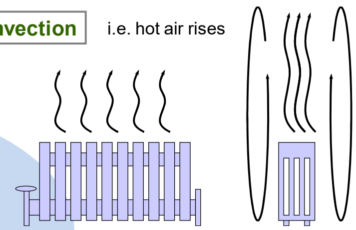
Michigan Tech

CM3110
Transport/Unit Ops I
Part II: Heat Transfer

Free Convection

i.e. hot air rises



Complex Heat Transfer – Dimensional Analysis

(Natural convection heat transfer)

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Complex Heat transfer Problems to Solve:

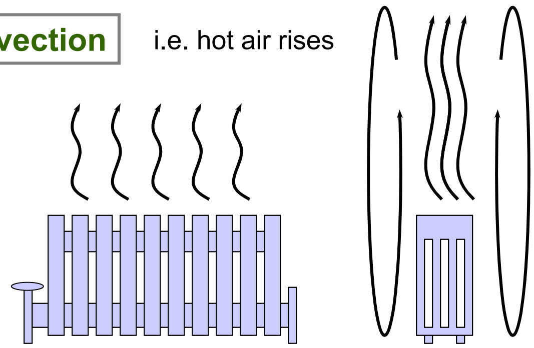
- Forced convection heat transfer from fluid to wall
  Solution: ?
- Natural convection heat transfer from fluid to wall
  Solution: ?
- Radiation heat transfer from solid to fluid
  Solution: ?

We started with a forced-convection pipe problem, did dimensional analysis, and found the dimensionless numbers.

To do a situation with different physics, we must start with a different starting problem.

Complex Heat Transfer – Dimensional Analysis—Free Convection

Free Convection i.e. hot air rises



- heat moves from hot surface to cold air (fluid) by radiation and conduction
- increase in fluid temperature decreases fluid density
- recirculation flow begins
- **recirculation adds to the heat-transfer** from conduction and radiation

$\Rightarrow$ coupled heat and momentum transport

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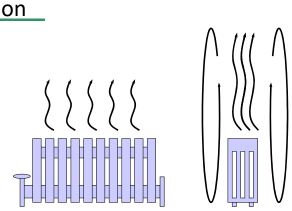
Complex Heat Transfer – Dimensional Analysis—Free Convection

Free Convection i.e. hot air rises



How can we solve **real** problems involving free (natural) convection?

We'll try this: Let's review how we approached solving real problems in *earlier* cases, i.e. in fluid mechanics, forced convection.

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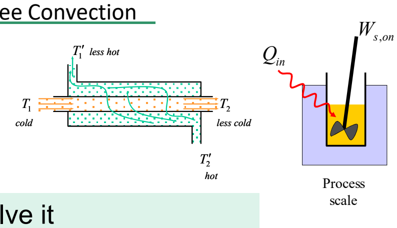
Complex Heat Transfer – Dimensional Analysis—Free Convection



Engineering Modeling

- Choose an idealized problem and solve it
- From insight obtained from **ideal** problem, identify governing equations of **real** problem
- Nondimensionalize the governing equations; deduce dimensionless scale factors (e.g. Re, Fr for fluids)
- Design experiments to test modeling thus far
- Revise modeling (structure of dimensional analysis, identity of scale factors, e.g. add roughness lengthscale)
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- Iterate until useful correlations result

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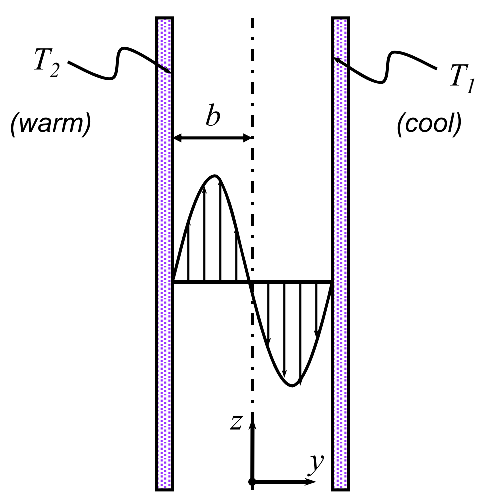
Complex Heat Transfer – Dimensional Analysis—Free Convection

Example: Free convection between long parallel plates *or* heat transfer through double-pane glass windows



$$T_2 > T_1$$

assumptions:
- long, wide slit
- steady state
- no source terms
- viscosity constant
- density varies with T

