As teachers we can choose between

(a) sentencing students to thoughtless mechanical operations and
(b) facilitating their ability to think.

If students' readiness for more involved thought processes is bypassed in favor of jamming more facts and figures into their heads, they will stagnate at the lower levels of thinking. But if students are encouraged to try a variety of thought processes in classes, then they can … develop considerable mental power. Writing is one of the most effective ways to develop thinking.

—Syrene Forsman

Reference: Forsman, S. (1985). "Writing to Learn Means Learning to Think." In A. R. Gere (Ed.), *Roots in the sawdust: Writing to learn across the disciplines* (pp. 162-174). Urbana, IL: National Council of Teachers of English.

Professor Faith A. Morrison
Department of Chemical Engineering
Michigan Technological University

1

Transport/Unit Operations

Professor Faith A. Morrison
Department of Chemical Engineering
Michigan Technological University

CM2120—Fundamentals of ChemE 2 (Steady Unit Operations Introduction, MEB)
CM3110—Transport/Unit Ops 1 (Momentum & Steady Heat Transport, Unit Operations)
CM3120—Transport/Unit Ops 2 (Unsteady Heat Transport, Mass Transport, Unit Operations

2

Why study transport/unit ops?

Michigan Tech

 3

Why study transport/unit ops?

Michigan Tech

•Modern engineering systems are complex and often cannot be operated and maintained without analytical understanding

•Design of new systems will come from high-tech innovation, which can only come from detailed, analytical understanding of how physics/nature works

Image: wikipedia.org

Image: planetforward.ca

 4

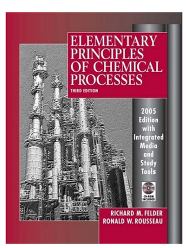
Where are we now?

Michigan Tech

Where are we now?

Michigan Tech

CM2110

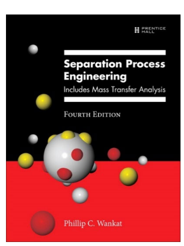


Summary

CM2110

1. Steady mass balances
2. Steady energy balances (how to calc. energy)
3. MEB-Mechanical Energy Balance (no friction)

CM2120



CM2120/CM3215

1. MEB-Mechanical Energy Balance (with friction)
2. Pumps
3. Introduction to Unit Operations
4. *Staged* Unit Operations (distillation, absorption)

Where are we now?

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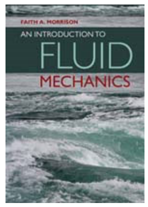
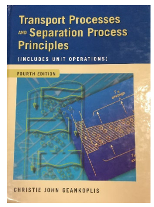
CM3110





Summary

CM3110

1. Steady *momentum* balances (macro and micro)
2. *Rate-based* heat transfer processes (Fourier's law, heat transfer coefficients, radiation)
3. Unit Operations involving heat transfer (Heat Exchangers)

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CM3110
Transport Processes and Unit Operations I

Professor Faith Morrison

Department of Chemical Engineering
Michigan Technological University

CM3110 - Momentum and Heat Transport

www.chem.mtu.edu/~fmorriso/cm310/cm310.html

8

TR Section

EMERGENCY EVACUATION PROCEDURES

Important: The Michigan Bureau of Fire Services has adopted new rules for colleges and universities effective 2015

1. Only residence halls are required to hold fire and tornado drills.
2. In lieu of fire drills in other university buildings all faculty and instructional staff are required to do the following on the first day of class:
   - Explain the university fire evacuation procedures to the class (see below).
   - Explain the locations of the primary and secondary exit routes for your class location.
   - Explain your designated safe location where the class will meet after evacuating the building.
3. The class instructor is responsible for directing the class during a building evacuation.

General evacuation procedure:

- Use the nearest safe exit route to exit the building. **The nearest safe exit from room 07-0100 is the back (south) entrance that is close to the Portage Canal and just outside our door, to the right. The secondary exit is the campus (north) exit, that exits near the Husky. The nearest safe exit from Fisher 139 is**
- Close all doors on the way out to prevent the spread of smoke and fire.
- After exiting, immediately proceed to a safe location at least 100 feet from the building. **Our designated safe location is in front of the MUB, near the circle drive.**
- Do not re-enter the building until the all-clear is given by Public Safety or the fire department.

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MW Section

EMERGENCY EVACUATION PROCEDURES

Important: The Michigan Bureau of Fire Services has adopted new rules for colleges and universities effective 2015

1. Only residence halls are required to hold fire and tornado drills.
2. In lieu of fire drills in other university buildings all faculty and instructional staff are required to do the following on the first day of class:
   - Explain the university fire evacuation procedures to the class (see below).
   - Explain the locations of the primary and secondary exit routes for your class location.
   - Explain your designated safe location where the class will meet after evacuating the building.
3. The class instructor is responsible for directing the class during a building evacuation.

General evacuation procedure:

- Use the nearest safe exit route to exit the building. **The nearest safe exit from room 15-139 is the front (south) entrance that is close to highway 41. The secondary exit is the campus (north) exit, that connects to the main path through campus.**
- Close all doors on the way out to prevent the spread of smoke and fire.
- After exiting, immediately proceed to a safe location at least 100 feet from the building. **Our designated safe location is east of Fisher, in the parking lot of the Center for Diversity and Inclusion.**
- Do not re-enter the building until the all-clear is given by Public Safety or the fire department.

10

Why study fluid mechanics?

Michigan Tech

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Why study fluid mechanics?

Michigan Tech

•It's required for my degree

12

Why study fluid mechanics?

Michigan Tech

~~•It's required for my degree~~ (too literal)

13

Why study fluid mechanics?

Michigan Tech

~~•It's required for my degree~~ (too literal)

•Fluids are involved in engineered systems

Image from: newegg.com

Image from: money.cnn.com

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Why study fluid mechanics?

Michigan Tech

~~•It's required for my degree~~ (too literal)

~~•Fluids are involved in engineering systems~~ (many devices that employ fluids can be operated and maintained and sometimes designed without detailed mathematical analysis)

15

Why study fluid mechanics?

Michigan Tech

•Modern engineering systems are complex and often cannot be operated and maintained without analytical understanding

•Design of new systems will come from high-tech innovation, which can only come from detailed, analytical understanding of how physics/nature works

Image: wikipedia.org

•It's all part of learning to think and perform like an engineer. We will be intentionally ordering our knowledge and practicing asking relevant questions.

