1.	/20
2.	/20
3.	/20
4.	/20
5.	/20

Exam 2

CM3120

Wednesday 3 March 2021

Name:

Rules:

- Closed book, closed notes.
- Two-page 8.5" by 11" study sheet allowed, double sided; you may use a calculator; you may not search the internet or receive help from anyone.
- Please text clarification questions to Dr. Morrison 906-487-9703. I will respond if I am able.
- All work submitted for the exam must be your own.
- Do not discuss the contents of the exam with anyone before midnight Wednesday 3 March 2021.
- ***Please copy the following Honors Pledge onto the first page of your exam submission and sign and date your agreement to it.***

Honor's Pledge:

On my honor, I agree to abide by the rules stated on the exam sheet.

Signature ______________________________

Date ____________________

Exam Instructions:

i. You may work on the exam for up to two hours and 30 minutes (150 minutes).
ii. Please be neat. Only neat answers will be granted partial credit. Please use a dark pencil or pen so that your work is readable once scanned.
iii. Significant figures always count.
iv. Please box your final answers.
v. Submit your work as a single PDF file; put your name on every page. (Genius Scan is a free app that can create a PDF from photos taken by your phone). If you take photos of your work, insert them into Word or Google Docs and create a PDF.
vi. Submit your exam study sheet as a separate PDF file; put your name on the first page (at a minimum)

1. (20 points) For the scenario described in the box below, answer the questions that follow. You do not need to solve the scenario; just answer the two questions.

Scenario: If a cube of lead of (volume= a^3) at uniform temperature T_0 is dropped into a large, stirred reservoir of oil (material properties known) at bulk temperature T_b, what is the temperature of the lead cube as a function of time?

   a. What is the formula for the Biot number for this scenario? Identify all quantities in the formula, including whether the quantity is a property of the lead or of the oil.
   b. The Biot number may be thought of as the ratio of two resistances. In words, what are these two resistances and what is the formula for each?

2. (20 points) The Sieder & Tate equation for Nusselt number Nu for turbulent flow in pipes is given by

$$\mathrm{Nu}_{lm} = \frac{h_{lm}D}{k} = 0.027\ \mathrm{Re}^{0.8}\mathrm{Pr}^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

Answer the following two questions (part a and part b):

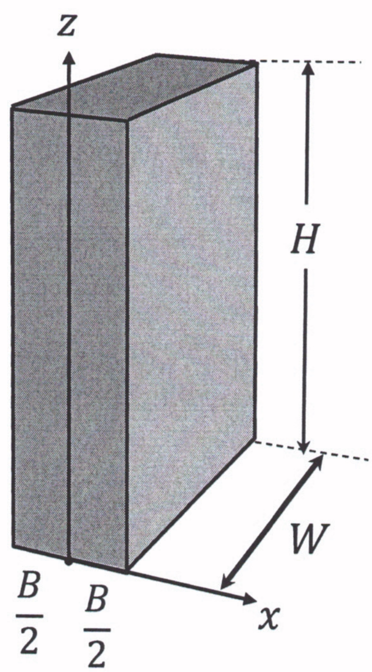
   a. Identify following symbols in this equation and provide SI units for the quantity:

Symbol	Name/formula	SI units
D		
k		
Re		
Pr		
μ_b/μ_w		

   b. Describe a situation when you would use this equation.

3. (20 points) What is the partial differential equation that we need to solve in order to determine the temperature as a function of time and position for the situation described in the box below? Indicate the assumptions you used to eliminate terms from the general microscopic energy balance. I am not asking for the boundary or initial conditions; I am only asking for the simplified partial differential equation and the reason for eliminating any terms. Use the coordinate system shown; y is the direction in the depth direction.

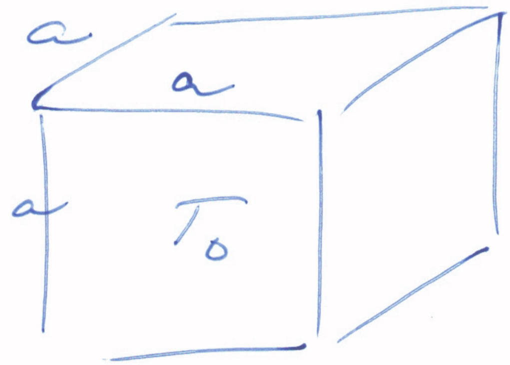
A tall and deep slab (height H, depth W, thickness B) is initially at uniform temperature T_0. Suddenly, a fan is switched on creating a convective stream of air on all surfaces at a bulk air temperature of T_b. What is the temperature distribution as a function of time and position through the thickness direction of the slab?



4. (20 points) A large, thick plate of lead is initially at a uniform temperature of $300.^o\ C$. One broad surface is suddenly exposed to a liquid coolant at $20.^o\ C$. The heat transfer coefficient in the coolant in this scenario is $1.00 \times 10^2\ W/m^2K$. At a distance $5.5\ cm$ into the plate from the surface exposed to coolant, calculate the lead temperature after three minutes.
5. (20 points) A copper sphere (outer diameter $2.0cm$) initially at a uniform temperature of 32^oC is placed in a precisely constructed spherical oven chamber (chamber inner diameter is $2.0cm$) that instantly and precisely holds the outer surface of the enclosed sphere at 82^oC. How long does it take for the center of the copper sphere to reach 81^oC? Show your supporting calculations.

