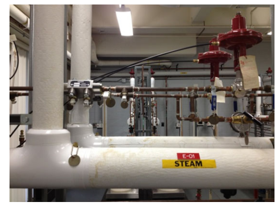
CM3110
Transport and Unit Ops I
Part II: Heat Transfer



One-Dimensional Heat Transfer

(part 2: radial heat transfer)

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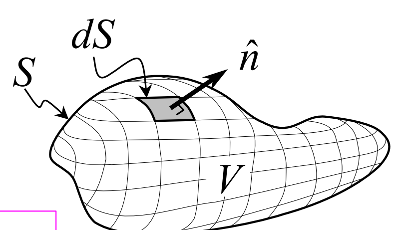
General Energy Transport Equation

(microscopic energy balance)

As for the derivation of the microscopic momentum balance, the microscopic energy balance is derived on an arbitrary volume, *V*, enclosed by a surface, *S*.



Gibbs notation:

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S_e$$

see handout for component notation

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1D Heat Transfer

General Energy Transport Equation

(microscopic energy balance)

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+\underline{v}\cdot\nabla T\right)=k\nabla^2 T+S_e$$

- rate of change: $\frac{\partial T}{\partial t}$
- convection: $\underline{v}\cdot\nabla T$
- conduction (all directions): $k\nabla^2 T$
- source: S_e (energy generated per unit volume per time)

velocity must satisfy equation of motion, equation of continuity

see handout for component notation

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1D Heat Transfer

The **Equation of Energy** for systems with **constant k**

Microscopic energy balance, constant thermal conductivity; Gibbs notation

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+\underline{v}\cdot\nabla T\right)=k\nabla^2 T+S$$

Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_x\frac{\partial T}{\partial x}+v_y\frac{\partial T}{\partial y}+v_z\frac{\partial T}{\partial z}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+S$$

Microscopic energy balance, constant thermal conductivity; cylindrical coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}+v_z\frac{\partial T}{\partial z}\right)=k\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}\right)+S$$

Microscopic energy balance, constant thermal conductivity; spherical coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial T}{\partial \phi}\right)$$
$$=k\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \phi^2}\right)+S$$

Note: this handout is also on the web

https://pages.mtu.edu/~fmorriso/cm310/energy.pdf

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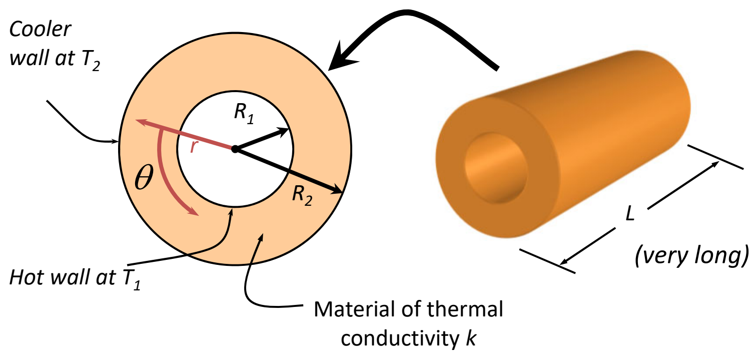
1D Heat Transfer – Radial

Example 3: Heat flux in a cylindrical shell – Temp BC

Assumptions:
- long pipe
- steady state
- k = thermal conductivity of wall

What is the steady state temperature profile in a cylindrical shell (pipe) if the ***inner wall*** *is at T_1 and the* ***outer wall*** *is at T_2? ($T_1 > T_2$)*



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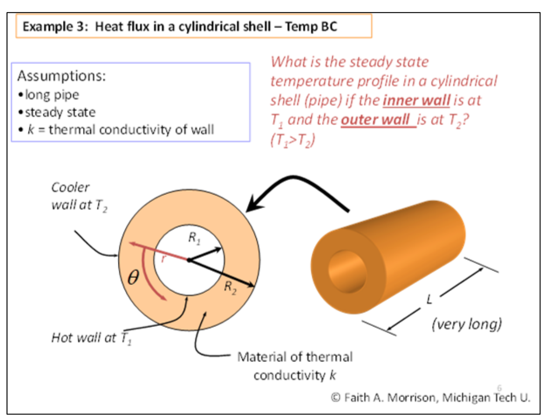
1D Heat Transfer – Radial

Example 3: Heat flux in a cylindrical shell – Temp BC

Let's try.



