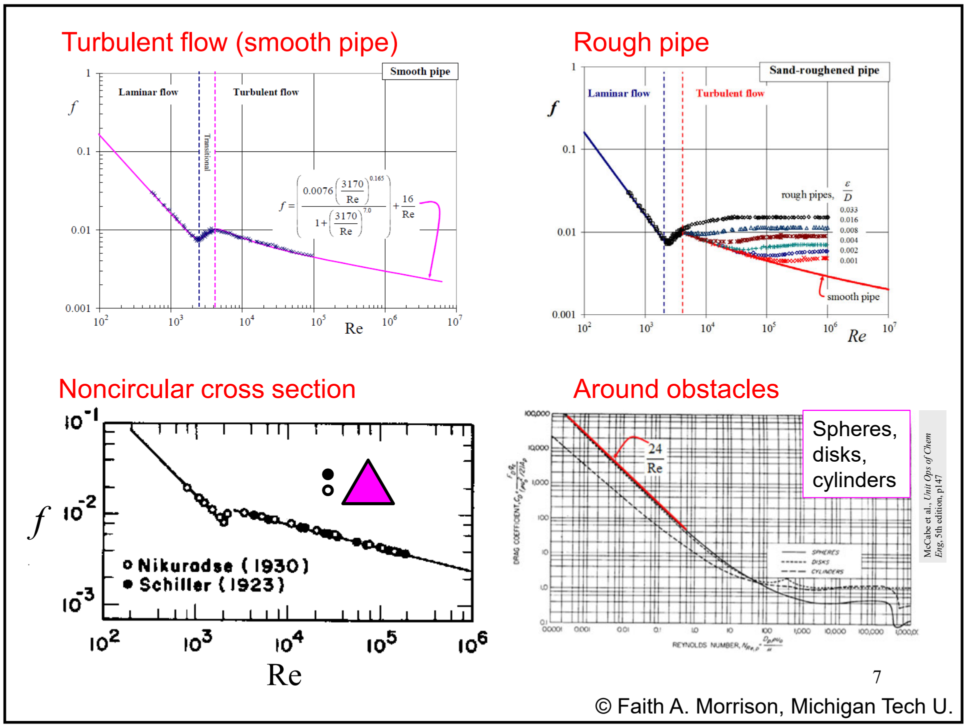
- Rough pipes ***Solution:*** add additional length scale; then nondimensionalize

- Non-circular conduits ***Solution:*** Use hydraulic diameter as the length scale of the flow to nondimensionalize

- Flow around obstacles (spheres, other complex shapes

  Solution: Navier-Stokes, Re, dimensionless drag $= C_D$; $C_D = C_D(\mathrm{Re})$

- Boundary layers ***Solution:*** Two components of velocity need independent lengthscales



Turbulent flow (smooth pipe)

$$f = \frac{0.0076\left(\frac{3170}{Re}\right)^{0.165}}{1+\left(\frac{3170}{Re}\right)^{7.0}} + \frac{16}{Re}$$

Rough pipe

Noncircular cross section

Around obstacles

Spheres, disks, cylinders

McCabe et al., *Unit Ops of Chem Eng*, 5th edition, p147

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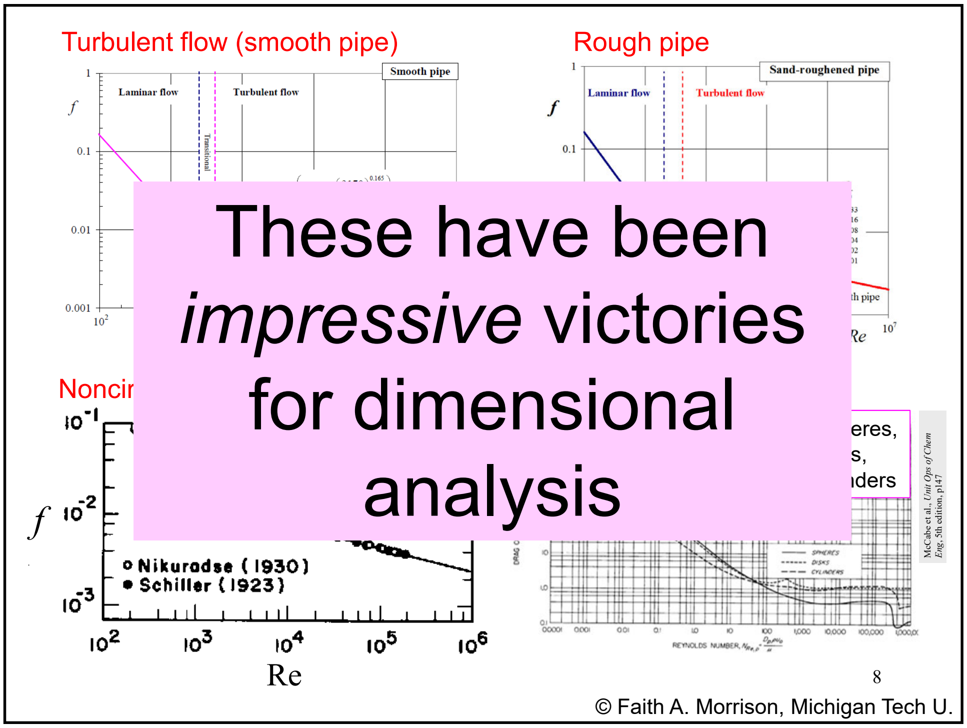
Turbulent flow (smooth pipe)

Rough pipe

These have been *impressive* victories for dimensional analysis

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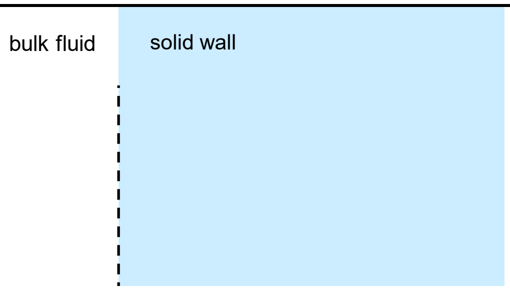
Complex Heat Transfer – Dimensional Analysis

Now, move to heat transfer:

- Forced convection heat transfer **from fluid to wall**
  Solution: ?
- Natural convection heat transfer from fluid to wall
  Solution: ?
- Radiation heat transfer from solid to fluid
  Solution: ?

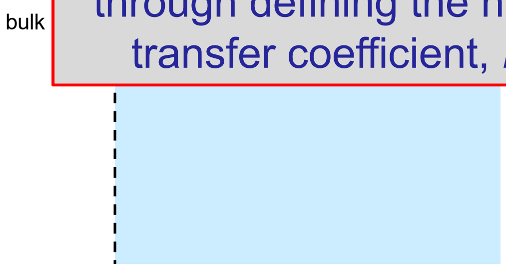


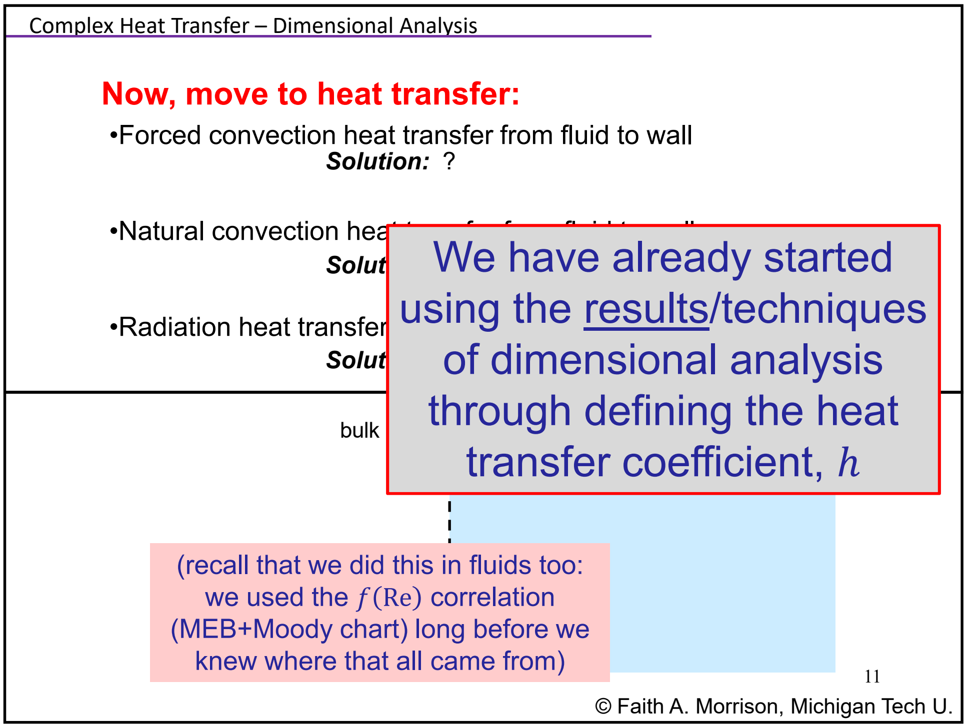
Complex Heat Transfer – Dimensional Analysis

Now, move to heat transfer:

- Forced convection heat transfer from fluid to wall
  Solution: ?
- Natural convection hea
  Solut
- Radiation heat transfer
  Solut

We have already started using the <u>results</u>/techniques of dimensional analysis through defining the heat transfer coefficient, h





Complex Heat Transfer – Dimensional Analysis

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We have already started using the results/techniques of dimensional analysis through defining the heat transfer coefficient, h

(recall that we did this in fluids too: we used the $f(\mathrm{Re})$ correlation (MEB+Moody chart) long before we knew where that all came from)