Image: planetforward.ca

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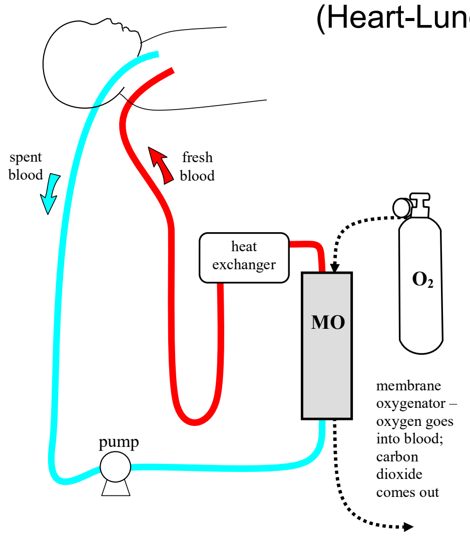
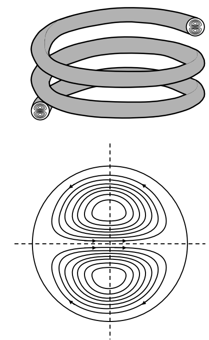
Membrane Oxygenator, MO (Heart-Lung Machine)





Secondary flow drives the O_2 mass transfer

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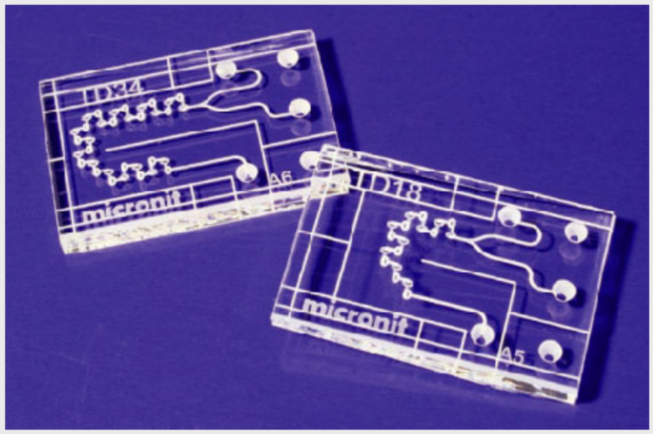
Artificial Heart

18

Image: health.howstuffworks.com/medicine/modern/artificial-heart.htm

Microfluidics – Lab on a Chip

Sensors, diagnostics



An example of a passive mixer in which fluids are mixed by chaotic advection. (Courtesy of Micronit Microfluidics.)

www.nature.com/nmeth/journal/v4/n8/full/nmeth0807-665.html

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And more. . . .

Helicopters
Airplanes
Quieter fans
Flexible body armor
Undersea oil drilling
Surgery
Food processing
Plastics
2D and 3D printing
Battery manufacture
Celestial exploration
Volcanos
Biomedical devices (stents, artificial organs, prosthetics)
Sensor development
…

Image from: en.wikipedia.org

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Where to start?

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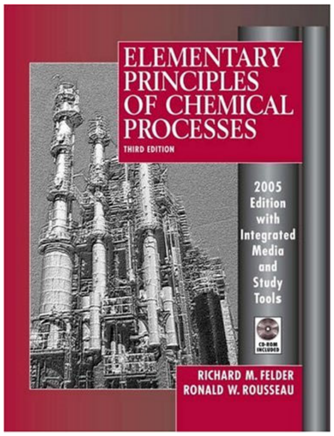
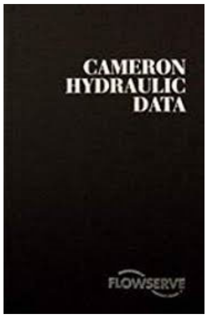
21

Where to start?

Michigan Tech

We've already started.



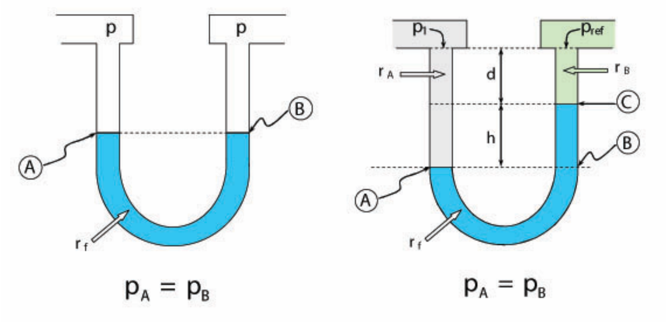


22

We've already started.

Michigan Tech

1. We've learned fluid **statics**.



DrMorrisonMTU on **YouTube**: On 3Sept19 #views >134,000!
Introduction to Manometers: Two Essential Rules

www.youtube.com/watch?v=zeNQOqr63cc

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We've already started.

Michigan Tech

2. There are **flow** problems that can be addressed with one type of macroscopic **energy** balance:

The Mechanical Energy Balance

$$\frac{\Delta p}{\rho} + \frac{\Delta \langle v \rangle^2}{2\alpha} + g\Delta z + F = \frac{W_{s,on}}{\dot{m}}$$

F = friction

$$\frac{p_2 - p_1}{\rho} + \frac{\langle v \rangle_2^2 - \langle v \rangle_1^2}{2\alpha} + g(z_2 - z_1) + F_{21} = \frac{W_{s,on,21}}{\dot{m}}$$

Assumptions:

1. single-input, single output
2. Steady state
3. Constant density (incompressible fluid)
4. Temperature approximately constant
5. No phase change, no chemical reaction
6. Insignificant amounts of heat transferred

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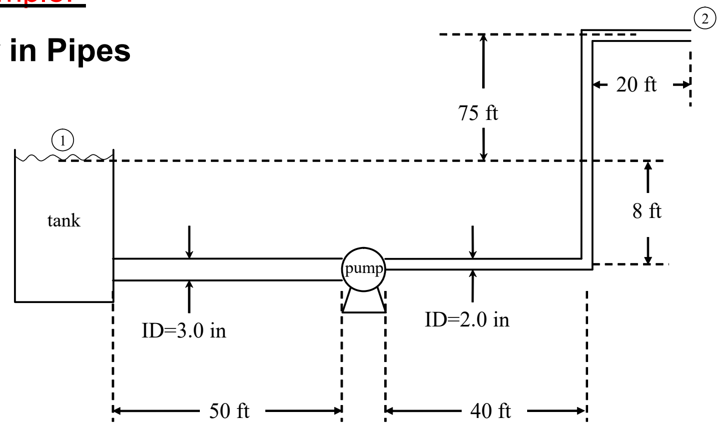
For example:

Flow in Pipes



1. Single-input, single output
2. Steady state
3. Constant density (incompressible fluid)
4. Temperature approximately constant
5. No phase change, no chemical reaction
6. Insignificant amounts of heat transferred