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1D Heat Transfer – Radial

Example 3: Heat flux in a cylindrical shell – Temp BC

Solution:

$$T = c_1 \ln(r) + c_2$$

Not linear

$$\frac{q_r}{A} = -k\left(\frac{dT}{dr}\right) = -k\left(\frac{c_1}{r}\right)$$

Fourier's law

Not constant

Boundary conditions?

Note: different integration constants c_1, c_2 are defined when we use the temperature version and the flux version of the microscopic energy balance; after boundary conditions are applied, the answer is the same.

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1D Heat Transfer – Radial

Example 3: Heat flux in a cylindrical shell – Temp BC

Solution for Cylindrical Shell:

NOT constant

$$\frac{q_r}{A} = \left(\frac{(T_1 - T_2)}{\frac{1}{k} \ln \frac{R_2}{R_1}} \right) \frac{1}{r}$$

The heat flux $\frac{q_r}{A}$ **DOES** depend on, k; also $\frac{q_r}{A}$ decreases as $1/r$

NOT linear

$$\frac{T_2 - T}{T_2 - T_1} = \frac{\ln \frac{R_2}{r}}{\ln \frac{R_2}{R_1}}$$

Note that $T(r)$ does not depend on the thermal conductivity, k (steady state)

Pipe with temperature BCs

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1D Heat Transfer – Radial

Example 3: Heat flux in a cylindrical shell – Temp BC

Solution for Cylindrical Shell:

NOT constant

$$\frac{q_r}{A} = \left(\frac{(T_1 - T_2)}{\frac{1}{k} \ln \frac{R_2}{R_1}} \right) \frac{1}{r}$$

Let: $\mathcal{R}_i \equiv \frac{1}{k_i} \ln \frac{R_{i+1}}{R_i}$

$$\frac{q_r}{A} = \left(\frac{(T_1 - T_2)}{\mathcal{R}_1} \right) \frac{1}{r} = \frac{\text{driving force}}{\text{resistance}}$$

Resistance due to finite thermal conductivity, radial

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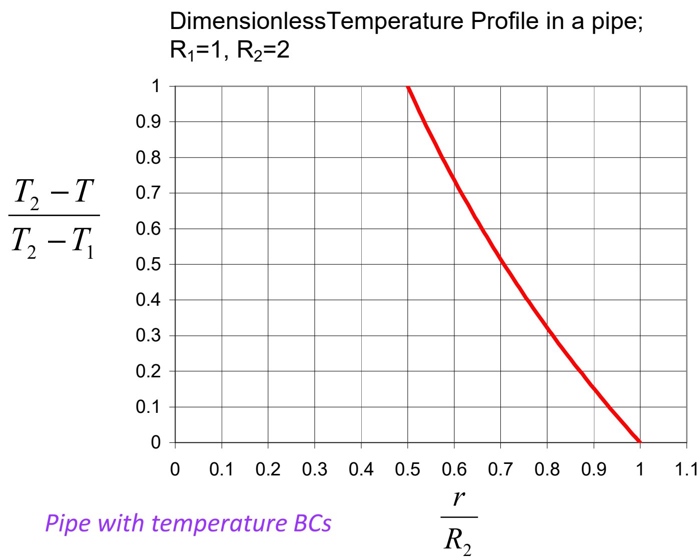
1D Heat Transfer – Radial

Example 3: Heat flux in a cylindrical shell – Temp BC



Pipe with temperature BCs

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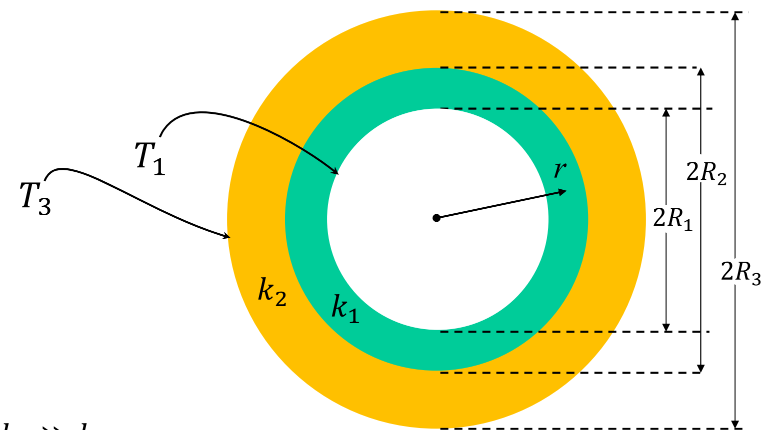
1D Radial Heat Transfer

Using the solution: Insulated Pipe (Composite, radial conduction)

For a metal pipe carrying a hot liquid (k_1) an insulation layer is added with thermal conductivity k_2. What is the temperature profile in the composite pipe at steady state? What is the flux? The inside temperature of the metal pipe is T_1 and the outside temperature of the insulation is T_3.