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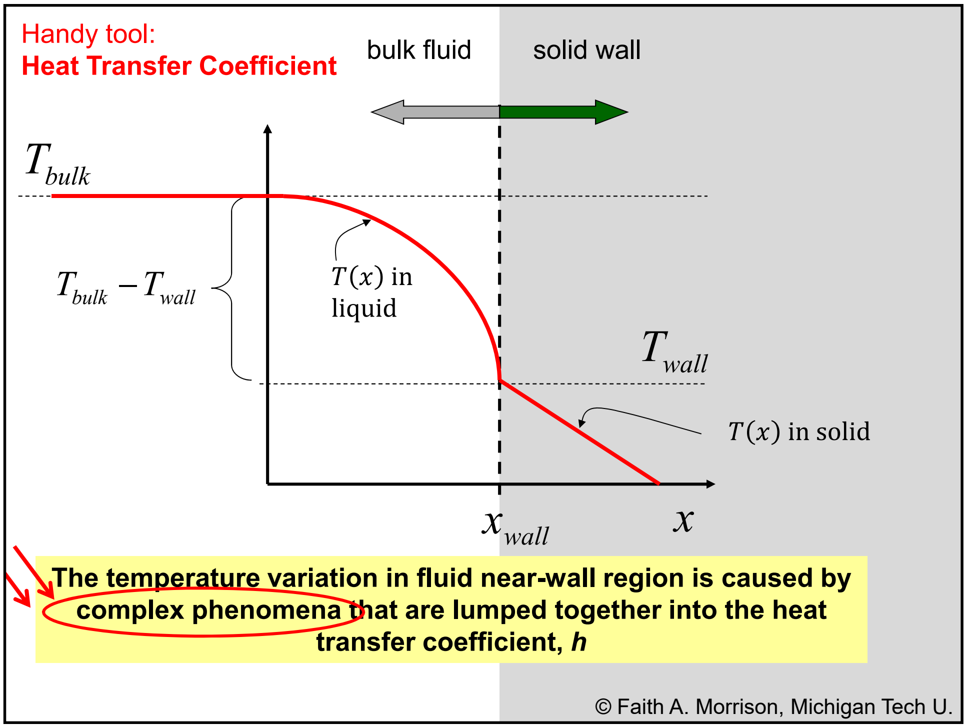

Handy tool:
Heat Transfer Coefficient

The temperature variation in fluid near-wall region is caused by complex phenomena that are lumped together into the heat transfer coefficient, *h*

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Complex Heat Transfer – Dimensional Analysis

The flux at the wall is given by the empirical expression known as **Newton's Law of Cooling**

This expression serves as the definition of the heat transfer coefficient.

$$\left|\frac{q_x}{A}\right| \equiv \textcircled{h}\left|T_{bulk} - T_{wall}\right|$$

***h* depends on:**

- geometry
- fluid velocity
- fluid properties
- temperature difference

Complex Heat Transfer – Dimensional Analysis

The flux at the wall is given by the empirical expression known as **Newton's Law of Cooling**

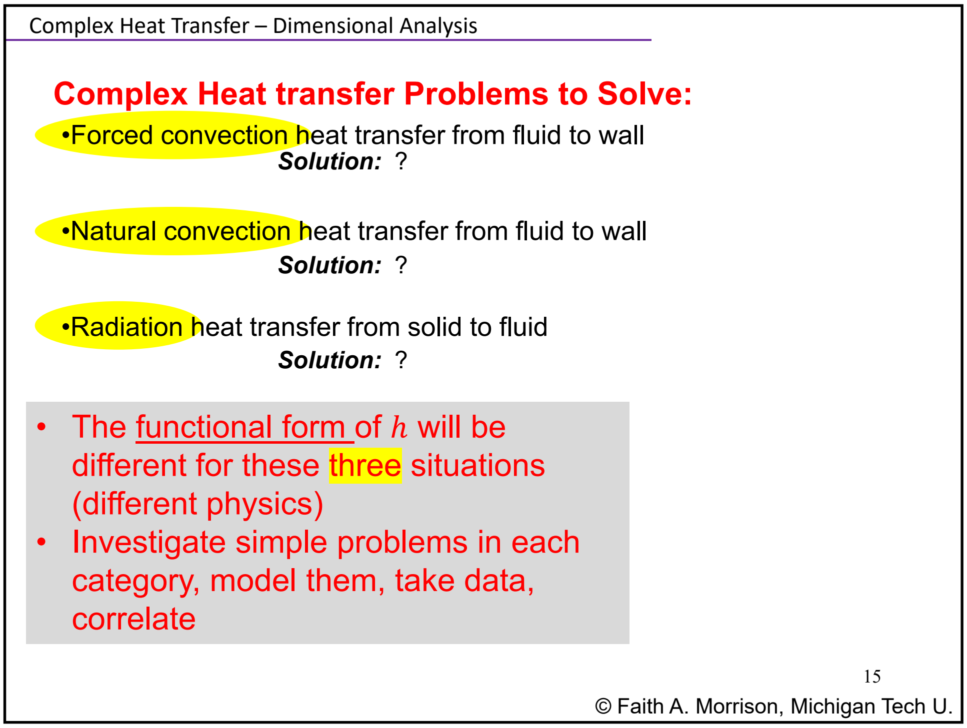
This expression serves as the definition of the heat transfer coefficient.

$$\left|\frac{q_x}{A}\right| \equiv \textcircled{h}\left|T_{bulk} - T_{wall}\right|$$

To get values of h for various situations, we need to measure data and create data correlations (**dimensional analysis**)

***h* depends on:**

- geometry
- fluid velocity
- fluid properties
- temperature difference



Complex Heat Transfer – Dimensional Analysis

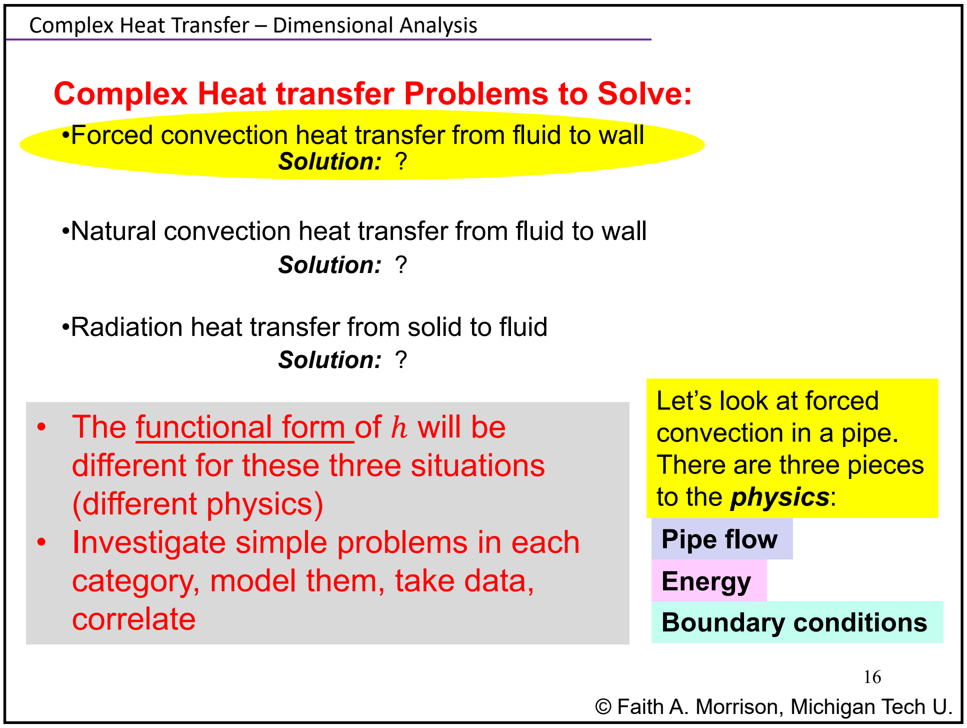
Complex Heat transfer Problems to Solve:

•Forced convection heat transfer from fluid to wall
Solution: ?

•Natural convection heat transfer from fluid to wall
Solution: ?

•Radiation heat transfer from solid to fluid
Solution: ?

- The functional form of h will be different for these three situations (different physics)
- Investigate simple problems in each category, model them, take data, correlate



Complex Heat Transfer – Dimensional Analysis

Complex Heat transfer Problems to Solve:

•Forced convection heat transfer from fluid to wall
Solution: ?

•Natural convection heat transfer from fluid to wall
Solution: ?

•Radiation heat transfer from solid to fluid
Solution: ?

- The functional form of h will be different for these three situations (different physics)
- Investigate simple problems in each category, model them, take data, correlate

Let's look at forced convection in a pipe. There are three pieces to the ***physics***:

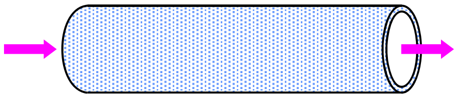
Pipe flow

Energy

Boundary conditions

Complex Heat Transfer – Dimensional Analysis

Chosen problem: Forced Convection Heat Transfer
Solution: Dimensional Analysis



Following procedure familiar from pipe flow,

- **What are governing equations?**
- **Scale factors (dimensionless numbers)?**
- **Quantity of interest?**

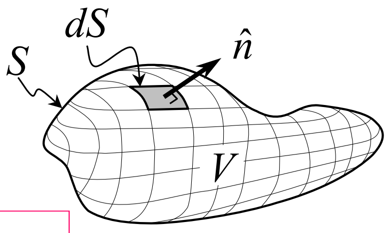
Answer: Heat flux at the wall

Complex Heat Transfer – Dimensional Analysis

General Energy Transport Equation

(microscopic energy balance)

As for the derivation of the microscopic momentum balance, the microscopic energy balance is derived on an arbitrary volume, *V*, enclosed by a surface, *S*.