→ Mechanical Energy Balance

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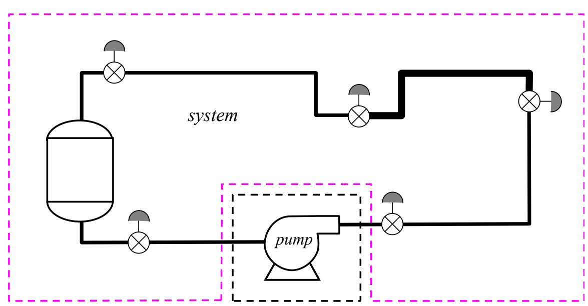
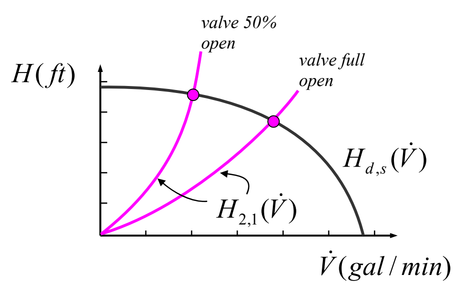
For example:

Centrifugal Pumps

What flow rate does a centrifugal pump produce?

Answer: Depends on how much work it is asked to do.





Calculate with the Mechanical Energy Balance (CM2110, CM2120, CM3215)

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We can apply the MEB to many important engineering systems


Image from: www.directindustry,com


Image from: www.processindustryforum.com


Image from: www.directindustry.com


Image from: www.epa.gov

MEB Assumptions:

1. single-input, single output
2. Steady state
3. Constant density (incompressible fluid)
4. Temperature approximately constant
5. No phase change, no chemical rxn
6. Insignificant amounts of heat transferred

Calculate: **Work, pressures, flows**

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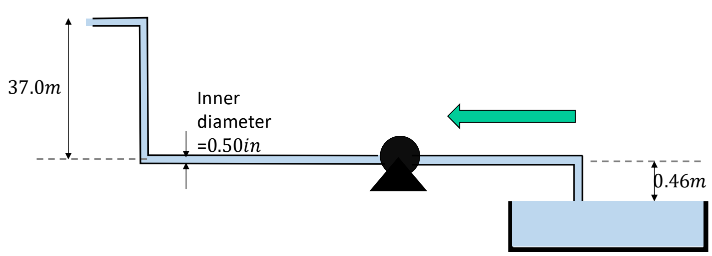
Example 1

How many horsepower (hp) will it take to raise water at $4.0\ gpm$ from a flooded basement to the surface (see figure)? The hose is smooth; as a first calculation we can neglect friction.



$W_{s,on}$ = shaft work

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Example 1

Michigan Tech

How many horsepower (hp) will it take to raise water at $4.0\ gpm$ from a flooded basement to the surface (see figure)? The hose is smooth; as a first calculation we can neglect friction.

ANSWER: 0.12 hp

(our TA has the solution: HW/Example help session Sunday 6:30-7:30, 19-211)

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Handy Sheet for Unit Conversions

Michigan Tech

Prof. Faith A. Morrison
Department of Chemical Engineering
MichiganTech

FACTORS FOR UNIT CONVERSIONS

Quantity	Equivalent Values
Mass	1 kg = 1000 g = 0.001 metric ton = 2.20462 lb_m = 35.27392 oz 1 lb_m = 16 oz = 5 x 10^{-4} ton = 453.593 g = 0.453593 kg
Length	1 m = 100 cm = 1000 mm = 10^6 microns (μm) = 10^{10} angstroms (Å) = 39.3701 in = 3.28084 ft = 1.09361 yd = 0.000621371 mile 1 ft = 12 in. = 1/3 yd = 0.3048 m = 30.48 cm
Volume	1 m^3 = 1000 liters = 10^6 cm^3 = 10^6 ml = 35.31467 ft^3 = 219.969 imperial gallons = 264.172 gal = 1056.69 qt 1 ft^3 = 1728 in^3 = 7.48052 gal = 0.028317 m^3 = 28.3168 liters = 28,316.8 cm^3
Force	1 N = 1 kg·m/s^2 = 10^5 dynes = 10^5 g·cm/s^2 = 0.22481 lb_f 1 lb_f = 32.174 lb_m·ft/s^2 = 4.4482 N = 4.4482 x 10^5 dynes
Pressure	1 atm = 1.01325 x 10^5 N/m^2 (Pa) = 101.325 kPa = 1.01325 bars = 1.01325 x 10^6 dynes/cm^2 = 760 mm Hg at 0° C (torr) = 10.333 m H_2O at 4° C = 14.696 lb_f/in^2 (psi) = 33.9 ft H_2O at 4°C 100 kPa = 1 bar
Energy	1 J = 1 N·m = 10^7 ergs = 10^7 dyne·cm = 2.778 x 10^{-7} kW·h = 0.23901 cal = 0.7376 ft·lb_f = 9.47817 x 10^{-4} Btu
Power	1 W = 1 J/s = 0.23885 cal/s = 0.7376 ft·lb_f/s = 9.47817 x 10^{-4} Btu/s = 3.4121 Btu/h = 1.341 x 10^{-3} hp (horsepower)
Viscosity	1 Pa·s = 1 N·s/m^2 = 1 kg/m·s = 10 poise = 10 dynes·s/cm^2 = 10 g/cm·s = 10^3 cp (centipoise) = 0.67197 lb_m/ft·s = 2419.088 lb_m/ft·h
Density	1 kg/m^3 = 10^{-3} g/cm^3 = 0.06243 lb_m/ft^3 10^3 kg/m^3 = 1 g/cm^3 = 62.428 lb_m/ft^3
Volumetric Flow	1 m^3/s= 35.31467 ft^3/s=15,850.32 gal/min (gpm) 1 gpm = 6.30902 x 10^{-5} m^3/s=2.228009 x 10^{-3} ft^3/s=3.7854 liter/min 1 liter/min=0.26417 gpm

Ver. 30-Oct-2014

Temperature	$T(^oC)=\frac{5}{9}\left[T(^oF)-32\right]$ $T(^oF)=\frac{9}{5}T(^oC)+32=1.8T(^oC)+32$
Absolute Temperature	T(K) = T(°C) + 273.15 T(°R) = T(°F) + 459.67
Temperature Interval (ΔT)	1 C° = 1 K = 1.8 F° = 1.8 R° 1F° = 1 R° = (5/9) C° = (5/9) K

USEFUL QUANTITIES

SG = ρ(20°C)/ρ_{water} (4°C)