$k_1 \gg k_2$

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1D Radial Heat Transfer

Recall our composite solution for rectangular heat conduction:

1D Heat Transfer

Example 1b: Composite Door (two equal width layers)

SOLUTION:

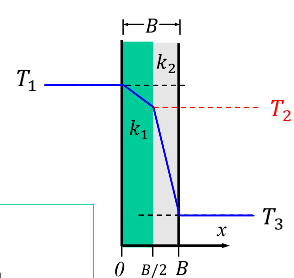
$$\frac{q_x}{A} = \frac{(T_1 - T_3)}{\left(\frac{\frac{B}{2}(k_1 + k_2)}{k_1 k_2}\right)}$$

k_1 material: $(0 \le x \le B/2)$

$$T(x) = \frac{(T_2 - T_1)}{B/2} x + T_1$$

k_2 material: $(B/2 \le x \le B)$

$$T(x) = \frac{(T_3 - T_2)}{B/2} x + (2T_2 - T_3)$$



$$T_2 = \frac{k_1 T_1 + k_2 T_3}{k_1 + k_2}$$

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1D Radial Heat Transfer

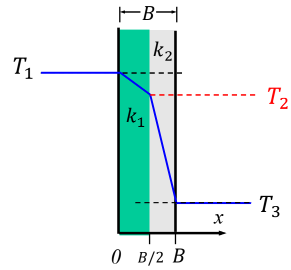
Recall our composite solution for rectangular heat conduction:

1D Heat Transfer

Example 1b: Composite Door (two equal width layers)

SOLUTION:

$$\frac{q_x}{A} = \frac{(T_1 - T_3)}{\left(\frac{\frac{B}{2}(k_1 + k_2)}{k_1 k_2}\right)}$$



Let: $\mathcal{R}_i \equiv \frac{\Delta x}{k_i}$

$$\frac{q_x}{A} = \frac{(T_1 - T_3)}{\mathcal{R}_1 + \mathcal{R}_2} = \frac{\text{driving force}}{\text{resistance}}$$

$$T_2 = \frac{k_1 T_1 + k_2 T_3}{k_1 + k_2}$$

Each of the layers contributes a resistance, added in series (like in electricity).

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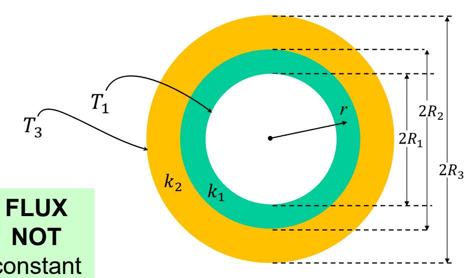
1D Heat Transfer – Radial

Example 3b: Insulated Pipe (Composite, radial conduction)

SOLUTION:



$$\frac{q_r}{A} = -k_i\left(\frac{dT}{dr}\right) = (\text{constant})\frac{1}{r}$$

FLUX NOT constant

k_1 material: $(R_1 \le r \le R_2)$

$$T(r) = a_1 \ln r + b_1$$

k_2 material: $(R_2 \le r \le R_3)$

$$T(r) = a_2 \ln r + b_2$$

$\boldsymbol{T(r)}$ **NOT** linear

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Example 3b: Insulated Pipe (Composite, radial conduction)

k_1 material: $(R_1 \leq r \leq R_2)$

$$T(r) = a_1 \ln r + b_1$$

k_2 material: $(R_2 \leq r \leq R_3)$

$$T(r) = a_2 \ln r + b_2$$

Flux is NOT constant

BUT, fluxes materials match at $r = R_2$:

k_1 material: $(R_1 \leq r \leq R_2)$ $\quad \frac{q_r}{A} = c_1 \frac{1}{r}$

k_2 material: $(R_2 \leq r \leq R_3)$ $\quad \frac{q_r}{A} = c_2 \frac{1}{r}$

At $r = R_2$: $\quad c_1 \frac{1}{R_2} = c_2 \frac{1}{R_2}$

$$\Rightarrow c_1 = c_2$$

$$r\frac{q_r}{A} = c_1$$

Same constant c_1 for all layers

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Example 3b: Insulated Pipe (Composite, radial conduction)