Gibbs notation:

$$\boxed{\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+\underline{v}\cdot\nabla T\right)=k\nabla^2 T+S}$$

see handout for component notation

Complex Heat Transfer – Dimensional Analysis

General Energy Transport Equation

(microscopic energy balance; **in the fluid**)

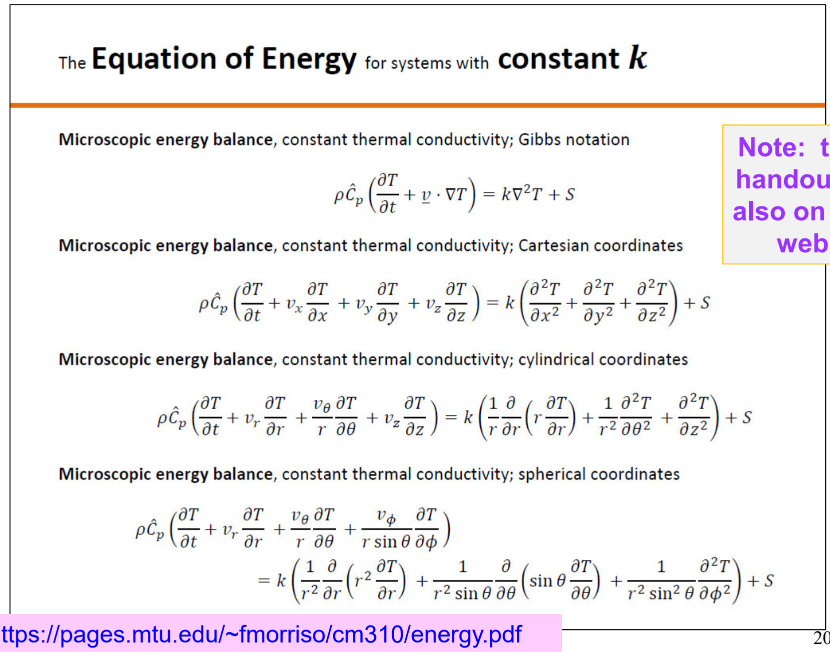
$$\rho\hat{C}_p\left(\underbrace{\frac{\partial T}{\partial t}}_{\text{rate of change}}+\overbrace{\underline{v}\cdot\nabla T}^{\text{convection}}\right)=\underbrace{k\nabla^2 T}_{\substack{\text{conduction}\\\text{(all directions)}}}+\overbrace{S}^{\text{source}}$$

source (energy generated per unit volume per time)

fluid velocity must satisfy equation of motion, equation of continuity

see handout for component notation

Complex Heat Transfer – Dimensional Analysis



The **Equation of Energy** for systems with **constant *k***

Microscopic energy balance, constant thermal conductivity; Gibbs notation

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+\underline{v}\cdot\nabla T\right)=k\nabla^2 T+S$$

Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_x\frac{\partial T}{\partial x}+v_y\frac{\partial T}{\partial y}+v_z\frac{\partial T}{\partial z}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+S$$

Microscopic energy balance, constant thermal conductivity; cylindrical coordinates

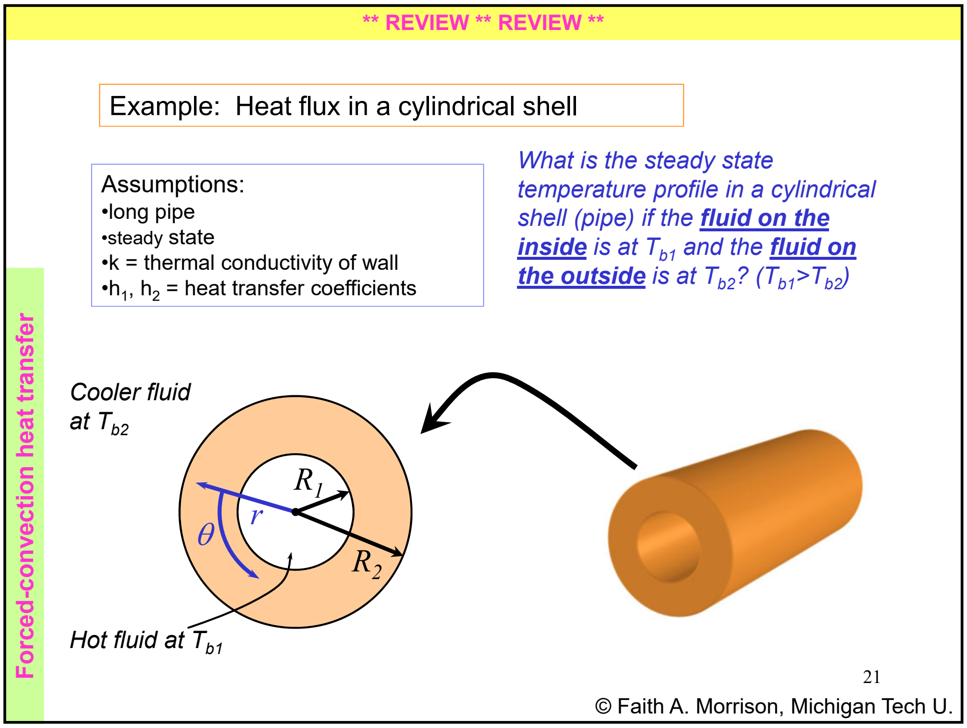
$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}+v_z\frac{\partial T}{\partial z}\right)=k\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}\right)+S$$

Microscopic energy balance, constant thermal conductivity; spherical coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial T}{\partial \phi}\right)$$
$$=k\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \phi^2}\right)+S$$

Note: this handout is also on the web

https://pages.mtu.edu/~fmorriso/cm310/energy.pdf



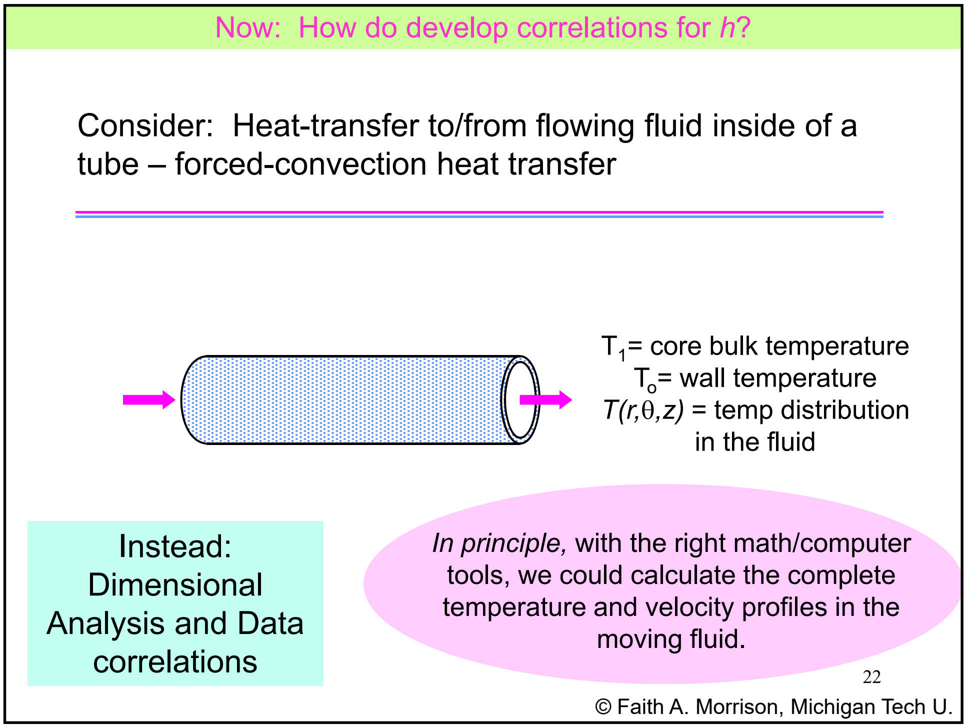
** REVIEW ** REVIEW **

Forced-convection heat transfer

Example: Heat flux in a cylindrical shell

Assumptions:
- long pipe
- steady state
- k = thermal conductivity of wall
- h_1, h_2 = heat transfer coefficients

*What is the steady state temperature profile in a cylindrical shell (pipe) if the **fluid on the inside** is at T_{b1} and the **fluid on the outside** is at T_{b2}? ($T_{b1} > T_{b2}$)*



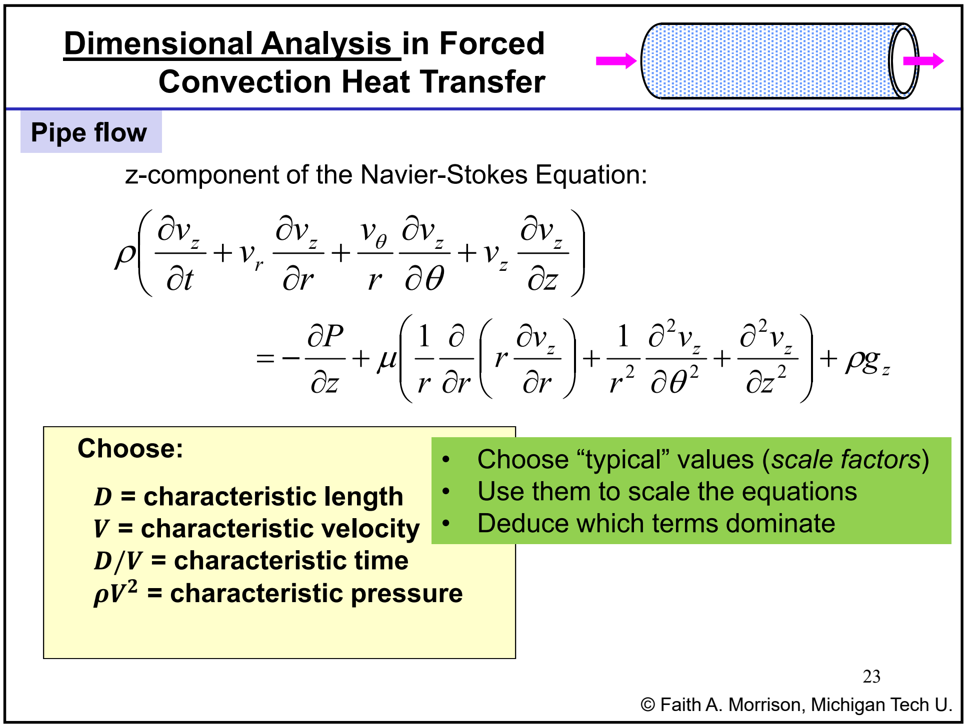
Now: How do develop correlations for *h*?

Consider: Heat-transfer to/from flowing fluid inside of a tube – forced-convection heat transfer

T_1= core bulk temperature
T_o= wall temperature
$T(r,\theta,z)$ = temp distribution in the fluid

Instead: Dimensional Analysis and Data correlations

In principle, with the right math/computer tools, we could calculate the complete temperature and velocity profiles in the moving fluid.



