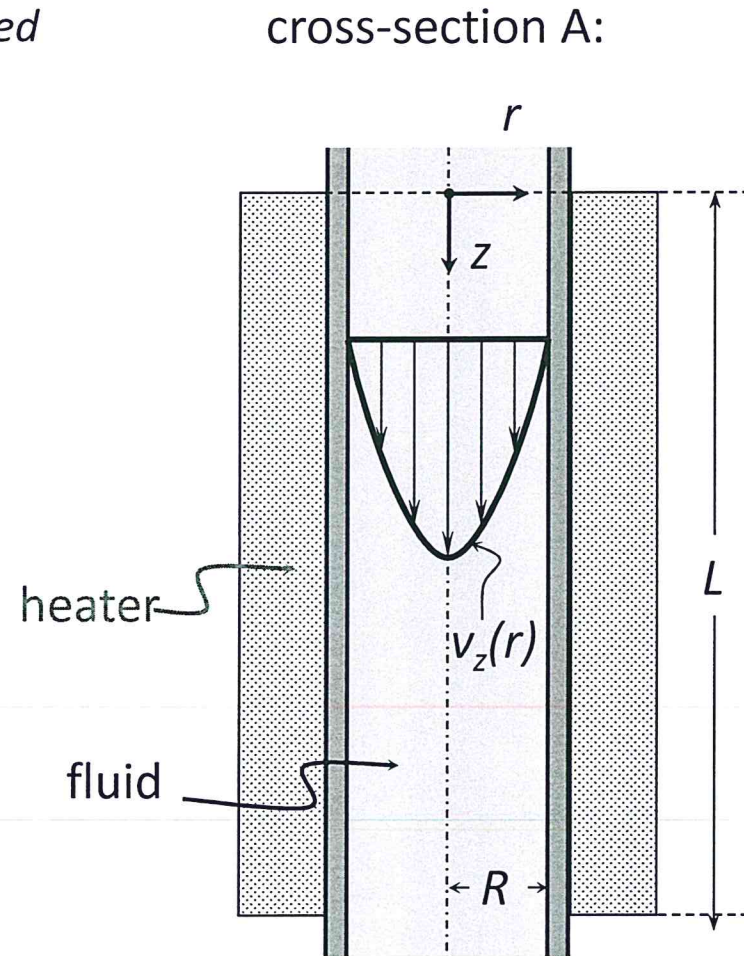
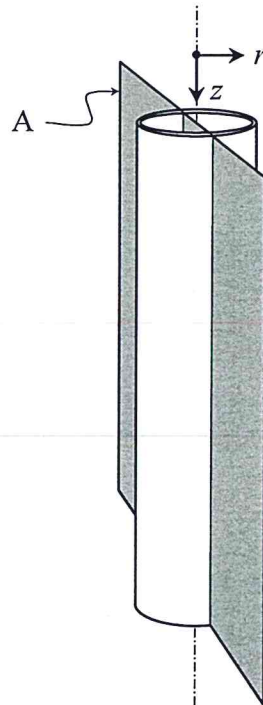


1D Heat Transfer – With Flow

Example 6: Wall heating of laminar flow. What is the steady state temperature profile in a flowing fluid in a tube if the walls are heated (constant flux, q_1/A) and if the fluid is a Newtonian fluid in laminar flow?

assume:
constant viscosity



The Equation of Energy for systems with constant k

Microscopic energy balance, constant thermal conductivity; Gibbs notation

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v \cdot \nabla T \right) = k \nabla^2 T + S_e$$

Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + S_e$$

Microscopic energy balance, constant thermal conductivity; cylindrical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + v_\theta \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right) + S_e$$

Microscopic energy balance constant thermal conductivity; spherical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + v_\theta \frac{\partial T}{\partial \theta} + v_\phi \frac{\partial T}{\partial \phi} \right) = k \left(\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \phi^2} \right) + S_e$$

Handwritten notes:
 $\rho \hat{C}_p v_i \frac{\partial T}{\partial z} = k \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + k \frac{\partial^2 T}{\partial z^2}$
 $T = T(r, z)$
no v_θ or v_ϕ terms

Reference: F. A. Morrison, "Web Appendix to An Introduction to Fluid Mechanics," Cambridge University Press, New York, 2013 (pages:mtu.edu/~fmorriso/cm310/FMWWebAppendixDMicroEBalanceMorrison.pdf). This worksheet is on the web at pages.mtu.edu/~fmorriso/cm310/energy.pdf.

Laminar flow in a tube:

(3)

$$U_z = U_{z,0} \left(1 - \frac{r^2}{R^2}\right)$$

$$U_{z,0} = \frac{\Delta P R^2}{4\mu L}$$

Can we "sketch" any more terms?

Candidate: $\left(\frac{\partial^2 T}{\partial z^2}\right)$ axial conduction

If k not too big, neglect.

BC: $r=0$ $T = \text{finite}$

$r=R$ $\frac{\partial T}{\partial r} = \text{constant} = -k \frac{\partial T}{\partial r}$

$z=0$ $T = T_0$

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