ρ_{water}(4°C) = 1000 kg/m^3 = 62.43 lb_m/ft^3 = 1.000 g/cm^3
ρ_{water}(25°C) = 997.08 kg/m^3 = 62.25 lb_m/ft^3 = 0.99709 g/cm^3

g = 9.8066 m/s^2 = 980.66 cm/s^2 = 32.174 ft/s^2

μ_{water} (25°C) = 8.937 x 10^{-4} Pa·s = 8.937 x 10^{-4} kg/m·s
= 0.8937 cp =0.8937 x 10^{-2} g/cm·s = 6.005 x 10^{-4} lb_m/ft·s

Composition of air:

N_2	78.03%
O_2	20.99%
Ar	0.94%
CO_2	0.03%
H_2, He, Ne, Kr, Xe	0.01%
	100.00%

M_{air} = 29 g/mol = 29 kg/kmol = 29 lb_m/lbmole

$\hat{C}_{p,water}$ (25°C) = 4.182 kJ/kg K = 0.9989 cal/g°C = 0.9997 Btu/lb_m°F

R = 8.314 m^3·Pa/mol·K = 0.08314 liter·bar/mol·K = 0.08206 liter·atm/mol·K
= 62.36 liter·mm Hg/mol·K = 0.7302 ft^3·atm/lbmole·°R
= 10.73 ft^3·psia/lbmole·°R
= 8.314 J/mol·K
= 1.987 cal/mol·K = 1.987 Btu/lbmole·°R

Ver. 30-Oct-2014

pages.mtu.edu/~fmorriso/cm310/convert.pdf

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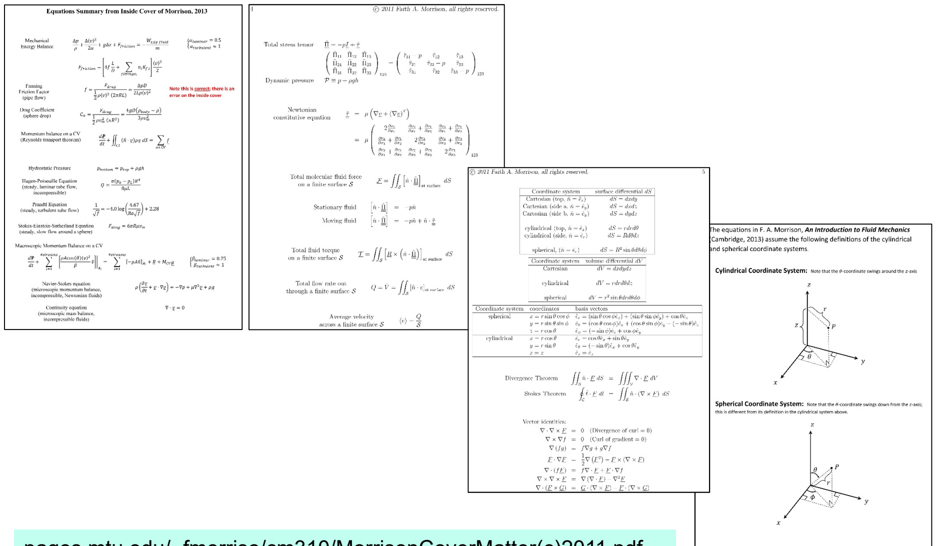
Handy Sheet of Fluid Mechanics Equations *from inside cover of Morrison*

Michigan Tech



pages.mtu.edu/~fmorriso/cm310/MorrisonCoverMatter(c)2011.pdf

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Example 2

What is the volumetric flow rate at the drain from a constant-head tank with a fluid level *h*? You may neglect frictional losses.

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Example 2

What is the volumetric flow rate at the drain from a constant-head tank with a fluid level *h*? You may neglect frictional losses.

ANSWER: $\pi R^2 \sqrt{2hg}$

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(Review)

For more examples: see CM2110/20 notes; HW1; Prerequisite review readings

HW1 (online and in Canvas)

number	Area	Topics	Assigned Problems	Stretch Problems
1	prereq	problem solving	1.2	
2	prereq	fluid statics/manometer	4.13	
3	prereq	fluid statics	C	
4	prereq	flow rate	1.10	
5	prereq	flow rate		1.19
6	prereq	flow rate	A	
7	prereq	siphon (neglect friction)		1.25
8	prereq	fluid statics	1.26	
9	prereq	math - vectors	1.41	
10	prereq	math - matrix	1.44	
11	prereq	math - dot product	1.45	
12	prereq	math matrices	B	
13	prereq	math - cyl coords		1.48
14	prereq	math - plot profile		1.58
15	prereq	math-integration	D	

Reading Recommendations CM3110
Fall Semester 2019
Dr. Faith A. Morrison

Prerequisite topics, suggested reference readings:	Source	Chapter	Pages	Section
Steady State Mass & Energy Balances	Felder and Rousseau	Ch 4.2-4.4, Ch7, Ch8.1-8.4a	pp 85-110, 313-381	
Mech Energy Balance	Felder and Rousseau	Ch 7.7	pp 333-337	
	Morrison	Ch 1	pp 8-93	
	Morrison (MEB parts only)	Ch 9	pp 766-800	
	Geankoplis	Ch 2.7F	pp 67-74	2.7F-G
	McCabe, Smith, Harriott	Ch 4	pp 86-94	
Fluid statics	Felder and Rousseau	Ch 3	pp 54-59	
	Morrison	Ch 4.2	pp 236-277	
	Geankoplis	Ch 2.1-2.2	pp 34-42	2.1-2.2
	McCabe, Smith, Harriott	Ch 2	pp 31-44	
Calc 1, 2, 3, 4	Your math text	*Topics*: differentiating, 1D, 2D, and 3D integrating, coordinate systems, vectors, vector fields, dot products		
Prerequisite topics, *YouTube* videos:	**Title (with link)**	**URL-link**	**Notes**	
Mechanical energy balance (single-input, single output, steady, no rxn, no phase change, little temperature change)	Short Intro to MEB	https://youtu.be/e4uEFCtuNic		
	Unit Conversion Issue with MEB	https://youtu.be/5E0i6qw6AeM		
	Analysis of a Pitot Tube	https://youtu.be/Af5RcQ18v_4	ends early	
Fluid statics (fluid velocity=0)	Intro to Manometers: Two rules	https://youtu.be/zeNQOqr63cc		
	Analysis of a Pitot Tube	https://youtu.be/Af5RcQ18v_4		

Exam 1: Next Tues 6:30-8:00pm, Dow 641

The exam and solution from 2017 is on the web. TA help session is Sunday night.
Exam topics: vectors, linear algebra, integration, balances, MEB, fluid statics