Calculate: T, $\underline{v}$ profiles

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Complex Heat Transfer – Dimensional Analysis—Free Convection

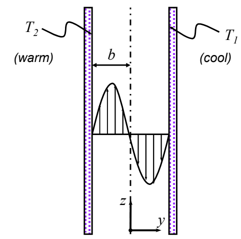
Example : Natural convection between vertical plates

Mass balance:

$$\frac{\partial \rho}{\partial t} + \underline{v} \cdot \nabla \rho + \rho(\nabla \cdot \underline{v}) = 0$$

Momentum balance:

$$\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v}\right) = -\nabla p + \mu \nabla^2 \underline{v} + \rho \underline{g}$$



$T_2 > T_1$

Equation of Motion for incompressible, Newtonian fluid (Navier-Stokes equation) 3 components in Cartesian coordinates

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial P}{\partial x} + \mu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right) + \rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial P}{\partial y} + \mu\left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right) + \rho g_y$$

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Equation of Motion for incompressible, Newtonian fluid (Navier-Stokes equation), 3 components in cylindrical coordinates

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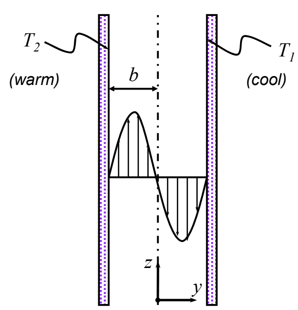
Complex Heat Transfer – Dimensional Analysis—Free Convection

Example : Natural convection between vertical plates

You try.



$T_2 > T_1$

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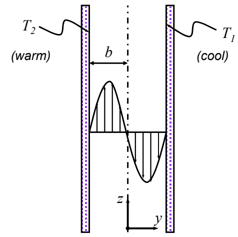
Complex Heat Transfer – Dimensional Analysis—Free Convection



$T_2 > T_1$

Mass balance:

$$\frac{\partial \rho}{\partial t} + \underline{v} \cdot \nabla \rho + \rho(\nabla \cdot \underline{v}) = 0$$

$$\frac{\partial \rho}{\partial t} + \left(v_x \frac{\partial \rho}{\partial x} + v_y \frac{\partial \rho}{\partial y} + v_z \frac{\partial \rho}{\partial z} \right) + \rho \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) = 0$$

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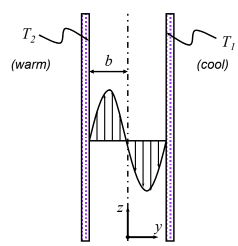
Complex Heat Transfer – Dimensional Analysis—Free Convection



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steady $\underline{v} = v_z(y)\hat{e}_z$ tall, wide

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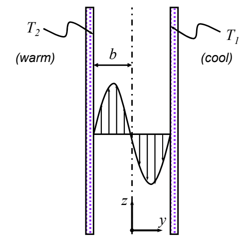
Complex Heat Transfer – Dimensional Analysis—Free Convection



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steady $\underline{v} = v_z(y)\hat{e}_z$

Conclusion: density must not vary with z.

tall, wide

$$\rho = \rho(x, y)$$

$$\rho = \rho(y)$$

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Complex Heat Transfer – Dimensional Analysis—Free Convection

Momentum balance:

$$\rho \left(\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v} \right) = -\nabla p + \mu \nabla^2 \underline{v} + \rho \underline{g}$$

$$\rho \left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} + \right) = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \right) + \rho g_x$$

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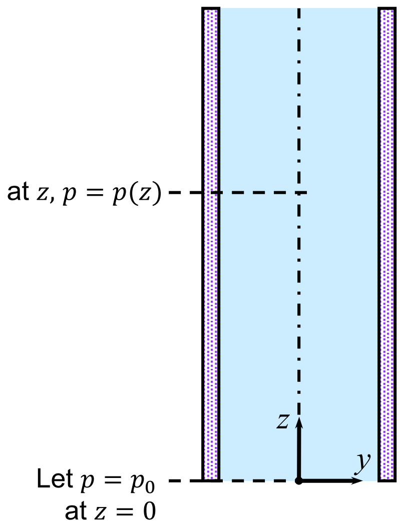
Is Pressure a function of z?