Dimensional Analysis in Forced Convection Heat Transfer

Pipe flow

z-component of the Navier-Stokes Equation:

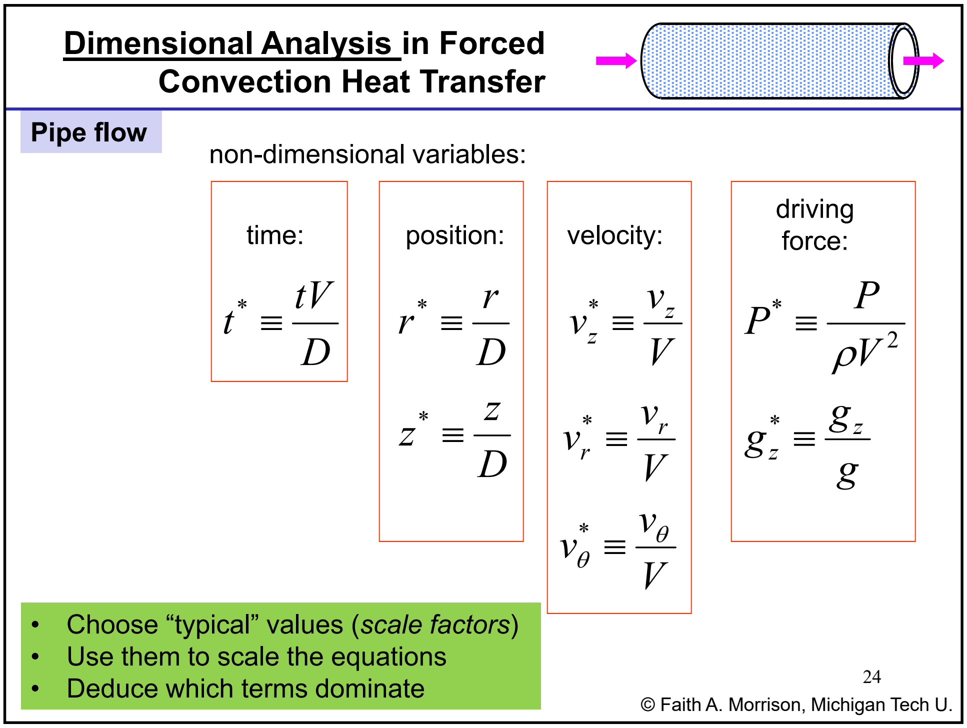
$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)$$

$$=-\frac{\partial P}{\partial z}+\mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right)+\rho g_z$$

Choose:

D = characteristic length
V = characteristic velocity
D/V = characteristic time
ρV^2 = characteristic pressure

- Choose “typical” values (*scale factors*)
- Use them to scale the equations
- Deduce which terms dominate



Dimensional Analysis in Forced Convection Heat Transfer

Pipe flow

non-dimensional variables:

time:

$$t^* \equiv \frac{tV}{D}$$

position:

$$r^* \equiv \frac{r}{D}$$

$$z^* \equiv \frac{z}{D}$$

velocity:

$$v_z^* \equiv \frac{v_z}{V}$$

$$v_r^* \equiv \frac{v_r}{V}$$

$$v_\theta^* \equiv \frac{v_\theta}{V}$$

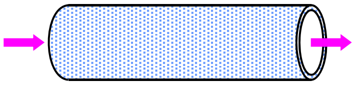
driving force:

$$P^* \equiv \frac{P}{\rho V^2}$$

$$g_z^* \equiv \frac{g_z}{g}$$

- Choose “typical” values (*scale factors*)
- Use them to scale the equations
- Deduce which terms dominate

Dimensional Analysis in Forced Convection Heat Transfer



Energy

Microscopic energy balance:

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}+v_z\frac{\partial T}{\partial z}\right)=k\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}\right)+S$$

non-dimensional variables:

position:

$$r^*\equiv\frac{r}{D}\qquad z^*\equiv\frac{z}{D}$$

temperature:

$$T^*\equiv\frac{T-T_o}{(T_1-T_o)}$$

source:

$$S^*\equiv\frac{S}{S_0}$$

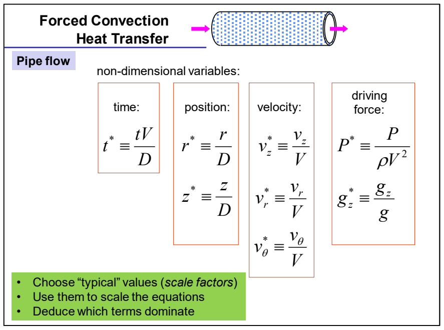
Choose:

T –use a characteristic **interval** (since distance from absolute zero is not part of this physics)

S – use a reference source, S_0

$$S_0\equiv\frac{(T_1-T_0)V\rho\hat{C}_p}{D}[=]\frac{W}{m^2}$$

Complex Heat Transfer – Dimensional Analysis Forced Convection



Forced Convection Heat Transfer

Pipe flow

non-dimensional variables:

time:

$$t^*\equiv\frac{tV}{D}$$

position:

$$r^*\equiv\frac{r}{D}\qquad z^*\equiv\frac{z}{D}$$

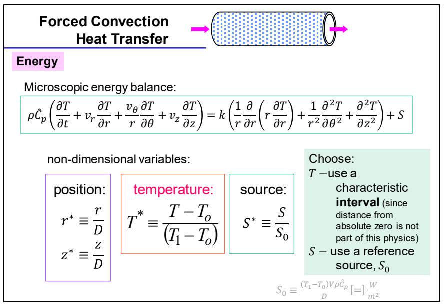
velocity:

$$v_z^*\equiv\frac{v_z}{V}\qquad v_r^*\equiv\frac{v_r}{V}\qquad v_\theta^*\equiv\frac{v_\theta}{V}$$

driving force:

$$P^*\equiv\frac{P}{\rho V^2}\qquad g_z^*\equiv\frac{g_z}{g}$$

- Choose “typical” values (*scale factors*)
- Use them to scale the equations
- Deduce which terms dominate



Forced Convection Heat Transfer

Energy

Microscopic energy balance:

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}+v_z\frac{\partial T}{\partial z}\right)=k\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}\right)+S$$

non-dimensional variables:

position: $r^*\equiv\frac{r}{D}$ $z^*\equiv\frac{z}{D}$

temperature: $T^*\equiv\frac{T-T_o}{(T_1-T_o)}$

source: $S^*\equiv\frac{S}{S_0}$

Choose:

T –use a characteristic **interval** (since distance from absolute zero is not part of this physics)

S – use a reference source, S_0

$S_0\equiv\frac{(T_1-T_0)V\rho\hat{C}_p}{D}[=]\frac{W}{m^2}$

Substitute all these definitions,

$$(t^*,r^*,z^*,p^*,g^*,\\ v_r^*,v_\theta^*,v_z^*,T^*,S^*)$$

into the ***governing equations*** and simplify…

Complex Heat Transfer – Dimensional Analysis Forced Convection

Non-dimensional Energy Equation

$$\left(\frac{\partial T^*}{\partial t^*} + v_r^*\frac{\partial T^*}{\partial r^*} + \frac{v_\theta^*}{r^*}\frac{\partial T^*}{\partial \theta} + v_z^*\frac{\partial T^*}{\partial z^*}\right) = \boxed{\frac{1}{\text{Pe}}}\left(\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial T^*}{\partial r^*}\right) + \frac{1}{r^{*2}}\frac{\partial^2 T^*}{\partial \theta^2} + \frac{\partial^2 T^*}{\partial z^2}\right) + S^*$$

$$\text{Pe} \equiv \text{Pr}\,\text{Re} = \frac{\hat{C}_p\mu}{k}\frac{\rho V D}{\mu}$$

Non-dimensional Navier-Stokes Equation

$$\frac{Dv_z^*}{Dt^*} = -\frac{\partial P^*}{\partial z^*} + \boxed{\frac{1}{\text{Re}}}\left(\nabla^2 v_z\right)^* + \boxed{\frac{1}{Fr}}g^*$$

$$\text{Pr} \equiv \frac{\hat{C}_p\mu}{k}$$

Non-dimensional Continuity Equation

$$\frac{\partial v_x^*}{\partial x^*} + \frac{\partial v_y^*}{\partial y^*} + \frac{\partial v_z^*}{\partial z^*} + = 0$$

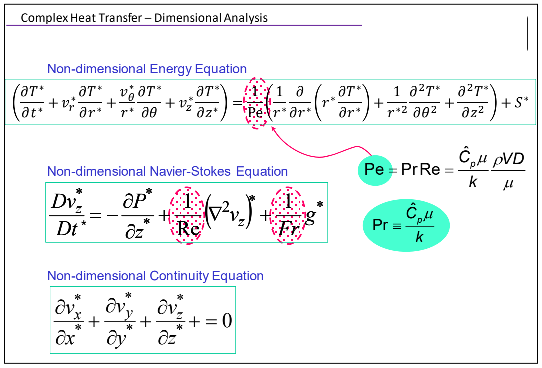
$$T^* = T^*(\text{Re}, \text{Pr})$$
$$\underline{v}^* = \underline{v}^*(\text{Re}, \text{Fr})$$

$$\frac{Dv_z}{Dt} \equiv \frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}$$

Complex Heat Transfer – Dimensional Analysis Forced Convection

How do we get to the "*engineering quantities of interest*" ?

What are the "*engineering quantities of interest*" ?