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CM3110 Fall 1019 Recommended Reading Topics and Pages

Lecture	Topics	Text	Sections
0	MEB, fluid statics, calc 1, 2, 3, & 4	Morrison	See first page
1	Why study fluids?	Morrison	All = 1.1, 1.2, stretch = Ch1
2,3	Fluid behavior, modeling	Morrison	All = Intro to Ch2, 2.1-2.4, 2.11, Intro to Ch3, 3.1, 3.2.1; Stretch = Ch 2&3
4,5	Fluid stresses	Morrison	All = 4.1, (4.3 lightly), 4.3.2; Stretch = Ch 4.2-4.3
6	Stress/velocity, microscopic balance equations, internal flows	Morrison	All = 5.1, 5.2, 5.4, 6.2, 6.3, 7.1, 7.2; Stretch = Ch 5,6,& 7
7,8	Stress/velocity, microscopic balance equations, internal flows	Morrison	All = 5.1, 5.2, 5.4, 6.2, 6.3, 7.1, 7.2; Stretch = Ch 5,6,& 7
9,10	Non-newtonian fluids, internal flows, correlations, dimensional analysis	Morrison	All = 5.3.1, 6.2, 6.3, 7.1, 7.2; Stretch = Ch 5,6,& 7
11	Macroscopic momentum balances	Morrison	All = 9.2, Stretch = Ch 9
12,13	External flows, dimensional analysis, boundary layers, compressible flows, numerical solutions	Morrison	All = 8.1, 8.2, 10.1-10.3, 10.6, 10.7; Stretch = Ch 8 & 10
14,15	Fourier's law, intro to heat transfer	Geankoplis, 4th ed.	All = 4.1, 4.2, 4.3; Stretch = Ch 4, Perry's Section 5
16,17	1D heat transfer, 2D heat transfer, unsteady state	Geankoplis, 4th ed.	All = 4.14, 5.1-5.3, 5.6; Stretch = Ch 4&5; Morrison 6.1.4, 9.1.3, Appendix D
18,19	Dimensional analysis; heat transfer coefficients (forced convection)	Geankoplis, 4th ed.	All = 4.5-4.7; Stretch = Ch 4&5; Perry's Section 11
20	Dimensional analysis (natural convection)	Geankoplis, 4th ed.	All = 4.5-4.7; Stretch = Ch 4&5; Perry's Section 11
21	Heat exchanger design/effectiveness/fouling	Geankoplis, 4th ed.	All = 4.9, 5.1-5.3; Stretch = Ch 4&5
22	Heat transfer with phase change, evaporators, radiation	Geankoplis, 4th ed.	All = 4.8, 4.10
23	Radiation	Geankoplis, 4th ed.	All = 4.10

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$$\frac{p_2 - p_1}{\rho} + \frac{\langle v\rangle_2^2 - \langle v\rangle_1^2}{2\alpha} + g(z_2 - z_1) + F_{21} = \frac{W_{s,on,21}}{\dot{m}}$$

F = friction

The *Mechanical Energy Balance* (MEB) is a macroscopic analysis.

- **It is limited in application:**
  1. single-input, single output
  2. Steady state
  3. Constant density (incompressible fluid)
  4. Temperature approximately constant
  5. No phase change, no chemical rxn
  6. Insignificant amounts of heat transferred
- **It cannot determine flow patterns**
- **It does not model momentum exchanges**
- **It cannot be adapted to systems other than those for which it was designed** (see list above)

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CM3110
Transport Processes and Unit Operations I

Part 1:

Professor Faith Morrison

Department of Chemical Engineering
Michigan Technological University

CM3110 - **Momentum** and Heat Transport
CM3120 – Heat and Mass Transport

www.chem.mtu.edu/~fmorriso/cm310/cm310.html

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Energy balances (the MEB) can only take us so far with fluids modeling (due to assumptions).

To understand complex flows, we must use the **MOMENTUM** balance.

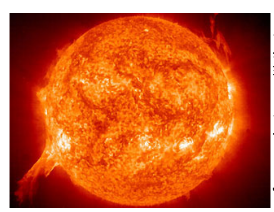
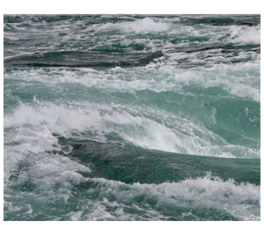


Image from: whatsupwiththat.com

Image from: www-math.mtu.edu

Image from: www.123rf.com



Naruto Whirlpools, Japan

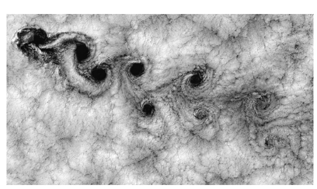


Image from: commons.wikipedia.org

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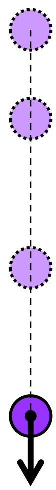
Momentum Balance: Newton's 2nd Law of Motion

$$\underline{f} = m\underline{a}$$



PH 2100: apply momentum conservation to individual bodies

CM 3110: apply momentum conservation to a continuum

Image from: www.texturex.com

See also: http://youtu.be/6KKNnjFpGto

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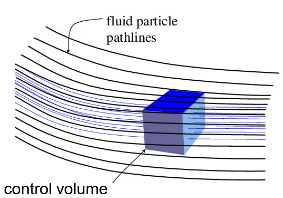
Fluid Mechanics



$$\underline{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}_{xyz}$$

$$\rho(x, y, z)$$

$$\underline{\underline{\tau}} = \begin{pmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{pmatrix}_{xyz}$$

- Continuum (density, velocity, stress fields) *(calc3, differential equations)*
- Control volume
- Stress in a fluid at a point (stress tensor)
- Stress and deformation (Newtonian constitutive equation)
- Microscopic and macroscopic momentum balances (engineering quantities of interest)
- Internal flows – pipes, conduits
- External flows – drag, boundary layers
- Advanced fluid mechanics – complex shapes

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Momentum . . . is a vector

$$\rho \underline{v} = \begin{pmatrix} \rho v_x \\ \rho v_y \\ \rho v_z \end{pmatrix}_{xyz}$$

Microscopic momentum balance

$$\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v}\cdot\nabla\underline{v}\right) = -\nabla P + \mu\nabla^2\underline{v} + \rho\underline{g}$$

Ch 6

Macroscopic momentum balance

$$\frac{d\mathbf{\underline{P}}}{dt} + \sum_{i=1}^{\#streams}\left[\frac{\rho A\cos\theta\langle v\rangle^2}{\beta}\hat{v}\right]\Bigg|_{A_i} = \sum_{i=1}^{\#streams}\left[-pA\hat{n}\right]\Big|_{A_i} + \underline{R} + M_{CV}\underline{g}$$

Ch 9

So we need vector math.

