Homework 2
CM3110 Morrison

Numbered problems are from the text; Lettered problems are on the pages that follow.

Module	Number	Topics	Assigned Problems	Stretch Problems
2	1	literature; Newton's law	1.2	
2	2	Newton's law of viscosity-*see sketch in Figure 5.5 p351	3.6	
2	3	Newton's law of viscosity		3.1
2	4	flow rate integral		3.14
2	5	flow rate integral	3.16	
2	6	average velocity thru S		3.22
2	7	average, max velocity	3.24	
2	8	vector components	3.31	
2	9	MEB versus NS	6.2	
2	10	sketch velocity profiles and velocity fields	B	
2	11	flow rate from velocity integral	6.21	
2	12	boundary conditions	6.30	
2	13	boundary conditions		6.33
2	14	**flow field problem: uphill slit**	6.39	
2	15	**flow field problem: wire coating**	6.43	
2	16	**flow field problem: complete tube flow**	7.6	
2	17	**flow problem: natural convection (We'll use this solution in the heat-transfer part of the course)**		7.40
2	18	Calculate torque given the flow field		A

Problem A:

a. A rod of radius R is rotated around its axis by application of a constant tangential force $\underline{f} = \Phi_0 \hat{e}_\theta$ at the rod's surface $(r = R)$. What is the torque $\underline{\mathcal{T}}$ on the rod? Recall that the fundamental definition of torque is $\underline{\mathcal{T}} = \underline{R} \times \underline{f}$, where $\underline{f}$ is the force applied and $\underline{R}$ is the lever arm, a vector from the axis of rotation to the point of application of the force. All quantities are written in the cylindrical coordinate system. Answer: $\underline{\mathcal{T}} = R\Phi_0 \hat{e}_z$

b. Tangential annular flow of a Newtonian fluid takes place between two concentric cylinders (radius of inner cylinder $= R$; radius of outer cylinder $= R_O$), the inner one of which is turning. The velocity field in this flow may be obtained from application of the microscopic momentum balance (Text 7.37) and is given by

$$\underline{v}(r) = \begin{pmatrix} 0 \\ ar + \frac{b}{r} \\ 0 \end{pmatrix}_{r\theta z}$$

where a and b are constants. The pressure is constant at P_0 throughout the fluid. What is the torque $\underline{\mathcal{T}}$ on the inner cylinder needed to sustain the flow? Note that the total torque due to a fluid in contact with a surface $\mathcal{S}$ is calculated from the velocity field as follows:

$$\underline{\mathcal{T}} = \iint_{\mathcal{S}} \underline{R} \times \left(\hat{n} \cdot \underline{\underline{\tilde{\Pi}}}\right)_{surface} dS$$

The stress tensor $\underline{\underline{\tilde{\Pi}}}$ in cylindrical coordinates may be found on this handout: https://pages.mtu.edu/~fmorriso/cm310/stress.pdf

Answer: $\underline{\mathcal{T}} = -4\pi\mu L b \, \hat{e}_z$

Problem B:

For each of the velocity distributions given, create the following two plots. First, create a plot of the scalar velocity component versus the dependent variable listed (1D plot). Second, sketch a vector plot of the vector field in the plane indicated (2D plot; vector lengths are to be proportional to the velocity at a location).

a. $\underline{v}(x) = U_\infty \hat{e}_x = \begin{pmatrix} U_\infty \\ 0 \\ 0 \end{pmatrix}_{xyz}$ where U_∞ is a positive constant. The flow takes place everywhere in space. Sketch the 2D plot in the xy-plane.

b. $\underline{v}(y) = -ay\hat{e}_x = \begin{pmatrix} -ay \\ 0 \\ 0 \end{pmatrix}_{xyz}$ where a is a positive constant. The flow takes place between two long and wide parallel plates over the thickness $0 \le y \le B$. Sketch the 2D plot in the xy-plane.

c. $\underline{v}(r) = V\left(1 - \frac{r^2}{R^2}\right)\hat{e}_z = \begin{pmatrix} 0 \\ 0 \\ V\left(1 - \frac{r^2}{R^2}\right) \end{pmatrix}_{r\theta z}$ where V, a, and R are positive constants. The flow takes place in a long circular tube of radius R, $0 \le r \le R$. Sketch the 2D plot in a rz-plane.