Complex Heat Transfer – Dimensional Analysis

Non-dimensional Energy Equation

$$\left(\frac{\partial T^*}{\partial t^*} + v_r^*\frac{\partial T^*}{\partial r^*} + \frac{v_\theta^*}{r^*}\frac{\partial T^*}{\partial \theta} + v_z^*\frac{\partial T^*}{\partial z^*}\right) = \boxed{\frac{1}{\text{Pe}}}\left(\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial T^*}{\partial r^*}\right) + \frac{1}{r^{*2}}\frac{\partial^2 T^*}{\partial \theta^2} + \frac{\partial^2 T^*}{\partial z^2}\right) + S^*$$

$$\text{Pe} \equiv \text{Pr}\,\text{Re} = \frac{\hat{C}_p\mu}{k}\frac{\rho V D}{\mu}$$

Non-dimensional Navier-Stokes Equation

$$\frac{Dv_z^*}{Dt^*} = -\frac{\partial P^*}{\partial z^*} + \boxed{\frac{1}{\text{Re}}}\left(\nabla^2 v_z\right)^* + \boxed{\frac{1}{Fr}}g^*$$

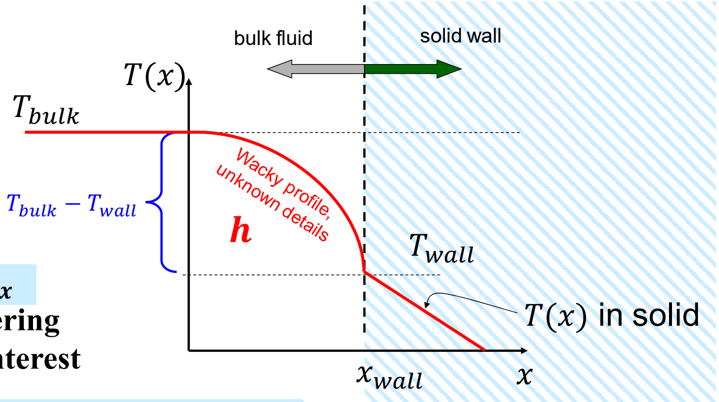
$$\text{Pr} \equiv \frac{\hat{C}_p\mu}{k}$$

Non-dimensional Continuity Equation

$$\frac{\partial v_x^*}{\partial x^*} + \frac{\partial v_y^*}{\partial y^*} + \frac{\partial v_z^*}{\partial z^*} + = 0$$

Microscopic Energy Balance Review—Boundary Conditions

The interface between the solid and the fluid calls for a new type of boundary condition, ***Newton's Law of Cooling***.

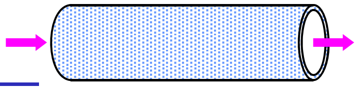


$Q = \boldsymbol{q_x}$

is the engineering quantity of interest

$$\boxed{\left|\frac{q_x}{A}\right| = h\left|T_{bulk} - T_{wall}\right|}$$

Forced Convection Heat Transfer



Linear driving force model

$$\left|\frac{q_x}{A}\right| = h|T_1 - T_0|$$

Apply at the surface (in fluid):

$$(2\pi RL)(h)(T_1 - T_0) = Q = \iint_S \left[\hat{e}_r \cdot \underline{\tilde{q}}\right]_{surface} dS$$

$$(2\pi RL)(h)(T_1 - T_0) = Q = \int_0^{2\pi}\int_0^{L} -k\left.\frac{\partial T}{\partial r}\right|_{r=R} Rdzd\theta$$

Now, non-dimensionalize this expression as well.

Complex Heat Transfer – Dimensional Analysis

Non-dimensionalize

non-dimensional variables:

position:

$$r^* \equiv \frac{r}{D}$$

$$z^* \equiv \frac{z}{D}$$

temperature:

$$T^* \equiv \frac{T-T_o}{(T_1-T_o)}$$

Complex Heat Transfer – Dimensional Analysis

$$h(\pi \cancel{D} L)\cancel{(T_1-T_o)} = \int_0^{2\pi}\int_0^{L/D} -k \left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1/2} \frac{\cancel{(T_1-T_o)}}{\cancel{D}}\,\frac{\cancel{D^2}}{2}\, dz^*\, d\theta$$

$$2\pi \underbrace{\left(\frac{hD}{k}\right)}\left(\frac{L}{D}\right) = \int_0^{2\pi}\int_0^{L/D} -\left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1/2} dz^*\, d\theta$$

Nusselt number, Nu
(dimensionless heat-transfer coefficient)

$$\boxed{Nu = Nu\left(T^*, \frac{L}{D}\right)}$$

one additional dimensionless group

Complex Heat Transfer – Dimensional Analysis

$$h(\pi \cancel{D}L)\cancel{(T_1-T_o)} = \int_0^{2\pi}\int_0^{L/D} -k\left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1/2} \frac{\cancel{(T_1-T_o)}}{\cancel{D}}\frac{\cancel{D^2}}{2}\,dz^*\,d\theta$$

This is a function of Re and Pr through fluid v^* distribution and energy balance

$$2\pi\underbrace{\left(\frac{hD}{k}\right)}\left(\frac{L}{D}\right) = \int_0^{2\pi}\int_0^{L/D} -\left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1/2} dz^*\,d\theta$$

Nusselt number, Nu
(dimensionless heat-transfer coefficient)

$$\boxed{Nu = Nu\left(T^*, \frac{L}{D}\right)}$$

one additional dimensionless group

Complex Heat Transfer – Dimensional Analysis

Non-dimensional Energy Equation

$$\left(\frac{\partial T^*}{\partial t^*} + v_r^*\frac{\partial T^*}{\partial r^*} + \frac{v_\theta^*}{r^*}\frac{\partial T^*}{\partial \theta} + v_z^*\frac{\partial T^*}{\partial z^*}\right) = \frac{1}{\mathrm{Pe}}\left(\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r\frac{\partial T^*}{\partial r^*}\right) + \frac{1}{r^{*2}}\frac{\partial^2 T^*}{\partial \theta^2} + \frac{\partial^2 T^*}{\partial z^2}\right) + S^*$$

$$\mathrm{Pe} = \mathrm{Pr}\,\mathrm{Re} = \frac{\hat{C}_p\mu}{k}\frac{\rho VD}{\mu}$$

Non-dimensional Navier-Stokes Equation

$$\frac{Dv_z^*}{Dt} = -\frac{\partial P^*}{\partial z^*} + \frac{1}{\mathrm{Re}}\left(\nabla^2 v_z\right)^* + \frac{1}{Fr}g^*$$

$$\mathrm{Pr} \equiv \frac{\hat{C}_p\mu}{k}$$

Non-dimensional Continuity Equation

$$\frac{\partial v_x^*}{\partial x^*} + \frac{\partial v_y^*}{\partial y^*} + \frac{\partial v_z^*}{\partial z^*} + = 0$$

Quantity of interest

$$Nu = \frac{1}{2\pi L/D}\int_0^{2\pi}\int_0^{L/D} -\left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1/2} dz^*\,d\theta$$

Complex Heat Transfer – Dimensional Analysis

According to our **dimensional analysis** calculations, the dimensionless heat transfer coefficient should be found to be a function of ~~four~~ dimensionless groups:

three

no free surfaces

$$\boxed{Nu = Nu\left(\mathrm{Re}, \mathrm{Pr}, \cancel{\mathrm{Fr}}, \frac{L}{D}\right)}$$

Peclet number

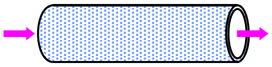
$$\mathrm{Pe} \equiv \frac{\rho \hat{C}_p V D}{k} = \frac{\hat{C}_p \mu}{k} \frac{\rho V D}{\mu}$$

Prandtl number

$$\mathrm{Pr} \equiv \frac{\hat{C}_p \mu}{k}$$ (fluid properties)

Now, do the experiments.

Complex Heat Transfer – Dimensional Analysis



Now, do the experiments.

Forced Convection Heat Transfer

- Build apparatus (*several* actually, with different D, L)
- Run fluid through the inside (at different $\underline{v}$; for different fluids $\rho, \mu, \hat{C}_p, k$)
- Measure T_{bulk} on inside; T_{wall} on inside
- Measure rate of heat transfer, Q
- Calculate h: $|Q| = hA|T_{bulk} - T_{wall}|$
- Report h values in terms of dimensionless correlation:

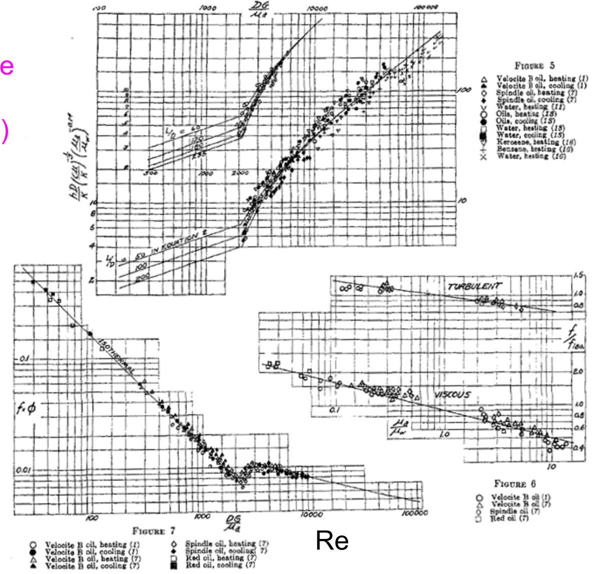
$$\mathrm{Nu} = \frac{hD}{k} = f\left(\mathrm{Re}, \mathrm{Pr}, \frac{L}{D}\right)$$

It should only be a function of these dimensionless numbers (***if*** our Dimensional Analysis is correct…..)

Complex Heat Transfer – Dimensional Analysis

Sieder and Tate equation
(turbulent flow)

Sieder and Tate equation
(laminar flow)



Re

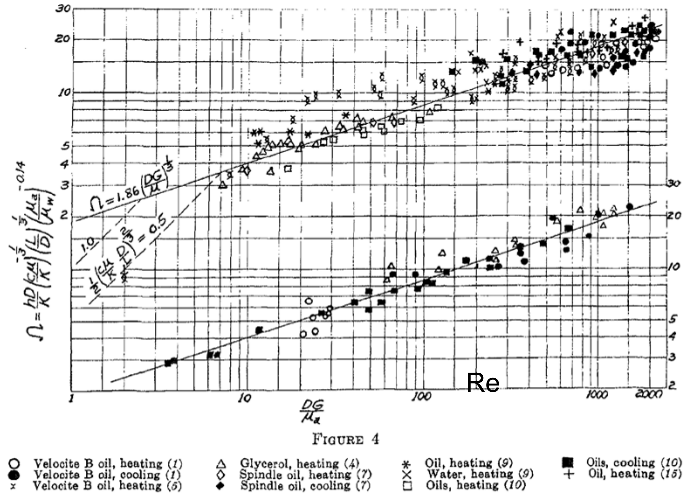
Re

Effect of viscosity ratio

Sieder & Tate, *Ind. Eng. Chem.* **28** 227 (1936)

Complex Heat Transfer – Dimensional Analysis

Sieder and Tate equation
(laminar flow)



Re

Sieder & Tate, *Ind. Eng. Chem.* **28** 227 (1936)