$(Calc\ 3)$

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Vectors

$$\underline{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}_{xyz} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}_{123} = \begin{pmatrix} v_r \\ v_\theta \\ v_z \end{pmatrix}_{r\theta z}$$

Note:
$v_x \neq v_1 \neq v_r$
(usually)

Same vector,
different coordinate systems,
different components.

$$|\underline{v}| = v = vector\ magnitude$$

$$\frac{(\underline{v})}{v} = \hat{v} = unit\ vector$$

We choose coordinate systems for convenience.

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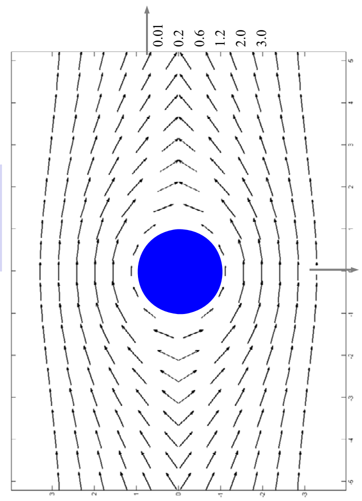
Fluid velocity is a vector field

(calc3)

$\underline{v} = v(x,y,z)$

Vector plot of the velocity field in slow flow around a sphere



creeping flow (sphere)

The flow is a steady upward flow; the length and direction of the vector indicates the velocity at that location.

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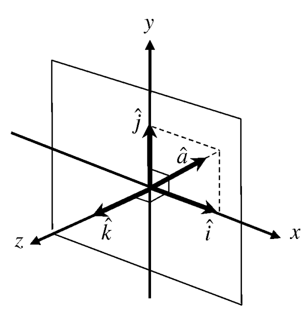
Vectors – Cartesian coordinate system

$$\underline{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}_{xyz} = v_x\hat{e}_x + v_y\hat{e}_y + v_z\hat{e}_z$$

$$\underline{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}_{123} = v_1\hat{e}_1 + v_2\hat{e}_2 + v_3\hat{e}_3$$

$$= v_1\hat{\imath} + v_2\hat{\jmath} + v_3\hat{k}$$



(three ways of writing the same thing, the Cartesian basis vectors)

- We do algebra with the basis vectors the same way as with other quantities
- The Cartesian basis vectors are constant (do not change with position)

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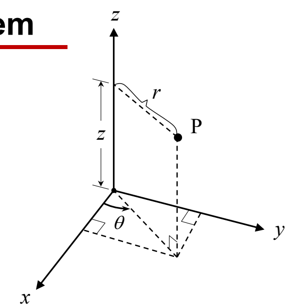
Vectors – Cylindrical coordinate system

$$\underline{v} = \begin{pmatrix} v_r \\ v_\theta \\ v_z \end{pmatrix}_{r\theta z} = v_r \hat{e}_r + v_\theta \hat{e}_\theta + v_z \hat{e}_z$$



- The cylindrical basis vectors are variable (depend on position)

$$x = r\cos\theta \qquad \hat{e}_r = \cos\theta\,\hat{e}_x + \sin\theta\,\hat{e}_y$$
$$y = r\sin\theta \qquad \hat{e}_\theta = -\sin\theta\,\hat{e}_x + \cos\theta\,\hat{e}_y$$
$$z = z \qquad \hat{e}_z = \hat{e}_z$$

(see inside back cover of text; also, supplemental handouts)

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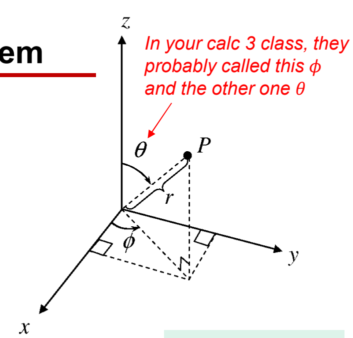
Vectors – Spherical coordinate system

$$\underline{v} = \begin{pmatrix} v_r \\ v_\theta \\ v_\phi \end{pmatrix}_{r\theta\phi} = v_r \hat{e}_r + v_\theta \hat{e}_\theta + v_z \hat{e}_\phi$$



Note: spherical coordinate system in use by the fluid mechanics community uses $0 < \theta < \pi$ as the angle from the z-axis to the point.

- The spherical basis vectors are variable (with position)

(see inside back cover of text; also, supplemental handouts)

$$x = r\sin\theta\cos\phi \qquad \hat{e}_r = \sin\theta\cos\phi\,\hat{e}_x + \sin\theta\sin\phi\,\hat{e}_y + \cos\theta\,\hat{e}_z$$
$$y = r\sin\theta\sin\phi \qquad \hat{e}_\theta = \cos\theta\cos\phi\,\hat{e}_x + \cos\theta\sin\phi\,\hat{e}_y + (-\sin\theta)\hat{e}_z$$
$$z = r\cos\theta \qquad \hat{e}_\phi = (-\sin\phi)\hat{e}_x + \cos\phi\,\hat{e}_y$$

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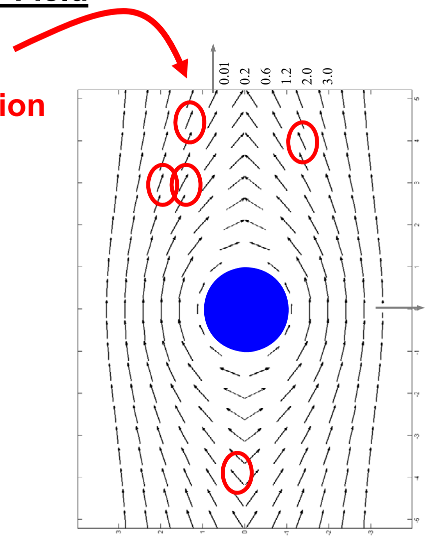
Fluid Velocity is a Vector Field

Velocity magnitude and direction vary with position

$$\underline{v} = v(x,y,z)$$



creeping flow (sphere)

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Example 3: At positions (1,45°,0) and (1,90°,0) in the r, θ, z coordinate system, the velocity vector of a fluid is given by

$$\underline{v} = \begin{pmatrix} v_r \\ v_\theta \\ v_z \end{pmatrix}_{r\theta z} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}_{r\theta z}$$

What is this vector in the usual xyz coordinate system?

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Example 3: At positions (1,45°,0) and (1,90°,0) in the r, θ, z coordinate system, the velocity vector of a fluid is given by

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What is this vector in the usual xyz coordinate system?