YES, there should be hydrostatic pressure (due to weight of fluid)

"Pressure at the bottom of a column of fluid = pressure at top $+\rho g h$."



average density

$$p_0 = p(z) + \bar{\rho} g z$$

$$p(z) = p_0 - \bar{\rho} g z$$

$$\Rightarrow \frac{dP}{dz} = -\bar{\rho} g$$

To account for the temperature variation of ρ:

(look up the physics in the literature)

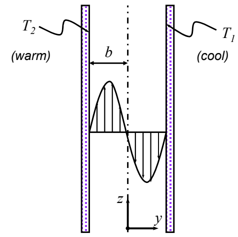
$$\rho = \bar{\rho} - \bar{\rho}\bar{\beta}(T - \bar{T})$$

$\bar{\rho} =$ mean density

$\bar{\beta} =$ volumetric coefficient of expansion at $\bar{T}$

$$\bar{T} = \frac{T_1 + T_2}{2}$$

Complex Heat Transfer – Dimensional Analysis—Free Convection



$T_2 > T_1$

Energy balance:

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S$$

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + S$$

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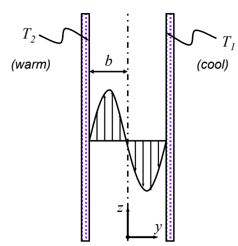
Complex Heat Transfer – Dimensional Analysis—Free Convection



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(solve) ...

$$T(y) = \frac{T_1 - T_2}{2b} y + \frac{T_1 + T_2}{2}$$

$$T(y) = \frac{T_1 - T_2}{2b} y + \bar{T}$$

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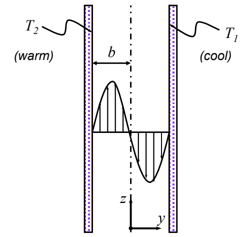
Complex Heat Transfer – Dimensional Analysis—Free Convection



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Energy balance:

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(solve)
...

$$T(y) = \frac{T_1 - T_2}{2b} y + \frac{T_1 + T_2}{2}$$

$$T(y) = \frac{T_1 - T_2}{2b} y + \bar{T}$$

$$\rho = \bar{\rho} - \bar{\rho}\bar{\beta}(T - \bar{T})$$

$$\rho = \bar{\rho} - \bar{\rho}\bar{\beta}\left(\frac{T_1 - T_2}{2b} y \right)$$

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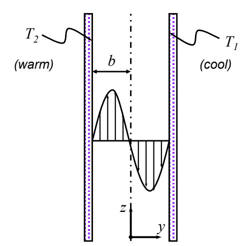
Complex Heat Transfer – Dimensional Analysis—Free Convection



$T_2 > T_1$

Solve

Energy balance:

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...

$$T(y) = \frac{T_1 - T_2}{2b} y + \frac{T_1 + T_2}{2}$$

$$T(y) = \frac{T_1 - T_2}{2b} y + \bar{T}$$

$$\rho = \bar{\rho} - \bar{\rho}\bar{\beta}(T - \bar{T})$$

$$\rho = \bar{\rho} - \bar{\rho}\bar{\beta}\left(\frac{T_1 - T_2}{2b} y \right)$$

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Complex Heat Transfer – Dimensional Analysis—Free Convection

Final Result: (free convection between two slabs)

$$v_z(y) = \frac{\bar{\rho}\bar{\beta}g(T_2 - T_1)b^2}{12\mu}\left[\left(\frac{y}{b}\right)^3 - \left(\frac{y}{b}\right)\right]$$

(see next slide for plot)

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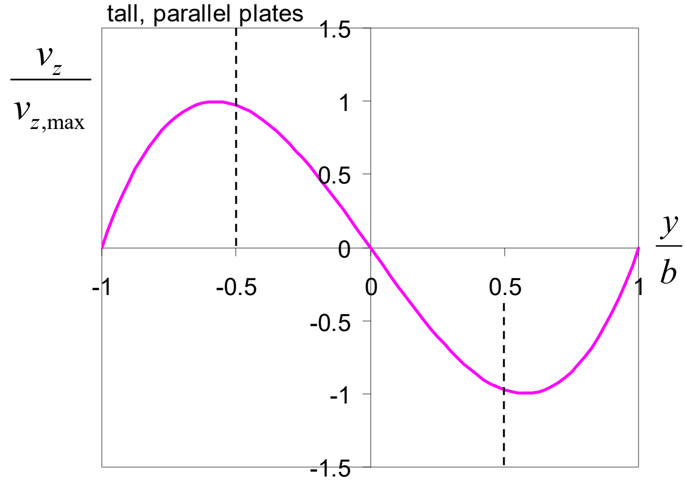
Complex Heat Transfer – Dimensional Analysis—Free Convection