Fourier's Law:

$$\frac{q_r}{A} = -k_i \frac{dT}{dr} = \frac{c_1}{r}$$

k_1 material: $(R_1 \leq r \leq R_2)$

$$T(r) = \frac{c_1}{-k_1} \ln r + b_1$$

k_2 material: $(R_2 \leq r \leq R_3)$

$$T(r) = \frac{c_1}{-k_2} \ln r + b_2$$

Apply BCs:

k_1 material: $(R_1 \leq r \leq R_2)$

$$T_1 = \frac{c_1}{-k_1} \ln R_1 + b_1$$

$$T_2 = \frac{c_1}{-k_1} \ln R_2 + b_1$$

SUBTRACT

$$T_1 - T_2 = c_1 \left(\frac{1}{k_1} \ln \frac{R_2}{R_1} \right)$$

k_2 material: $(R_2 \leq r \leq R_3)$

$$T_2 = \frac{c_1}{-k_2} \ln R_2 + b_2$$

$$T_3 = \frac{c_1}{-k_2} \ln R_3 + b_2$$

SUBTRACT

$$T_2 - T_3 = c_1 \left(\frac{1}{k_2} \ln \frac{R_3}{R_2} \right)$$

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1D Heat Transfer – Radial

Example 3b: Insulated Pipe (Composite, radial conduction)

k_1 material: $(R_1 \le r \le R_2)$

$$T(r) = \frac{c_1}{-k_1}\ln r + b_1$$

k_2 material: $(R_2 \le r \le R_3)$

$$T(r) = \frac{c_1}{-k_2}\ln r + b_2$$

k_1 material: $(R_1 \le r \le R_2)$

$$T_1 = \frac{c_1}{-k_1}\ln R_1 + b_1$$

$$T_2 = \frac{c_1}{-k_1}\ln R_2 + b_1$$

$$T_1 - T_2 = c_1\left(\frac{1}{k_1}\ln\frac{R_2}{R_1}\right)$$

k_2 material: $(R_2 \le r \le R_3)$

$$T_2 = \frac{c_1}{-k_2}\ln R_2 + b_2$$

$$T_3 = \frac{c_1}{-k_2}\ln R_3 + b_2$$

$$T_2 - T_3 = c_1\left(\frac{1}{k_2}\ln\frac{R_3}{R_2}\right)$$

ADD

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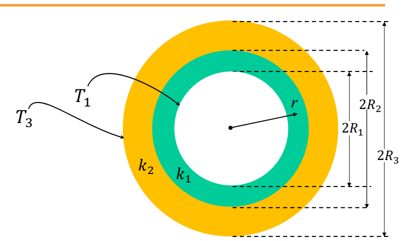
1D Heat Transfer – Radial

Example 3b: Insulated Pipe (Composite, radial conduction)



$$T_1 - T_2 = c_1\left(\frac{1}{k_1}\ln\frac{R_2}{R_1}\right)$$

$$T_2 - T_3 = c_1\left(\frac{1}{k_2}\ln\frac{R_3}{R_2}\right)$$

ADD

$$T_1 - T_3 = c_1\left(\frac{1}{k_1}\ln\frac{R_2}{R_1} + \frac{1}{k_2}\ln\frac{R_3}{R_2}\right)$$

Note that we could add more layers by repeating this step

$$\frac{q_r}{A} = c_1\frac{1}{r} = \left(\frac{T_1 - T_3}{\left(\frac{1}{k_1}\ln\frac{R_2}{R_1} + \frac{1}{k_2}\ln\frac{R_3}{R_2}\right)}\right)\frac{1}{r}$$

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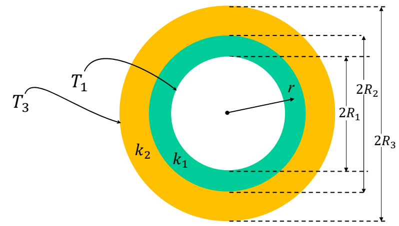
1D Heat Transfer – Radial

Example 3b: Insulated Pipe (Composite, radial conduction)

SOLUTION:



$$\frac{q_r}{A}=\left(\frac{(T_1-T_3)}{\frac{1}{k_1}\ln\frac{R_2}{R_1}+\frac{1}{k_2}\ln\frac{R_3}{R_2}}\right)\frac{1}{r}$$

Let: $\mathcal{R}_i \equiv \frac{1}{k_i}\ln\frac{R_{i+1}}{R_i}$

$$\frac{q_r}{A}=\left(\frac{(T_1-T_3)}{\mathcal{R}_1+\mathcal{R}_2}\right)\frac{1}{r}=\frac{\text{driving force}}{\text{resistance}}$$

Note that we can continue to add layers in terms of resistance

Each of the layers contributes a resistance, added in series (like in electricity).