Complex Heat Transfer – Dimensional Analysis

Sieder and Tate equation
effect of viscosity ratio

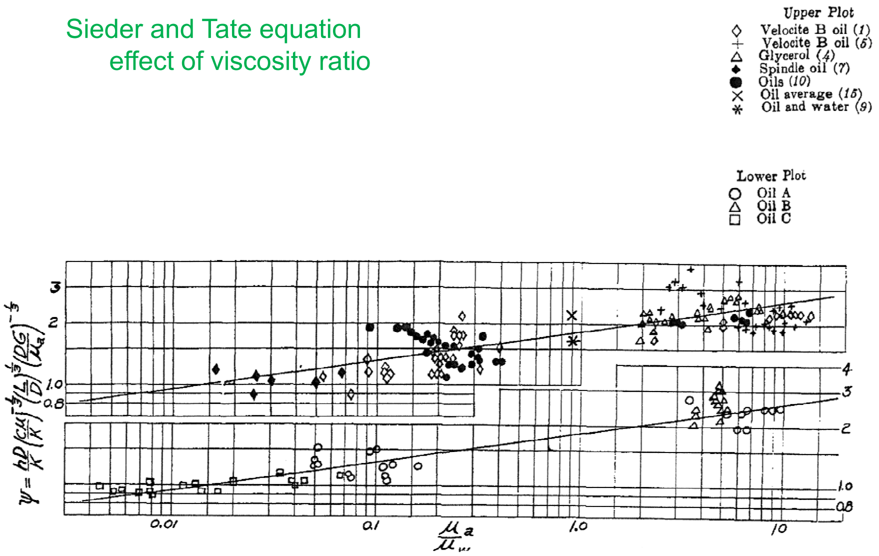
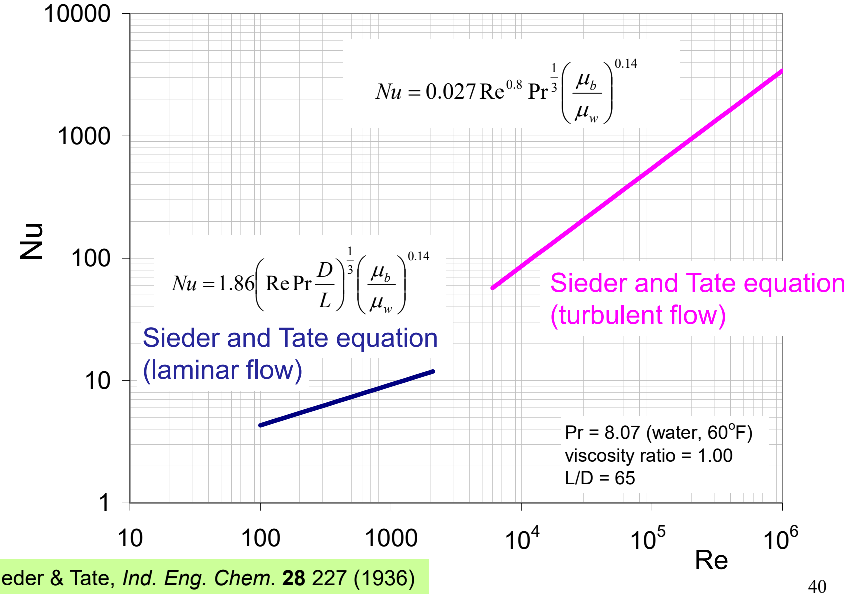


FIGURE 3

Sieder & Tate, *Ind. Eng. Chem.* **28** 227 (1936)

Complex Heat Transfer – Dimensional Analysis

Correlations for Forced Convection Heat Transfer Coefficients



$$Nu = 0.027\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

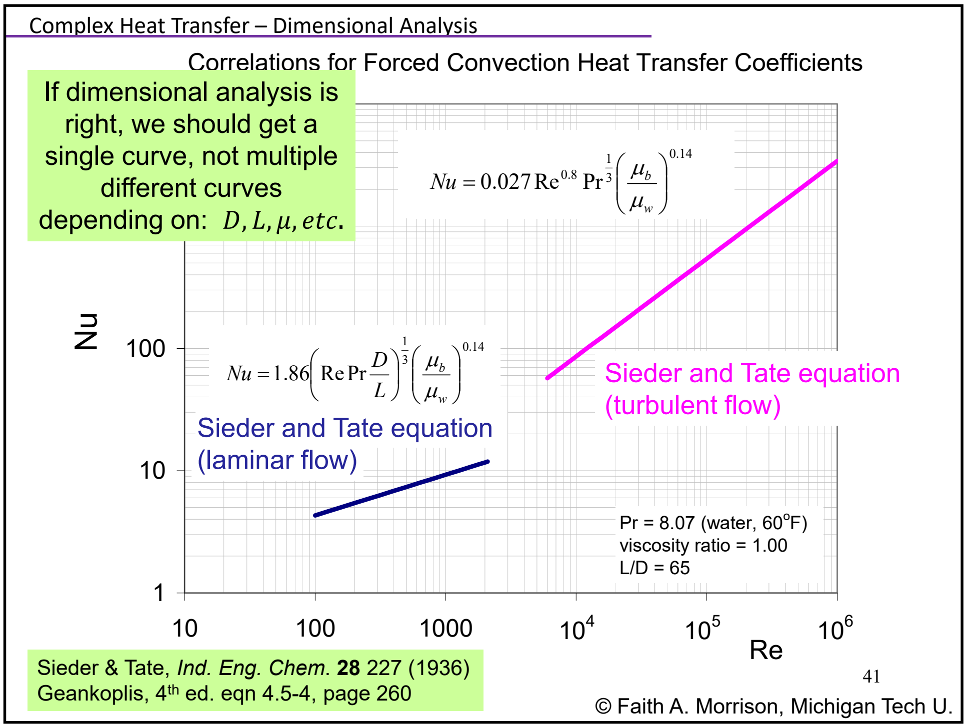
$$Nu = 1.86\left(\mathrm{Re}\,\mathrm{Pr}\frac{D}{L}\right)^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

Sieder and Tate equation
(turbulent flow)

Sieder and Tate equation
(laminar flow)

Pr = 8.07 (water, 60°F)
viscosity ratio = 1.00
L/D = 65

Sieder & Tate, *Ind. Eng. Chem.* **28** 227 (1936)
Geankoplis, 4th ed. eqn 4.5-4, page 260



Complex Heat Transfer – Dimensional Analysis

Correlations for Forced Convection Heat Transfer Coefficients

If dimensional analysis is right, we should get a single curve, not multiple different curves depending on: $D, L, \mu, etc.$

$$Nu = 0.027\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

Sieder and Tate equation (turbulent flow)

$$Nu = 1.86\left(\mathrm{Re}\,\mathrm{Pr}\frac{D}{L}\right)^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

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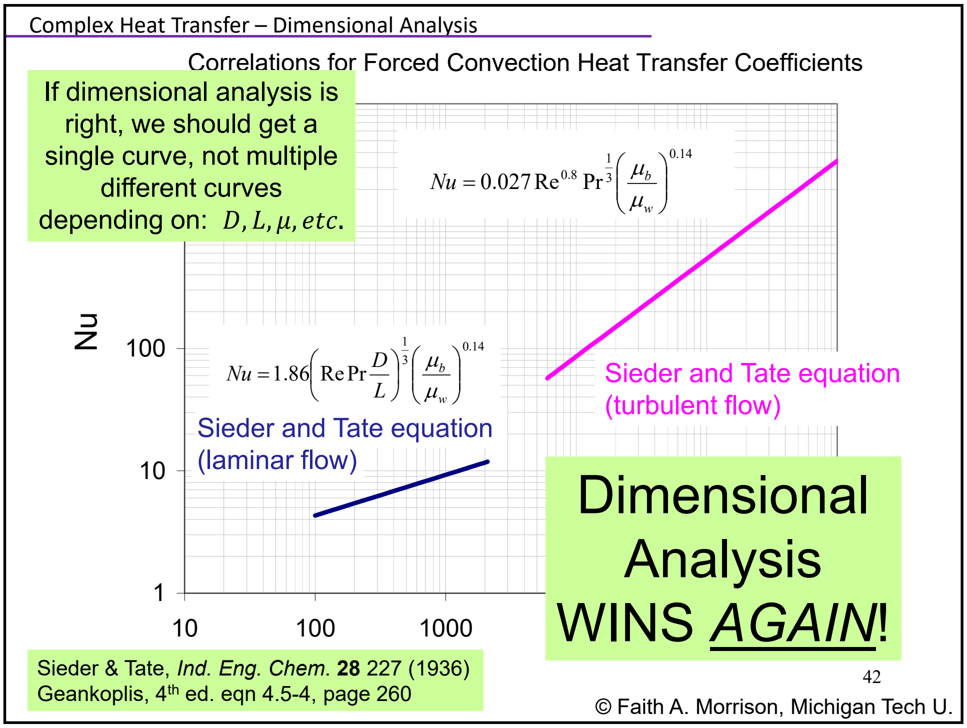

Complex Heat Transfer – Dimensional Analysis

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$$Nu = 1.86\left(\mathrm{Re}\,\mathrm{Pr}\frac{D}{L}\right)^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

Sieder and Tate equation (laminar flow)

Dimensional Analysis WINS *AGAIN*!

Sieder & Tate, *Ind. Eng. Chem.* **28** 227 (1936)
Geankoplis, 4th ed. eqn 4.5-4, page 260

Complex Heat Transfer – Dimensional Analysis

Heat Transfer in **Laminar** flow in pipes:

data correlation for *forced convection heat transfer coefficients*

$$Nu_a = \frac{h_a D}{k} = 1.86\left(RePr\frac{D}{L}\right)^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

Sieder-Tate equation (laminar flow)

Sieder & Tate, Ind. Eng. Chem. ***28*** *227 (1936)*

the subscript "*a*" refers to *the type of average temperature* used in calculating the heat flow, *q*

$$q = h_a A \Delta T_a$$

$$\Delta T_a = \frac{(T_w - T_{bi}) + (T_w - T_{bo})}{2}$$

Geankoplis, 4th ed. eqn 4.5-4, page 260

$Re < 2100$, $(RePr\frac{D}{L}) > 100$, horizontal pipes; all physical properties evaluated at the mean temperature of the bulk fluid except μ_w which is evaluated at the (constant) wall temperature.