Velocity profile for free convection between two wide, tall, parallel plates



(Note that the temperature maxima are not centered)

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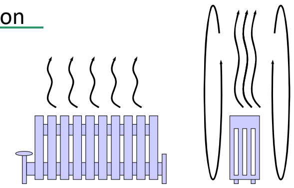
Complex Heat Transfer – Dimensional Analysis—Free Convection

Free Convection i.e. hot air rises



Engineering Modeling

- ✓ Choose an idealized problem and solve it
- **From insight obtained from ideal problem, identify governing equations of real problem**
- Nondimensionalize the governing equations; deduce dimensionless scale factors (e.g. Re, Fr for fluids)
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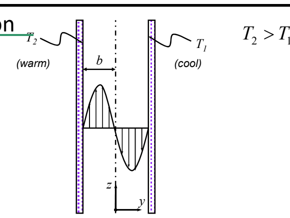
Complex Heat Transfer – Dimensional Analysis—Free Convection

Free Convection i.e. hot air rises



Mass balance:

$$\frac{\partial \rho}{\partial t} + \left(v_x \frac{\partial \rho}{\partial x} + v_y \frac{\partial \rho}{\partial y} + v_z \frac{\partial \rho}{\partial z} \right) + \rho \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) = 0$$

Momentum balance:

$$\rho \left(\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} + \right) = -\frac{\partial p}{\partial z} + \mu \left(\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2} \right) + \rho g_z$$

Energy balance:

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + S$$

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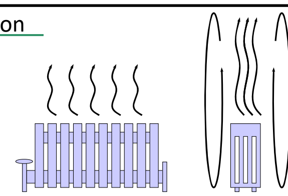
Complex Heat Transfer – Dimensional Analysis—Free Convection

Free Convection i.e. hot air rises



Engineering Modeling

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- **Nondimensionalize the governing equations; deduce dimensionless scale factors (e.g. Re, Fr for fluids)**
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Return to Dimensional Analysis…

Nondimensionalize the governing equations; deduce dimensionless scale factors

To nondimensionalized the Navier-Stokes for free convection problems, we follow the simple problem we just completed: $\rho = \rho(T), \langle v_z \rangle = 0$.

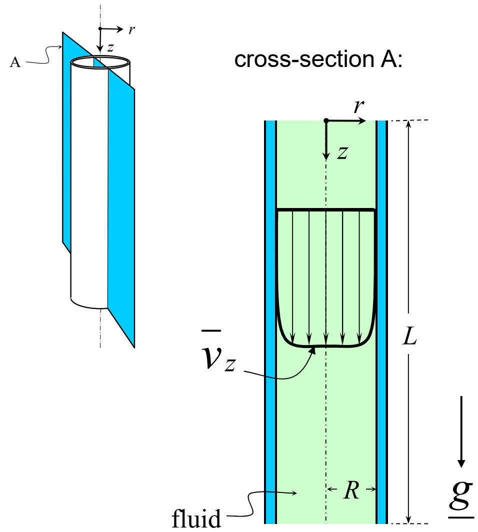
density not constant

driving the flow

$$\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v}\cdot\nabla\underline{v}\right) = -\nabla P + \mu\nabla^2\underline{v} + \rho\underline{g}$$

there was a trick for this

How did we nondimensionalized the Navier-Stokes before?



FORCED CONVECTION

EXAMPLE I: Pressure-driven flow of a Newtonian fluid in a tube:

- steady state
- well developed
- long tube

There was an average velocity used as the characteristic velocity

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FORCED CONVECTION *FORCED CONVECTION* *FORCED CONVECTION*

z-component of the Navier-Stokes Equation:

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)$$

$$=-\frac{\partial P}{\partial z}+\mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right)+\rho g_z$$

Choose:

D = characteristic length
V = characteristic velocity
D/V = characteristic time
ρV^2 = characteristic pressure