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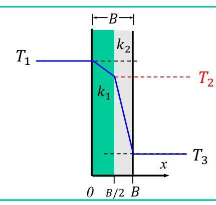
1D Heat Transfer – Composite Structures

Let: $\mathcal{R}_i \equiv \frac{\Delta x}{k_i}$

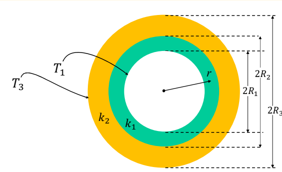
$$\frac{q_x}{A}=\frac{(T_1-T_3)}{\mathcal{R}_1+\mathcal{R}_2}=\frac{\text{driving force}}{\text{resistance}}$$



Note: Geankoplis uses a different resistance. For rectangular heat flux:
$R_{Geankoplis}=\mathcal{R}/LW$

Let: $\mathcal{R}_i \equiv \frac{1}{k_i}\ln\frac{R_{i+1}}{R_i}$

$$\frac{q_r}{A}=\left(\frac{(T_1-T_3)}{\mathcal{R}_1+\mathcal{R}_2}\right)\frac{1}{r}=\frac{\text{driving force}}{\text{resistance}}$$



Note: Geankoplis uses a different resistance. For radial heat flux:
$R_{Geankoplis}=\mathcal{R}/2\pi L$

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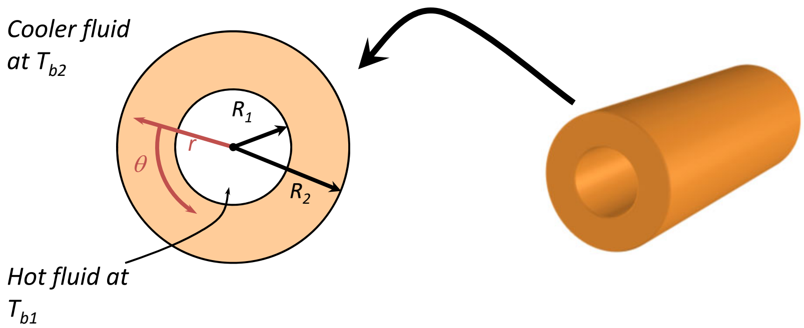
1D Heat Transfer – Radial

Example 4: Heat flux in a cylindrical shell – Newton's law of cooling

Assumptions:
- long pipe
- steady state
- k = thermal conductivity of wall
- h_1, h_2 = heat transfer coefficients

*What is the steady state temperature profile in a cylindrical shell (pipe) if the **fluid on the inside** is at T_{b1} and the **fluid on the outside** is at T_{b2}? (T_{b1}>T_{b2})*



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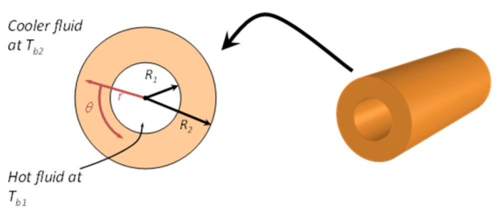
1D Heat Transfer – Radial

Example 4: Heat flux in a cylindrical shell



You try.

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1D Heat Transfer – Radial

Example 4: Heat flux in a cylindrical shell

Solution:

$$T = c_1 \ln(r) + c_2$$

$$\frac{q_r}{A} = -k\left(\frac{dT}{dr}\right) = -k\left(\frac{c_1}{r}\right)$$

Fourier's law

Boundary conditions?

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1D Heat Transfer – Radial

Example 4: Heat flux in a cylindrical shell

Solution:

$$T = c_1 \ln(r) + c_2$$

$$\frac{q_r}{A} = -k\left(\frac{dT}{dr}\right) = -k\left(\frac{c_1}{r}\right)$$

Fourier's law

Boundary conditions?