Complex Heat Transfer – Dimensional Analysis

Heat Transfer in **Turbulent** flow in pipes:

data correlation for *forced convection heat transfer coefficients*

$$\mathrm{Nu}_{lm} = \frac{h_{lm} D}{k} = 0.027\mathrm{Re}^{0.8}\mathrm{Pr}^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

Sieder-Tate equation (turbulent flow)

Sieder & Tate, Ind. Eng. Chem. ***28*** *227 (1936)*

the subscript "*lm*" refers to *the type of average temperature* used in calculating the heat flow, *q*

$$q = h_{lm} A \Delta T_{lm}$$

$$\Delta T_{lm} = \frac{\Delta T_{w-bi} - \Delta T_{w-bo}}{\ln\left(\frac{\Delta T_{w-bi}}{\Delta T_{w-bo}}\right)}$$

Geankoplis, 4th ed. section 4.5

Complex Heat Transfer – Dimensional Analysis

Forced convection
Heat Transfer in Laminar flow in pipes

$$\mathrm{Nu}_a = \frac{h_a D}{k} = 1.86\left(\mathrm{RePr}\frac{D}{L}\right)^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

Sieder-Tate equation (laminar flow)

$$q = h_a A \Delta T_a$$

$$\Delta T_a = \frac{(T_w - T_{bi}) + (T_w - T_{bo})}{2}$$

Forced convection
Heat Transfer in Turbulent flow in pipes

$$\mathrm{Nu}_{lm} = \frac{h_{lm} D}{k} = 0.027\mathrm{Re}^{0.8}\mathrm{Pr}^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

Sieder-Tate equation (turbulent flow)

$$q = h_{lm} A \Delta T_{lm}$$

$$\Delta T_{lm} = \frac{\Delta T_{w-bi} - \Delta T_{w-bo}}{\ln\left(\frac{\Delta T_{w-bo}}{\Delta T_{w-bo}}\right)}$$

Physical Properties (except μ_w) evaluated at:

$$\frac{T_{b,in} + T_{b,out}}{2}$$

*May have to be estimated

$$\frac{T_{b,in} + T_{b,out}}{2}$$

Bulk mean temperature

Fine print matters!

Complex Heat Transfer – Dimensional Analysis

Forced convection Heat Transfer in Laminar flow in pipes

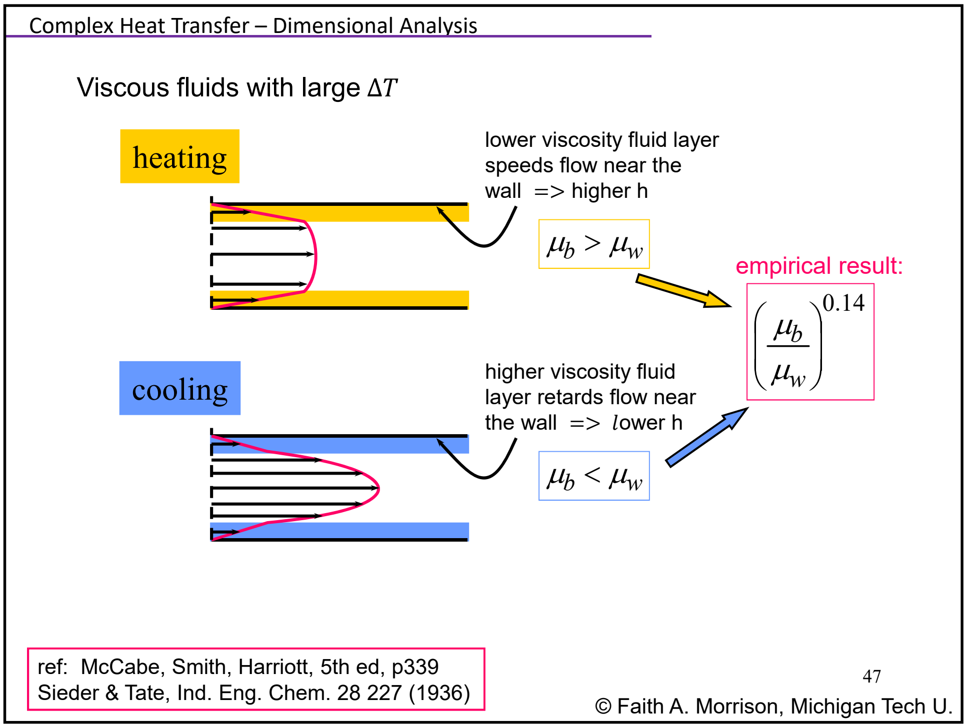
? In our dimensional analysis, we assumed constant ρ, k, μ, etc. Therefore we did not predict a viscosity-temperature dependence. If viscosity is not assumed constant, the dimensionless group shown below is predicted to appear in correlations.

$$\mathrm{Nu}_a = \frac{h_a D}{k} = 1.86\left(\mathrm{RePr}\frac{D}{L}\right)^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

Sieder & Tate, Ind. Eng. Chem. ***28*** *227 (1936)*

Sieder-Tate equation (laminar flow)

(reminiscent of pipe wall roughness; needed to modify dimensional analysis to correlate on roughness)



Complex Heat Transfer – Dimensional Analysis

Viscous fluids with large ΔT

heating

lower viscosity fluid layer speeds flow near the wall => higher h

$\mu_b > \mu_w$

empirical result:

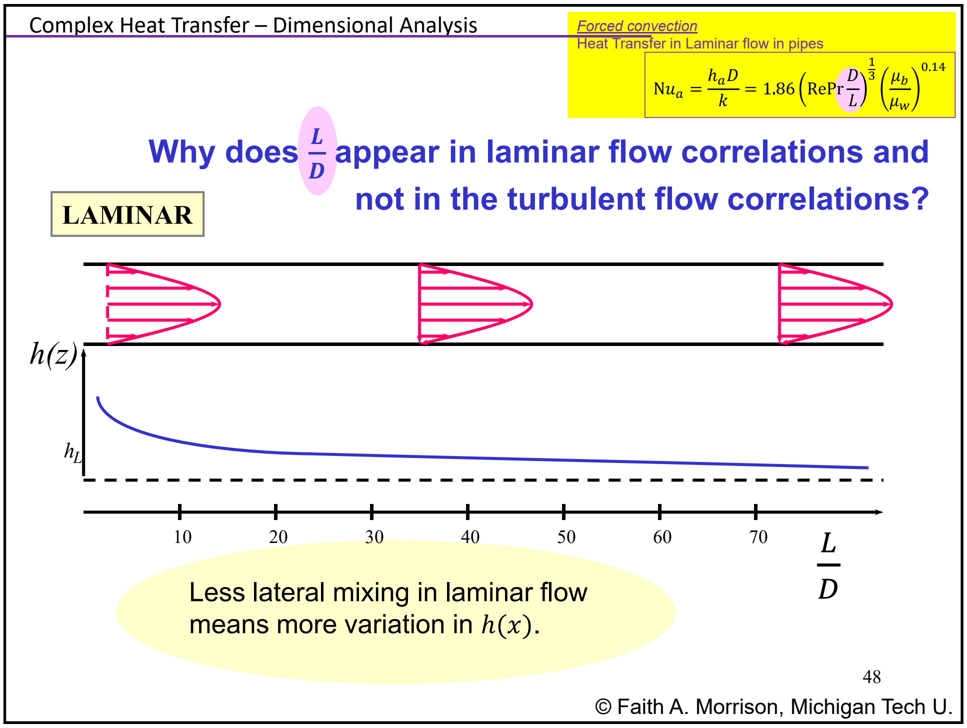
$$\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

cooling

higher viscosity fluid layer retards flow near the wall => lower h

$\mu_b < \mu_w$

ref: McCabe, Smith, Harriott, 5th ed, p339
Sieder & Tate, Ind. Eng. Chem. 28 227 (1936)



Complex Heat Transfer – Dimensional Analysis

Forced convection
Heat Transfer in Laminar flow in pipes

$$Nu_a = \frac{h_a D}{k} = 1.86\left(\mathrm{RePr}\frac{D}{L}\right)^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

Why does $\frac{L}{D}$ appear in laminar flow correlations and not in the turbulent flow correlations?

LAMINAR

Less lateral mixing in laminar flow means more variation in $h(x)$.

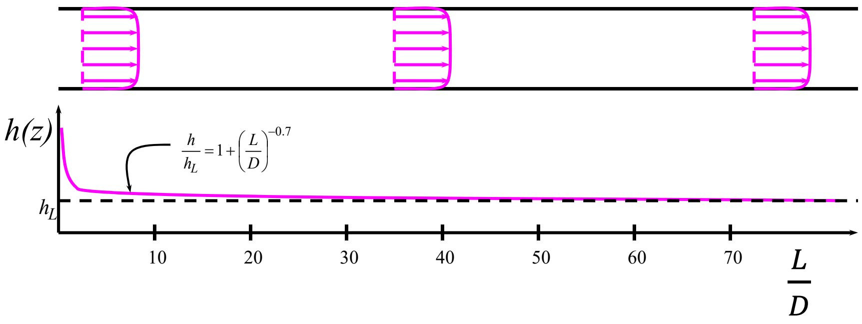
Complex Heat Transfer – Dimensional Analysis

Forced convection
Heat Transfer in Turbulent flow in pipes

$$Nu_{lm} = \frac{h_{lm}D}{k} = 0.027\mathrm{Re}^{0.8}\mathrm{Pr}^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

Sieder & Tate eqn, turbulent flow

TURBULENT



In turbulent flow, good lateral mixing reduces the variation in h along the pipe length.