This velocity is an imposed (forced) average velocity

We do not have such an imposed velocity in natural convection

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FORCED CONVECTION FORCED CONVECTION FORCED CONVECTION

non-dimensional variables:

time:

$$t^* \equiv \frac{tV}{D}$$

position:

$$r^* \equiv \frac{r}{D}$$

$$z^* \equiv \frac{z}{D}$$

velocity:

$$v_z^* \equiv \frac{v_z}{V}$$

$$v_r^* \equiv \frac{v_r}{V}$$

$$v_\theta^* \equiv \frac{v_\theta}{V}$$

driving force:

$$P^* \equiv \frac{P}{\rho V^2}$$

$$g_z^* \equiv \frac{g_z}{g}$$

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FORCED CONVECTION FORCED CONVECTION FORCED CONVECTION

z-component of the nondimensional Navier-Stokes Equation:

$$\frac{1}{\mathrm{Re}} \qquad \frac{1}{Fr}$$

$$\frac{Dv_z^*}{Dt} = -\frac{\partial P^*}{\partial z^*} + \frac{\mu}{\rho V D}\left(\nabla^2 v_z\right)^* + \frac{gD}{V^2} g^*$$

$$\left(\nabla^2 v_z\right)^* \equiv \frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^* \frac{\partial v_z^*}{\partial r^*}\right) + \frac{1}{r^{*2}}\frac{\partial^2 v_z^*}{\partial \theta^2} + \frac{\partial^2 v_z^*}{\partial z^{*2}}$$

$$\frac{Dv_z^*}{Dt} \equiv \left(\frac{\partial v_z^*}{\partial t^*} + v_r^* \frac{\partial v_z^*}{\partial r^*} + \frac{v_\theta^*}{r^*}\frac{\partial v_z^*}{\partial \theta} + v_z \frac{\partial v_z^*}{\partial z^*}\right)$$

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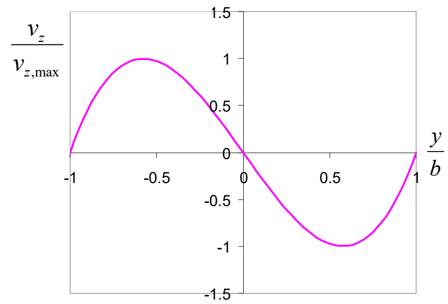
FREE CONVECTION ***FREE CONVECTION***

We do not have such an imposed velocity in natural convection

For free convection, what is the average velocity?

for forced convection we used: $v_z^* = \frac{v_z}{V}$ $V \equiv \langle v \rangle$



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FREE CONVECTION ***FREE CONVECTION***

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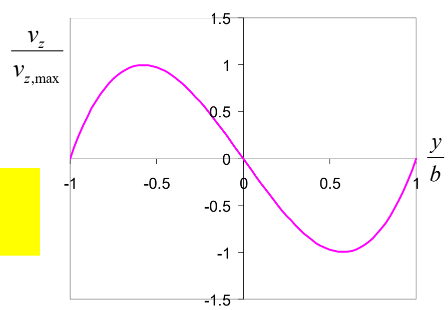

ZERO

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FREE CONVECTION ***FREE CONVECTION***

We do not have such an imposed velocity in natural convection

For free convection, what is the average velocity?
Answer: zero!

for forced convection we used: $v_z^* = \frac{v_z}{V}$ $\quad V \equiv \langle v \rangle$

For free convection $\langle v \rangle = 0$; what V should we use for free convection?

Solution: use a Reynolds-number type expression so that no characteristic velocity imposes itself (we'll see now how that works):

$$v_z^* = \frac{v_z}{V} = \frac{\bar{\rho} v_z D}{\mu} \qquad \Rightarrow V \equiv \frac{\mu}{D\bar{\rho}}$$

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FREE CONVECTION ***FREE CONVECTION*** ***FREE CONVECTION***

When non-dimensionalizing the Navier-Stokes, what do I use for ρ? (answer from idealized problem)

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z}\right) = -\frac{\partial P}{\partial z} + \mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right) + \rho g_z$$

here we use $\bar{\rho}$ because the issue is volumetric flow rate

as before, for pressure gradient we use $-\bar{\rho} g$

here we use $\rho(T)$ because the issue is driving the flow by density differences affected by gravity

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FREE CONVECTION ***FREE CONVECTION*** ***FREE CONVECTION***

non-dimensional variables:

time:

$$t^* \equiv \frac{t\mu}{D^2\bar{\rho}}$$

position:

$$r^* \equiv \frac{r}{D}$$

$$z^* \equiv \frac{z}{D}$$

velocity:

$$v_z^* \equiv \frac{v_z D\bar{\rho}}{\mu}$$

$$v_r^* \equiv \frac{\bar{\rho} v_r D}{\mu}$$

$$v_\theta^* \equiv \frac{\bar{\rho} v_\theta D}{\mu}$$

driving force:

$$T^* = \frac{T-\bar{T}}{T_2-\bar{T}}$$

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FREE CONVECTION ***FREE CONVECTION*** ***FREE CONVECTION***

SOLUTION: *z*-component of the **nondimensional** Navier-Stokes Equation (free convection):

$$\frac{Dv_z^*}{Dt} = \left(\nabla^2 v_z\right)^* + \left[\frac{gD^3\bar{\rho}^2\bar{\beta}\left(T_2-\bar{T}\right)}{\mu^2}\right]T^*$$