Newton's law of cooling
(the one with the heat transfer coefficient)

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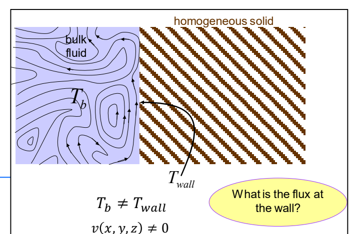
The flux at the wall is given by the empirical expression known as **Newton's Law of Cooling**



This expression serves as the definition of the heat transfer coefficient.

$$\left|\frac{q_x}{A}\right| = h\left|T_{bulk} - T_{wall}\right|$$

***h* depends on:**
- geometry
- fluid velocity field
- fluid properties
- temperature difference

For now, we'll "hand" you h; later, you'll get it from literature data correlations, i.e. from experiments.

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1D Heat Transfer – Radial

Example 4: Heat flux in a cylindrical shell

$$h_1(T_{b1} - T_{w1}) = -kc_1\left(\frac{1}{R_1}\right)$$

$$h_2(T_{w2} - T_{b2}) = -kc_1\left(\frac{1}{R_2}\right)$$

$$T_{w1} = T(R_1) = c_1 \ln(R_1) + c_2$$

$$T_{w2} = T(R_2) = c_1 \ln(R_2) + c_2$$

4 equations

4 unknowns;

SOLVE

c_1, T_{w1}, c_2, T_{w2}

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1D Heat Transfer – Radial

Example 4: Heat flux in a cylindrical shell

Newton's law of cooling boundary conditions

Solution: Radial Heat Flux in an Annulus

$T(r)$

$$T - T_{b2} = \frac{(T_{b1} - T_{b2})\left(\ln\left(\frac{R_2}{r}\right) + \frac{k}{h_2 R_2}\right)}{\frac{k}{h_2 R_2} + \ln\left(\frac{R_2}{R_1}\right) + \frac{k}{h_1 R_1}}$$

$q_r(r)$

$$\frac{q_r}{A} = \frac{(T_{b1} - T_{b2})}{\frac{1}{h_2 R_2} + \frac{1}{k}\ln\left(\frac{R_2}{R_1}\right) + \frac{1}{h_1 R_1}}\left(\frac{1}{r}\right)$$

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Example 4: Heat flux in a cylindrical shell

Newton's law of cooling boundary conditions

Solution: Radial Heat Flux in an Annulus

$T(r)$

$$T - T_{b2} = \frac{(T_{b1} - T_{b2})\left(\ln\left(\frac{R_2}{r}\right) + \frac{k}{h_2 R_2}\right)}{\frac{k}{h_2 R_2} + \ln\left(\frac{R_2}{R_1}\right) + \frac{k}{h_1 R_1}}$$

$q_r(r)$

$$\frac{q_r}{A} = \frac{(T_{b1} - T_{b2})}{\frac{1}{h_2 R_2} + \frac{1}{k}\ln\left(\frac{R_2}{R_1}\right) + \frac{1}{h_1 R_1}}\left(\frac{1}{r}\right)$$

Resistance $\mathcal{R}$ due to heat transfer at boundary
Resistance $\mathcal{R}$ due to finite thermal conductivity

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Solution: Radial Heat Flux in an Annulus

$$\frac{q_r}{A} = \frac{(T_{b1} - T_{b2})}{\frac{1}{h_2 R_2} + \frac{1}{k}\ln\left(\frac{R_2}{R_1}\right) + \frac{1}{h_1 R_1}}\left(\frac{1}{r}\right)$$

Note that we can continue to add layers in terms of *resistance*

Resistance $\mathcal{R}$ due to heat transfer at boundary, radial
Resistance $\mathcal{R}$ due to finite thermal conductivity, radial

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1D Heat Transfer

So far, common assumptions are:

1. Steady state
2. ??
3. ??
4. ??
5. ??

You try.