ANSWERS:

$$\underline{v}_{45^o} = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}_{xyz}$$

$$\underline{v}_{90^o} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}_{xyz}$$

$hint: \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}_{r\theta z} = \hat{e}_\theta$

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We use Calculus in Fluid Mechanics to:

1. Calculate flow rate, Q
2. Calculate average velocity, $\langle v \rangle$
3. Express forces on surfaces due to fluids (vectors)
4. Express torques on surfaces due to fluids (vectors)

These are quantities of interest.
These items are what we are learning to calculate.

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1. Calculate Flow rate: Q or $\dot{V}$

General:

$$Q = \iint_{area} (\underline{v} \cdot \hat{n})\, d(area)$$

Tube flow:

$$Q = \int_0^{2\pi} \int_0^R v_z(r)\, r dr d\theta$$

$(\underline{v} \cdot \hat{n})$ is the component of $\underline{v}$ in the direction normal to the area

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Common surface shapes in the standard coordinate systems:

$$\begin{aligned}
&\textit{rectangular}: && d(area) = dxdy \\
&\textit{circular}: && d(area) = r\, dr d\theta \\
&\textit{surface of cylinder}: && d(area) = R d\theta dz \\
&\textit{spherical}: && d(area) = (r d\theta)(r \sin\theta\, d\phi) = r^2 \sin\theta\, d\theta d\phi
\end{aligned}$$

(see inside back cover of text; also, supplemental handouts)

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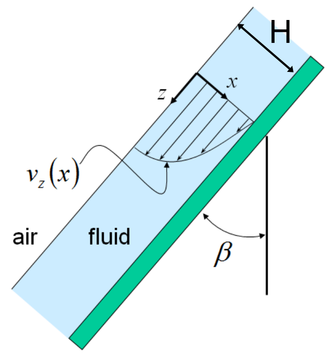
Example 4: *Calculate the flow rate in flow down an incline plane of width W.*

Momentum balance calculation gives:

$$v_z(x) = \frac{\rho g \cos(\beta)}{2\mu}\left(H^2 - x^2\right)$$

(we will learn how to get this equation for $v_z(x)$; here it is given)



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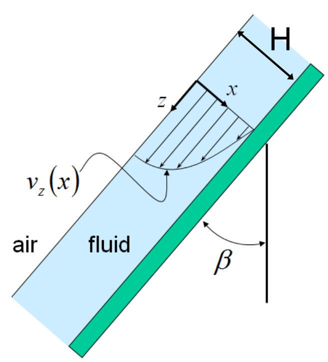

ANSWER:

$$Q = \frac{H^2 \rho g cos\beta}{3\mu}$$

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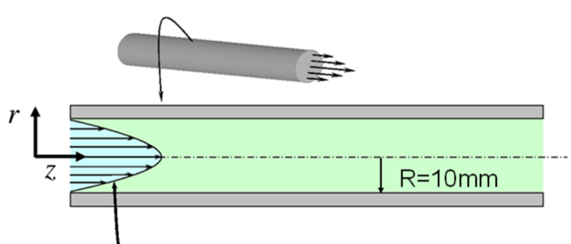
2. Calculate Average velocity: $\langle v \rangle$

General: $$\langle v \rangle = \frac{Q}{area}$$

Tube flow: $$\langle v \rangle = \frac{Q}{\pi R^2}$$

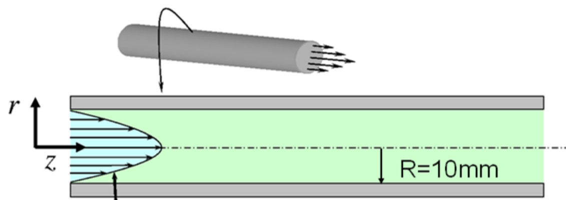
"*area*" is the cross-sectional area normal to flow

Example 5: *The shape of the velocity profile for a steady flow in a tube is found to be given by the function below. Over the range 0 <r<10 mm, (R=10mm), what is the average value of the velocity?*



$$\frac{v_z}{v_{max}} = f(r) = 1 - \left(\frac{r}{10}\right)^2$$

Example 5: *The shape of the velocity profile for a steady flow in a tube is found to be given by the function below. Over the range 0 <r<10 mm, (R=10mm), what is the average value of the velocity?*



$$\frac{v_z}{v_{max}} = f(r) = 1 - \left(\frac{r}{10}\right)^2$$

ANSWER: $\frac{v_{max}}{2}$

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3. Express forces on surfaces due to fluids

Total fluid force on a surface $= \underline{F} = \iint [\hat{n} \cdot \underline{\underline{\Pi}}]_{surface} dS$

(calc3)

$\underline{\underline{\Pi}} \equiv \underline{\underline{\tau}} - p\underline{\underline{I}} =$ Total stress tensor

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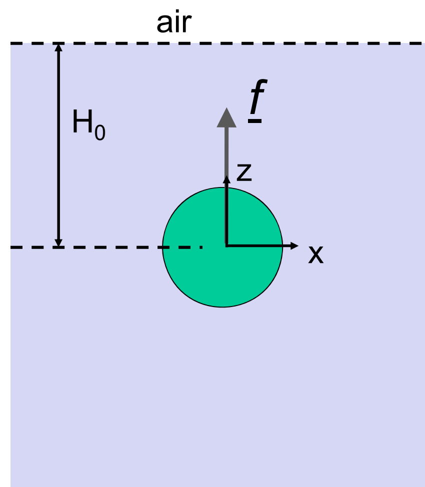
(p81)

Example 6: In a liquid of density ρ, what is the net fluid force on a submerged sphere (a ball or a balloon)? What is the direction of the force and how does the magnitude of the fluid force vary with fluid density?



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Solution: We will be able to do this in this course (Ch4, p257).

From expression for force due to fluid, obtain:
(in spherical coordinates)

Total fluid force on a surface $= \underline{F} = \iint [\hat{n} \cdot \underline{\underline{\Pi}}]_{surface} dS$

$$\underline{F} = -\rho g R^2 \int_0^{2\pi} \int_0^{\pi} (H_0 - R\cos\theta)\hat{e}_r \sin\theta \, d\theta d\phi$$

We can do the math from here. (Calc 3)

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Solution: We will be able to do this in this course (Ch4, p257).