$(T_2-\bar{T})$: Or any appropriate characteristic ΔT

$\frac{gD^3\bar{\rho}^2\bar{\beta}(T_2-\bar{T})}{\mu^2}$ ≡Grashof number

$$\left(\nabla^2 v_z\right)^* \equiv \frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial v_z^*}{\partial r^*}\right) + \frac{1}{r^{*2}}\frac{\partial^2 v_z^*}{\partial \theta^2} + \frac{\partial^2 v_z^*}{\partial z^{*2}}$$

$$\frac{Dv_z^*}{Dt} \equiv \left(\frac{\partial v_z^*}{\partial t^*} + v_r^*\frac{\partial v_z^*}{\partial r^*} + \frac{v_\theta^*}{r^*}\frac{\partial v_z^*}{\partial \theta} + v_z\frac{\partial v_z^*}{\partial z^*}\right)$$

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FREE CONVECTION ***FREE CONVECTION***

$$\text{Gr} \equiv \frac{gD^3\bar{\rho}^2\bar{\beta}\Delta T}{\mu^2}$$

Dimensionless Equation of Motion (free convection)

$$\frac{Dv_z^*}{Dt^*} = \left(\nabla^2 v_z\right)^* + \text{Gr}T^*$$

Dimensionless Energy Equation (free convection; Re = 1)

$$\left(\frac{\partial T^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* T^*\right) = \frac{1}{\text{Pr}}\nabla^{*2}T^*$$

No Pe
No Re

$$\text{Nu} = \text{Nu}\left(T^*, \frac{L}{D}\right) \Rightarrow \text{Nu} = \text{Nu}\left(\text{Pr}, \text{Gr}, \frac{L}{D}\right)$$

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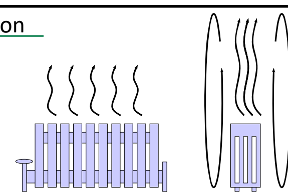
Complex Heat Transfer – Dimensional Analysis—Free Convection

Free Convection i.e. hot air rises



Engineering Modeling

- ✓ •Choose an idealized problem and solve it
- ✓ •From insight obtained from **ideal** problem, identify governing equations of **real** problem
- ✓ •Nondimensionalize the governing equations; deduce dimensionless scale factors (e.g. Re, Fr for fluids)
- **•Design experiments to test modeling thus far**
- •Revise modeling (structure of dimensional analysis, identity of scale factors, e.g. add roughness lengthscale)
- •Design additional experiments
- •Iterate until useful correlations result

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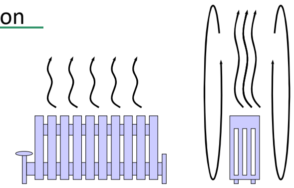
Complex Heat Transfer – Dimensional Analysis—Free Convection

Free Convection i.e. hot air rises



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- •Revise modelin… …ysis identity of scale … …thsc…
- •Design addition…
- •Iterate until use…

Done (see literature)

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Complex Heat Transfer – Dimensional Analysis—Free Convection

Literature Results:

$$\text{Gr} \equiv \frac{gD^3\bar{\rho}^2\bar{\beta}\Delta T}{\mu^2}$$

Example: Natural convection from vertical planes and cylinders

$$\text{Nu} = \frac{hL}{k} = a\text{Gr}^m\,\text{Pr}^m$$

- •*a*,*m* are given in Table 4.7-1, page 255 Geankoplis for several cases
- •*L* is the height of the plate
- •all physical properties evaluated at the **film temperature**, T_f

Free convection correlations use the film temperature for calculating the physical properties

$$T_f = \frac{T_w + T_b}{2}$$

Free convection correlations use the film temperature for calculating the physical properties

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Complex Heat Transfer – Correlations for Nu