The **Equation of Energy** for systems with **constant k**

Microscopic energy balance, constant thermal conductivity; Gibbs notation

$$\rho \hat{C}_p\left(\frac{\partial T}{\partial t} + \underline{v}\cdot\nabla T\right) = k\nabla^2 T + S$$

Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

$$\rho \hat{C}_p\left(\frac{\partial T}{\partial t} + v_x\frac{\partial T}{\partial x} + v_y\frac{\partial T}{\partial y} + v_z\frac{\partial T}{\partial z}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + S$$

Microscopic energy balance, constant thermal conductivity; cylindrical coordinates

$$\rho \hat{C}_p\left(\frac{\partial T}{\partial t} + v_r\frac{\partial T}{\partial r} + \frac{v_\theta}{r}\frac{\partial T}{\partial \theta} + v_z\frac{\partial T}{\partial z}\right) = k\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2}\right) + S$$

Microscopic energy balance, constant thermal conductivity; spherical coordinates

$$\rho \hat{C}_p\left(\frac{\partial T}{\partial t} + v_r\frac{\partial T}{\partial r} + \frac{v_\theta}{r}\frac{\partial T}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial T}{\partial \phi}\right) = k\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \phi^2}\right) + S$$

What would problems look like with other modeling assumptions?

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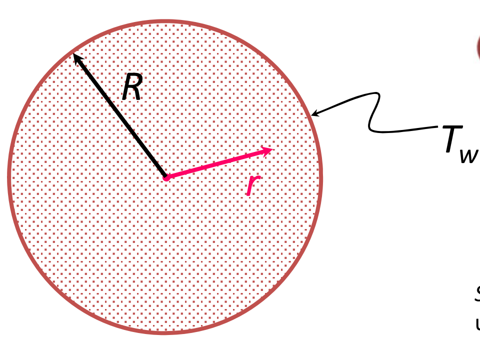
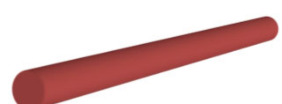
1D Heat Transfer – With Generation

Example 5: Heat Conduction with Generation

What is the steady state temperature profile in a wire if heat is generated uniformly throughout the wire at a rate of S_e W/m³ and the outer radius is held at T_w? What is the heat flux at the wire surface?





S_e = energy production per unit volume

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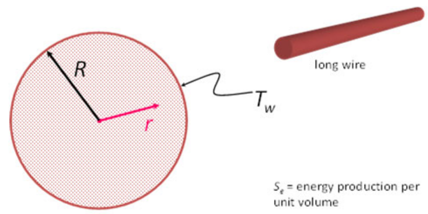
1D Heat Transfer – With Generation

Example 5: Heat conduction with generation



You try.

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See handwritten notes.

https://pages.mtu.edu/~fmorriso/cm310/selected_lecture_slides.html

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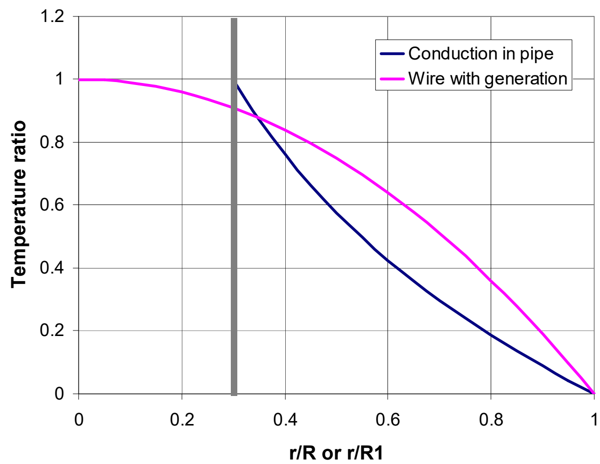
1D Heat Transfer – With Generation

Compare solutions



$$\frac{T-T_1}{T_1-T_2}\frac{1}{\ln\frac{R_1}{R_2}}=\ln\frac{r}{R_1}$$

$$\frac{T-T_w}{S_eR^2/4k}=1-\left(\frac{r}{R}\right)^2$$

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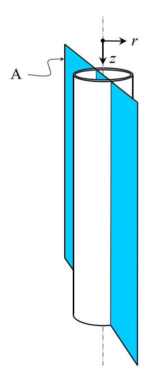
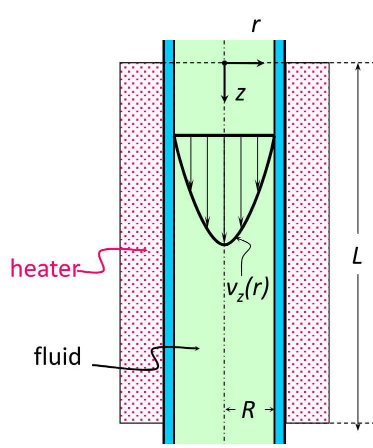
1D Heat Transfer – With Flow

Example 6: Wall heating of laminar flow. *What is the steady state temperature profile in a flowing fluid in a tube if the walls are heated (constant flux, q_1/A) and if the fluid is a Newtonian fluid in laminar flow?*

assume:
constant viscosity



cross-section A:



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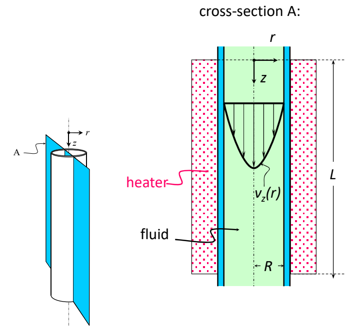
2D Heat Transfer – With Flow

Example 5: Wall heating of laminar flow



You try.