From expression for force due to fluid, obtain:
(in spherical coordinates)

Total fluid force on a surface $= \underline{F} = \iint [\hat{n} \cdot \underline{\underline{\Pi}}]_{surface} dS$

$$\underline{F} = -\rho g R^2 \int_0^{2\pi} \int_0^{\pi} (H_0 - R\cos\theta)\hat{e}_r \sin\theta \, d\theta d\phi$$

ANSWER: (see p83)

$$\underline{f} = \begin{pmatrix} 0 \\ 0 \\ \frac{4\pi R^3 \rho g}{3} \end{pmatrix}_{xyz}$$

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4. Express torques on surfaces due to fluids

total fluid torque on a surface $= \underline{T} = \iint_S \left[\underline{R} \times \left[\hat{n} \cdot \underline{\underline{\Pi}} \right] \right]_{at\ surface} dS$

$\underline{R} = lever\ arm$ (Points from axis of rotation to position where torque is applied)

$\underline{\underline{\Pi}} = \underline{\underline{\tau}} - p\underline{\underline{I}} = total\ stress\ tensor$

We will learn to write the stress tensor for our systems; then we can calculate stresses, torques.

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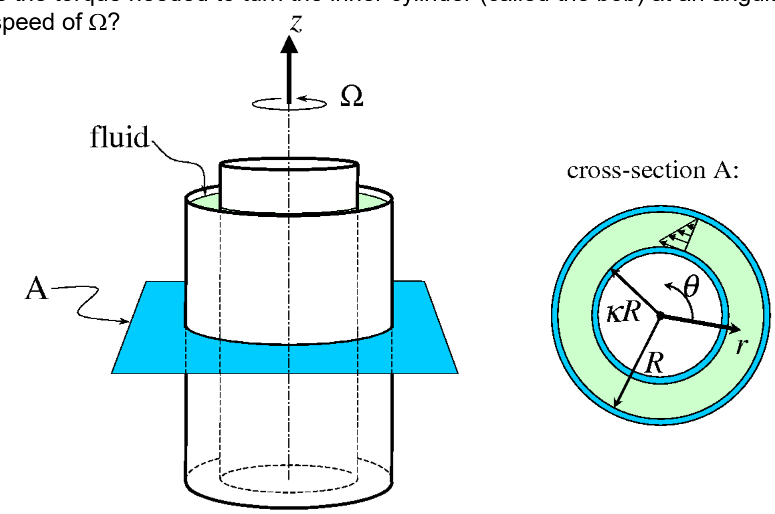
Example 7, Torque in Couette Flow: A cup-and-bob apparatus is widely used to measure viscosities for fluids. For the apparatus below, what is the torque needed to turn the inner cylinder (called the bob) at an angular speed of Ω?



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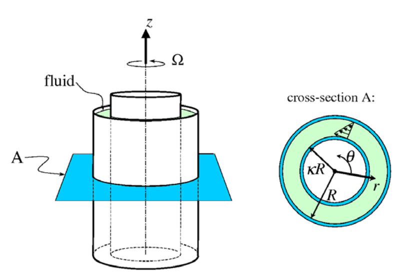
Torque in Couette Flow
Solution:

1. Solve for velocity field (microscopic momentum balance)
2. Calculate stress tensor
3. Formulate equation for torque (an integral)
4. Integrate
5. Apply boundary conditions



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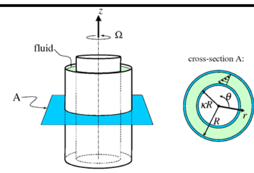
See problem 6.22 p487

Torque in Couette Flow Solution:



Velocity solution:

$$\underline{v} = \begin{pmatrix} 0 \\ \left(\frac{\kappa^2 \Omega R}{\kappa^2 - 1}\right)\left(\frac{r}{R} - \frac{R}{r}\right) \\ 0 \end{pmatrix}_{r\theta z}$$

$$\underline{\underline{\tau}} = \mu\left(\nabla \underline{v} + (\nabla \underline{v})^T\right)$$

$$\underline{\underline{\Pi}} = \underline{\underline{\tau}} - p\underline{\underline{I}}$$

$$\underline{\underline{\tau}} = \begin{pmatrix} \tau_{rr} & \tau_{r\theta} & \tau_{rz} \\ \tau_{\theta r} & \tau_{\theta\theta} & \tau_{\theta z} \\ \tau_{zr} & \tau_{z\theta} & \tau_{zz} \end{pmatrix}_{r\theta z}$$

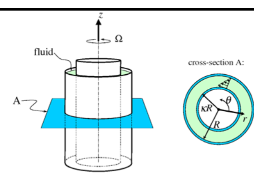
$$\textit{total fluid torque on a surface} = \underline{T} = \iint_S \left[\underline{R} \times \left[\hat{n} \cdot \underline{\underline{\Pi}}\right]\right]_{\textit{at surface}} dS$$

What is *lever arm*, $\underline{R}$?

etc…

See problem 6.22 p487

Torque in Couette Flow Solution:



$$\textit{total fluid torque on a surface} = \underline{T} = \iint_S \left[\underline{R} \times \left[\hat{n} \cdot \underline{\underline{\Pi}}\right]\right]_{\textit{at surface}} dS$$

ANSWER: (see p308)

$$\underline{T} = \frac{4\pi R^2 \kappa^2 L \mu \Omega}{(\kappa^2 - 1)} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}_{r\theta z}$$

F = friction

$$\frac{\Delta p}{\rho}+\frac{\Delta\langle v\rangle^2}{2\alpha}+g\Delta z+F=\frac{W_{s,on}}{\dot{m}}$$

$$\frac{p_2-p_1}{\rho}+\frac{\langle v\rangle_2^2-\langle v\rangle_1^2}{2\alpha}+g(z_2-z_1)+F_{21}=\frac{W_{s,on,21}}{\dot{m}}$$

Summary of Quick Start

A: Mechanical Energy Balance

1. SI-SO, steady, incompressible, no rxn, no ΔT, no Q
2. Macroscopic, based on **energy** (not momentum)
3. Choose points 1 and 2 wisely
4. Solve for F or $W_{s,on}$ or p, velocity, elevation

B): Use Calculus in Fluid Mechanics to

1. Calculate flow rate
2. Calculate average velocity
3. Express forces on surfaces due to fluids
4. Express torques on surfaces due to fluids

Summary of Quick Start

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End of **Quick Start**.

We have reviewed:

- MEB (an energy balance)
- Math tools

Now, on to **Fluid Mechanics**, i.e. momentum transport.

NEXT: Fluid Behavior

CM3110
Transport Processes and Unit Operations I

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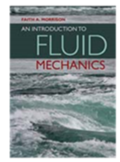
How do fluids behave?

1. Viscosity
2. Drag
3. Boundary Layers
4. Laminar versus Turbulent Flow
5. Lift
6. Supersonic
7. Surface Tension
8. Curved Streamlines
9. Magnetohydrodynamics



(Ch2)

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