Natural convection
Vertical planes and cylinders

$$Nu = \frac{hL}{k} = a Gr^m Pr^m$$

Physical Properties evaluated at:

$$T_f = \frac{T_w + T_b}{2}$$

•all physical properties evaluated at the **film temperature**, T_f

compare with:

Forced convection
Heat Transfer in Laminar flow in pipes

$$Nu_a = \frac{h_a L}{k} = 1.86 \left(RePr \frac{D}{L} \right)^{\frac{1}{3}} \left(\frac{\mu_b}{\mu_w} \right)^{0.14}$$

Sieder-Tate equation (laminar flow)
Ind. Eng. Chem. 28 227 (1936)

Physical Properties evaluated at:

$$\frac{T_{b,in} + T_{b,out}}{2}$$

•all physical properties (except μ_w) evaluated at the **bulk mean temperature**
•(true also for turbulent flow correlation)

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Complex Heat Transfer – Correlations for Nu

Film **temperature vs.** ***Bulk Mean*** **Temperature**

Natural convection
Vertical planes and cylinders

•all physical properties evaluated at the **film temperature**, T_f

Physical Properties evaluated at **film** temperature:

$$T_f = \frac{T_w + T_b}{2}$$

compare with:

Forced convection
Heat Transfer in Laminar flow in pipes
Sieder-Tate equation (laminar flow)

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Physical Properties evaluated at **bulk mean** temperature:

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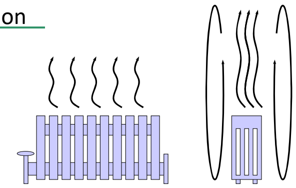
Complex Heat Transfer – Dimensional Analysis—Free Convection

Free Convection i.e. hot air rises



Engineering Modeling

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- ~~•Design additional experiments~~
- ✓ ~~•Iterate until~~ Useful correlations result

Success!
(Dimensional Analysis wins again)

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Practice Heat-Transfer Problems:

Forced Convection
Free Convection

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Practice 1: A wide, deep rectangular oven ($1.0\ ft$ tall) is used for baking loaves of bread. During the baking process the temperature of the air in the oven reaches a stable value of 100^oF. The oven side-wall temperature is measured at this time to be a stable 450^oF. Please estimate the *natural convection* heat flux from the wall per unit width. (The other contribution will be *radiation*, coming up soon)

Reference: Geankoplis Ex. 4.7-1 page 279

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Practice 2: A hydrocarbon oil enters a pipe ($0.0303\ ft$ inner diameter; $15.0\ ft$ long) at a flow rate of $80\ lb_m/h$. Steam condenses on the outside of the pipe, keeping the inside pipe surface at a constant 350^oF. If the temperature of the entering oil is 150^oF, what is temperature of the oil at the outlet of the pipe?

Hydrocarbon oil properties:

Mean heat capacity $= 0.50\,\frac{BTU}{lb_m{}^oF}$

Thermal conductivity $= 0.083\,\frac{BTU}{h\,ft\,^oF}$

Viscosity =

$6.50\ cp, 150^oF$
$5.05\ cp\ 200^oF$
$3.80 cp\ 250^oF$
$2.82\ cp\ 300^oF$
$1.95\ cp\ 350^oF$

(*forced convection* heat transfer; laminar Sieder-Tate)

Reference: Geankoplis Ex. 4.5-5 page 269

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Practice 3: Air flows through a tube ($25.4\ mm$ inside diameter, long tube) at $7.62\ m/s$. Steam condenses on the outside of the tube such that the inside surface temperature of the tube is $488.7\ K$. If the air pressure is $206.8\ kPa$ and the mean bulk temperature of the air is $(T_{out}+T_{in})/2 = 477.6\ K$, what is the steady-state heat flux to the air?

(*forced convection* heat transfer; turbulent Sieder-Tate)

Reference: Geankoplis Ex. 4.5-1 page 262

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Practice 4: Hard rubber tubing (inside radius $= 5.0mm$; outside radius $= 20.0mm$) is used as a cooling coil in a reaction bath. Cold water is flowing rapidly inside the tubing; the inside wall temperature is $274.9\ K$ and the outside wall temperature is $297.1\ K$. To keep the reaction in the bath under control, the required cooling rate is $14.65\ W$. What is the minimum length of tubing needed to accomplish this cooling rate? What length would be needed if the coil were copper?

Hard rubber properties:

Density $= 1198 \frac{kg}{m^3}$

Thermal conductivity $(0^oC) = 0.151 \frac{W}{mK}$

(steady, radial heat transfer)

Reference: Geankoplis Ex. 4.2-1 page 243, but don't do it his way—follow class methods.