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See handwritten notes (in class and on web).

https://pages.mtu.edu/~fmorriso/cm310/Details_laminiar_flow_plus_heater.pdf

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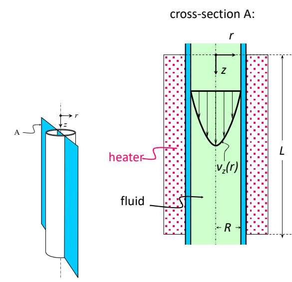
2D Heat Transfer – With Flow

Example 5: Wall heating of laminar flow



We need to solve this partial differential equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\partial T}{\partial r}r\right)+\frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right)-\frac{\rho\hat{C}_p}{k}v_z(r)\frac{\partial T}{\partial z}=0$$

with the appropriate boundary conditions. To see the solution see:

- R. Siegel, E. M. Sparrow, T. M. Hallman, *Appl. Science Research* A7, 386-392 (1958)
- R. B. Bird, W. Stewart, and E. Lightfoot, *Transport Phenomena*, Wiley, 1960, p295.

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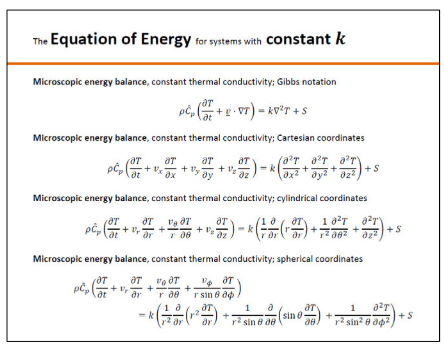
2D Heat Transfer

Common simplifying assumptions are:

1. Steady state; unsteady state
2. No flow; laminar flow
3. Long, wide
4. Long, θ-symmetric, other symmetries
5. no current in problem domain; constant current in problem domain
6. Conduction more dominant in one direction than another
7. others



The **Equation of Energy** for systems with **constant k**

Microscopic energy balance, constant thermal conductivity; Gibbs notation

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+\underline{v}\cdot\nabla T\right)=k\nabla^2 T+S$$

Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_x\frac{\partial T}{\partial x}+v_y\frac{\partial T}{\partial y}+v_z\frac{\partial T}{\partial z}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+S$$

Microscopic energy balance, constant thermal conductivity; cylindrical coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}+v_z\frac{\partial T}{\partial z}\right)=k\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}\right)+S$$

Microscopic energy balance, constant thermal conductivity; spherical coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial T}{\partial \phi}\right)$$
$$=k\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \phi^2}\right)+S$$

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1D-2D Heat Transfer – Various Cases

SUMMARY

Steady State Heat Transfer

Example 1: Heat flux in a rectangular solid – Temperature BC

Example 2: Heat flux in a rectangular solid – Newton's law of cooling

Example 3: Heat flux in a cylindrical shell – Temperature BC

Example 4: Heat flux in a cylindrical shell – Newton's law of cooling

Example 5: Heat conduction with generation

Example 6: Wall heating of laminar flow

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1D-2D Heat Transfer – Various Cases

SUMMARY

Steady State Heat Transfer

Example 1: Heat flux in a rectangular solid – Temperature BC

Example 2: Heat flux in a rectangular solid – Newton's law of cooling

Example 3: Heat flux in a cylindrical shell – Temperature BC

Example 4: Heat flux in a cylindrical shell – Newton's law of cooling

Example 5: Heat conduction with generation

Example 6: Wall heating of laminar flow

Conclusion: When we can simplify geometry, assume steady state, assume symmetry, the solutions are easily obtained

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CM3110
Transport I
Part II: Heat Transfer

Michigan Tech

Complex Heat Transfer – Dimensional Analysis

Professor Faith Morrison

Department of Chemical Engineering
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Next

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