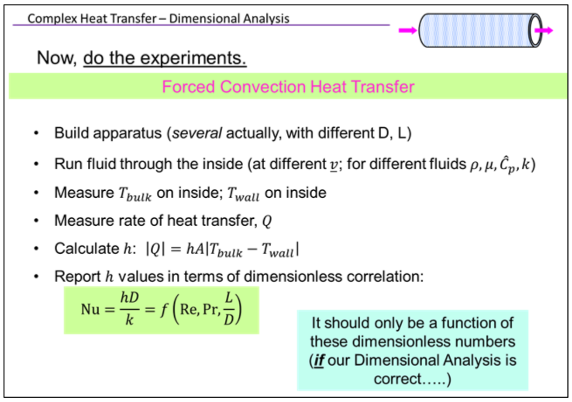
(Exam 4 2016)

Example:

Water flows at $0.0522\ kg/s$ (turbulent) in the inside of a double pipe heat exchanger (inside steel pipe, inner diameter= $0.545\ inches$, length unknown, physical properties given on page 1); the water enters at 30.0^oC and exits at 65.6^oC. In the shell of the heat exchanger, steam condenses at an unknown saturation pressure. What is the heat transfer coefficient, h_{lm} (based on log mean temperature driving force) in the water flowing in the pipe? You may neglect the effect on heat-transfer coefficient of the temperature-dependence of viscosity. Please give your answer in W/m^2K.

Physical properties of steel:
thermal conductivity = $16.3\ W/mK$
heat capacity = $0.49\ kJ/kg\ K$
density = $8050\ kg/m^3$

This is shown to work for *internal* flow (Seider-Tate).



Complex Heat Transfer – Dimensional Analysis

Now, do the experiments.

Forced Convection Heat Transfer

- Build apparatus (*several* actually, with different D, L)
- Run fluid through the inside (at different $\underline{v}$; for different fluids $\rho, \mu, \hat{C}_p, k$)
- Measure T_{bulk} on inside; T_{wall} on inside
- Measure rate of heat transfer, Q
- Calculate h: $|Q| = hA|T_{bulk} - T_{wall}|$
- Report h values in terms of dimensionless correlation:

$$\mathrm{Nu} = \frac{hD}{k} = f\left(\mathrm{Re}, \mathrm{Pr}, \frac{L}{D}\right)$$

It should only be a function of these dimensionless numbers (***if*** our Dimensional Analysis is correct…..)

It also works for ***external*** flows (flows past spheres, plates, cylinders, banks of cylinders (see Geankoplis, section 4.6)

Complex Heat Transfer – Dimensional Analysis

Heat Transfer in **External** flow around objects:
data correlation for *forced convection heat transfer coefficients*

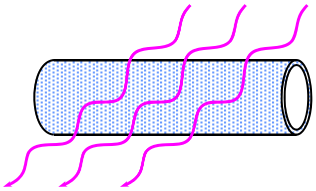
$$\boxed{\mathrm{Nu} = \frac{hD_{ext}}{k} = C\mathrm{Re}^m\mathrm{Pr}^{\frac{1}{3}}}$$

$$q = hA(T_w - T_b)$$

D_{ext} = external diameter

External flow, perpendicular to cylinder
Geankoplis, 4th ed. section 4.6, p272

Re	m	C
$1 - 4$	0.330	0.989
$4 - 40$	0.385	0.911
$40 - 4{,}000$	0.466	0.683
$4{,}000 - 4 \times 10^4$	0.618	0.193
$4 \times 10^4 - 2.5 \times 10^5$	0.805	0.0266



Dimensional Analysis

These numbers tell us about the relative importance of the terms they precede.

Dimensionless numbers from the
Equations of Change *(microscopic balances)*

momentum

Non-dimensional Navier-Stokes Equation

$$\left(\frac{\partial v_z^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* v_z^*\right) = -\frac{\partial P^*}{\partial z^*} + \frac{1}{\mathrm{Re}}(\nabla^{*2} v_z^*) + \frac{1}{\mathrm{Fr}} g^*$$

Re – Reynolds
Fr – Froude

energy

Non-dimensional Energy Equation

$$\left(\frac{\partial T^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* T^*\right) = \frac{1}{\mathrm{RePr}}(\nabla^{*2} T^*) + S^*$$

Pe – Péclet$_h$ = RePr
Pr – Prandtl

mass

Non-dimensional Continuity Equation (species A)

$$\left(\frac{\partial x_A^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* x_A^*\right) = \frac{1}{\mathrm{ReSc}}(\nabla^{*2} x_A^*)$$

Pe – Péclet$_m$ = ReSc
Sc – Schmidt

ref: BSL1, p581, 644

Dimensionless Numbers

Dimensionless numbers from the
Equations of Change

Re – Reynolds $= \dfrac{\rho V D}{\mu} = \dfrac{VD}{\nu}$

Fr – Froude $= \dfrac{V^2}{gD}$

Pe – Péclet$_h$ $= \mathrm{RePr} = \dfrac{\hat{C}_p \rho V D}{k} = \dfrac{VD}{\alpha}$

Pe – Péclet$_m$ $= \mathrm{ReSc} = \dfrac{VD}{D_{AB}}$

These numbers tell us about the **relative importance of the terms** they precede in the microscopic balances (*scenario properties*).

Pr – Prandtl $= \dfrac{\hat{C}_p \mu}{k} = \dfrac{\nu}{\alpha}$

Sc – Schmidt $= \mathrm{LePr} = \dfrac{\mu}{\rho D_{AB}} = \dfrac{\nu}{D_{AB}}$

Le – Lewis $= \dfrac{\alpha}{D_{AB}}$

These numbers compare the **magnitudes of the diffusive transport coefficients** ν, α, D_{AB} (*material properties*).

Dimensionless Numbers

Dimensionless numbers from the
Equations of Change

$\text{Re} - \text{Reynolds} = \frac{\rho V D}{\mu} = \frac{VD}{\nu}$

$\text{Fr} - \text{Froude} = \frac{V^2}{gD}$

$\text{Pe} - \text{Péclet}_h = \text{RePr} = \frac{\hat{C}_p \rho V D}{k} = \frac{VD}{\alpha}$

$\text{Pe} - \text{Péclet}_m = \text{ReSc} = \frac{VD}{D_{AB}}$

These numbers tell us about the **relative importance of the terms** they precede in the microscopic balances (*scenario properties*).

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$\text{Le} - \text{Lewis} = \frac{\alpha}{D_{AB}}$

These numbers compare the **magnitudes of the diffusive transport coefficients** ν, α, D_{AB} (*material properties*).

Transport coefficients

Dimensional Analysis

These numbers are defined to help us build **transport data correlations** based on the fewest number of grouped (dimensionless) variables (*scenario property*).

Dimensionless numbers from the
Engineering Quantities of Interest

momentum

Dimensionless Force on the Wall (Drag)

$$f = \frac{1}{\pi}\frac{D}{L}\frac{1}{\text{Re}} \int_0^{\frac{L}{D}} \int_0^{2\pi} \left(\frac{\partial v_z^*}{\partial r^*}\right)\Bigg|_{r^*=\frac{1}{2}} d\theta dz^*$$

(Fanning)

f — Friction Factor

$\frac{L}{D}$ — Aspect Ratio

$$f = \frac{\mathcal{F}_{drag}}{\left(\frac{1}{2}\rho V^2\right) A_c}$$

energy

Newton's Law of Cooling

$$Nu = \frac{1}{2\pi L / D} \int_0^{2\pi} \int_0^{L/D} -\frac{\partial T^*}{\partial r^*}\Bigg|_{r^*=1/2} dz^* d\theta$$

Nu — Nusselt

$\frac{L}{D}$ — Aspect Ratio

$$\text{Nu} = \frac{hD}{k}$$

mass xfer

Dimensionless Mass Transfer Coefficient

$$Sh = \frac{1}{2\pi}\frac{D}{L} \int_0^{\frac{L}{D}} \int_0^{2\pi} \left(-\frac{\partial x_A^*}{\partial r^*}\right)\Bigg|_{r^*=\frac{1}{2}} d\theta dz^*$$

Sh — Sherwood

$\frac{L}{D}$ — Aspect Ratio

$$\text{Sh} = \frac{k_m D}{D_{AB}}$$

momentum
energy
mass

Dimensionless Numbers

Re – Reynolds $= \frac{\rho V D}{\mu} = \frac{VD}{\nu}$

Fr – Froude $= \frac{V^2}{gD}$

Pe – Péclet$_h$ = RePr $= \frac{\hat{C}_p \rho V D}{k} = \frac{VD}{\alpha}$

These numbers from the governing equations tell us about the relative importance of the terms they precede in the

Pe – Péclet$_m$ = ReSc $= \frac{VD}{D_{AB}}$

Pr – Prandtl $= \frac{\hat{C}_p \mu}{k} = \frac{\nu}{\alpha}$

These numbers compare the magnitudes of the diffusive transport coefficients ν, α, D_{AB}

Sc – Schmidt = LePr $= \frac{\mu}{\rho D_{AB}} = \frac{\nu}{D_{AB}}$

Le – Lewis $= \frac{\alpha}{D_{AB}}$

These numbers are defined to help us build transport data correlations based on the fewest number of grouped (dimensionless) variables (***scenario***

f – Friction Factor =

Complex Heat transfer Problems to Solve:

✓ •Forced convection heat transfer from fluid to wall
Solution: ?

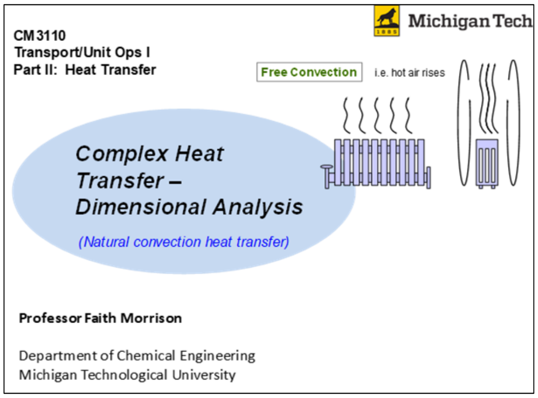
➡ •Natural convection heat transfer from fluid to wall
Solution: ?

•Radiation heat transfer from solid to fluid
Solution: ?

We started with a forced-convection pipe problem, did dimensional analysis, and found the dimensionless numbers.

To do a situation with different physics, we must start with a different starting problem.

Next:



Michigan Tech

CM3110
Transport/Unit Ops I
Part II: Heat Transfer

Free Convection

i.e. hot air rises

Complex Heat Transfer – Dimensional Analysis

(Natural convection heat transfer)

Professor Faith Morrison

Department of Chemical Engineering
Michigan Technological University