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Practice 5: A cold-storage room is constructed of an inner layer of pine (thickness $= 12.7\ mm$), a middle layer of cork board (thickness $= 101.6\ mm$), and an outer layer of concrete (thickness $= 76.2\ mm$). The inside wall surface temperature is $255.4\ K$ and the outside wall surface temperature is $297.1\ K$. What is the heat loss per square meter through the walls and what is the temperature at the interface between the wood and the cork board?

Material properties:

Thermal conductivity pine $= 0.151 \frac{W}{mK}$

Thermal conductivity $\text{cork board} = 0.0433 \frac{W}{mK}$

Thermal conductivity concrete $= 0.762 \frac{W}{mK}$

(steady, rectangular heat transfer, with insulation)

Reference: Geankoplis Ex. 4.3-1 page 245, but don't do it his way—follow class methods. 47

Practice 6: A thick-walled tube (stainless steel; $0.0254\ m$ inner diameter; $0.0508\ m$ outer diameter; length $0.305\ m$) is covered with a $0.0254\ m$ thickness of insulation. The inside-wall temperature of the pipe is $811.0\ K$ and the outside surface temperature of the insulation is $310.8\ K$. What is the heat loss and the temperature at the interace between the steel and the insulation?

Material properties of stainless steel:

Thermal conductivity $= 21.63 \frac{W}{mK}$

Density $= 7861 \frac{kg}{m^3}$

Heat Capacity $= 490 \frac{J}{kg\ K}$

Material properties of insulation:

Thermal conductivity $= 0.2423 \frac{W}{mK}$

(steady, radial heat transfer, with insulation)

Reference: Geankoplis Ex. 4.3-2 page 247, but don't do it his way—follow class methods. 48

Complex Heat Transfer – Dimensional Analysis

Experience with Dimensional Analysis thus far:

✓•Flow in pipes at all flow rates (laminar and turbulent)

Solution: Navier-Stokes, Re, Fr, L/D, dimensionless wall force = f; f=f(Re, L/D)

✓•Flow around obstacles (spheres, other complex shapes

Solution: Navier-Stokes, Re, dimensionless drag= C_D; $C_D = C_D(Re)$

✓•Forced convection heat transfer from fluid to wall

Solution: Microscopic energy, Navier-Stokes, Re, Pr, L/D, heat transfer coefficient=h; h = h(Re,Pr,L/D)

✓•Natural convection heat transfer from fluid to wall

Solution: Microscopic energy, Navier-Stokes, Gr, Pr, L/D, heat transfer coefficient=h; h = h(Gr,Pr,L/D)

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✓•Natural convection heat transfer from fluid to wall

Solution: Microscopic energy, Navier-Stokes, Gr, Pr, L/D, heat transfer coefficient=h; h = h(Gr,Pr,L/D)

Now, move to last heat-transfer mechanism:

•Radiation heat transfer from solid to fluid?

Solution: ?

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Experience with Dimensional Analysis thus far:

- ✓ •Flow in pipes at all flow rates (laminar and turbulent)
  Solution: Navier-Stokes, Re, Fr, L/D, dimensionless wall force = f; f=f(Re, L/D)
- ✓ •
- ✓ • … L/D,
- ✓ •Natural convection heat transfer from fluid to wall
  Solution: Microscopic energy, Navier-Stokes, Gr, Pr, L/D, heat transfer coefficient=h; h = h(Gr,Pr,L/D)

Actually, we'll hold off on radiation and spend some time on heat exchangers and other practical concerns

Now, move to last heat-transfer mechanism:

•Radiation heat transfer from solid to fluid?
Solution: ?

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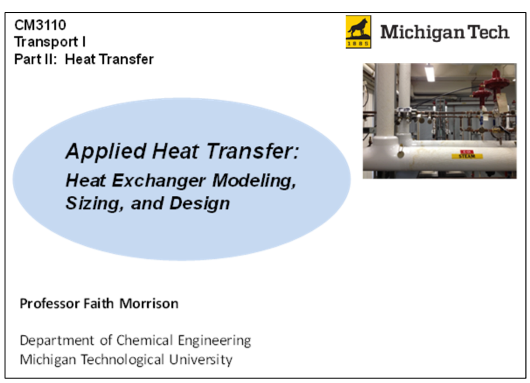
Next:



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