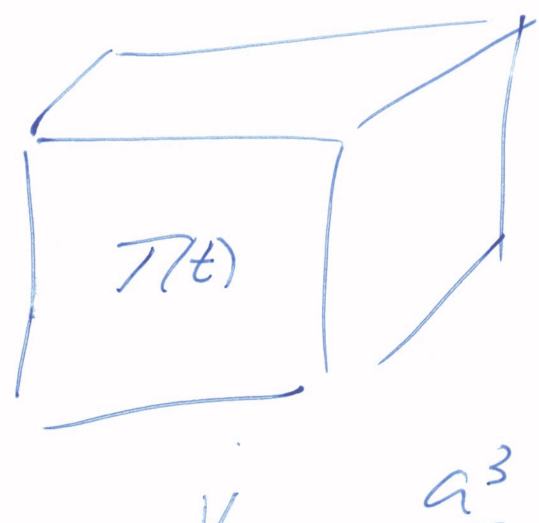
SOLUTION
EXAM 2
CM3120
3 MAR 2021
Morrison

1)



"Suddenly,"

- Oil
- Stirred
- T_b, h



a) $Bi = \dfrac{hD}{k_{lead}}$ — $\dfrac{V}{A_{surf}} = \dfrac{a^3}{6a^2} = \dfrac{a}{6}$ if $Bi < 0.1$

thermal conductivity of lead

b.) $Bi = \dfrac{D/k}{1/h} = \dfrac{\text{internal}}{\text{external}}$ resistances.

heat xfer coef $\rightarrow 1/h$

2)

D	pipe diameter	m
k	fluid thermal conductivity	$\frac{W}{mK}$
Re	Reynolds # $= \frac{\rho V D}{\mu}$	dimensionless

ρ fluid density
μ fluid viscosity
V average fluid velocity
D pipe diameter

Pr	Prandtl #	dimensionless

$$Pr = \frac{C_p \mu}{k}$$

C_p - fluid heat capacity
μ - fluid viscosity
k - fluid thermal conductivity

μ_b/μ_w	fluid μ at T_b / μ at T_{wall}	dimensionless

2b: We would use the Sieder & Tate eqn to determine heat xfer coef h for heat xfer in turbulent pipe flow. The eqn is a data correlation. It represents experimental data.

$$\left|\frac{q_r}{A}\right| = h\left|\left(T_b - T_{wall}\right)_{lm}\right|$$

3)

4

The **Equation of Energy** for systems with **constant k**

Microscopic energy balance, constant thermal conductivity; Gibbs notation

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S_e$$

no reaction
no elec current

Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \cancel{v_x \frac{\partial T}{\partial x}} + \cancel{v_y \frac{\partial T}{\partial y}} + \cancel{v_z \frac{\partial T}{\partial z}} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \cancel{\frac{\partial^2 T}{\partial y^2}} + \cancel{\frac{\partial^2 T}{\partial z^2}} \right) + \cancel{S_e}$$

deep

tall

Microscopic energy balance, constant thermal conductivity; cylindrical coordinates

v=0

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right) + S_e$$

the domain is a solid

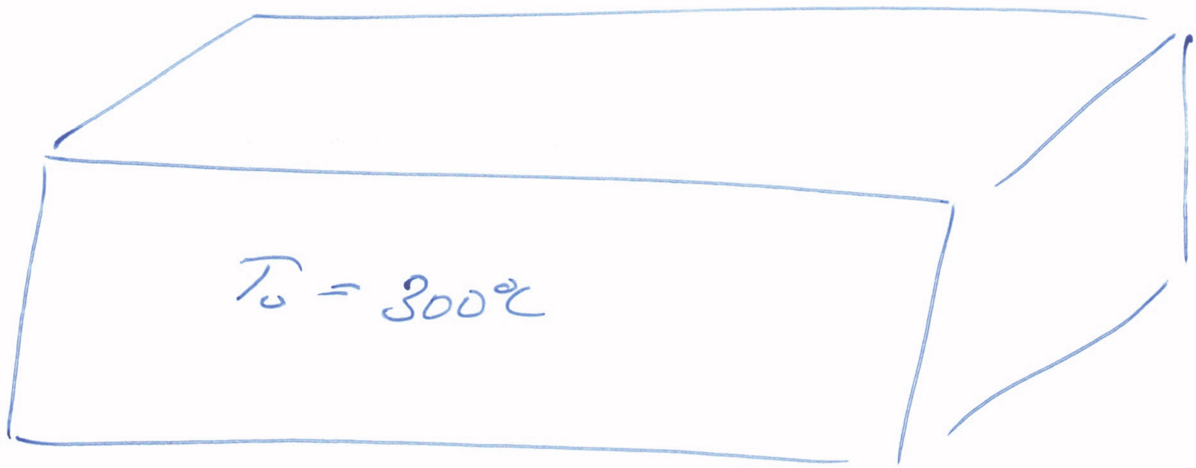
Microscopic energy balance, constant thermal conductivity; spherical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial T}{\partial \phi} \right)$$
$$= k \left(\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \phi^2} \right) + S_e$$

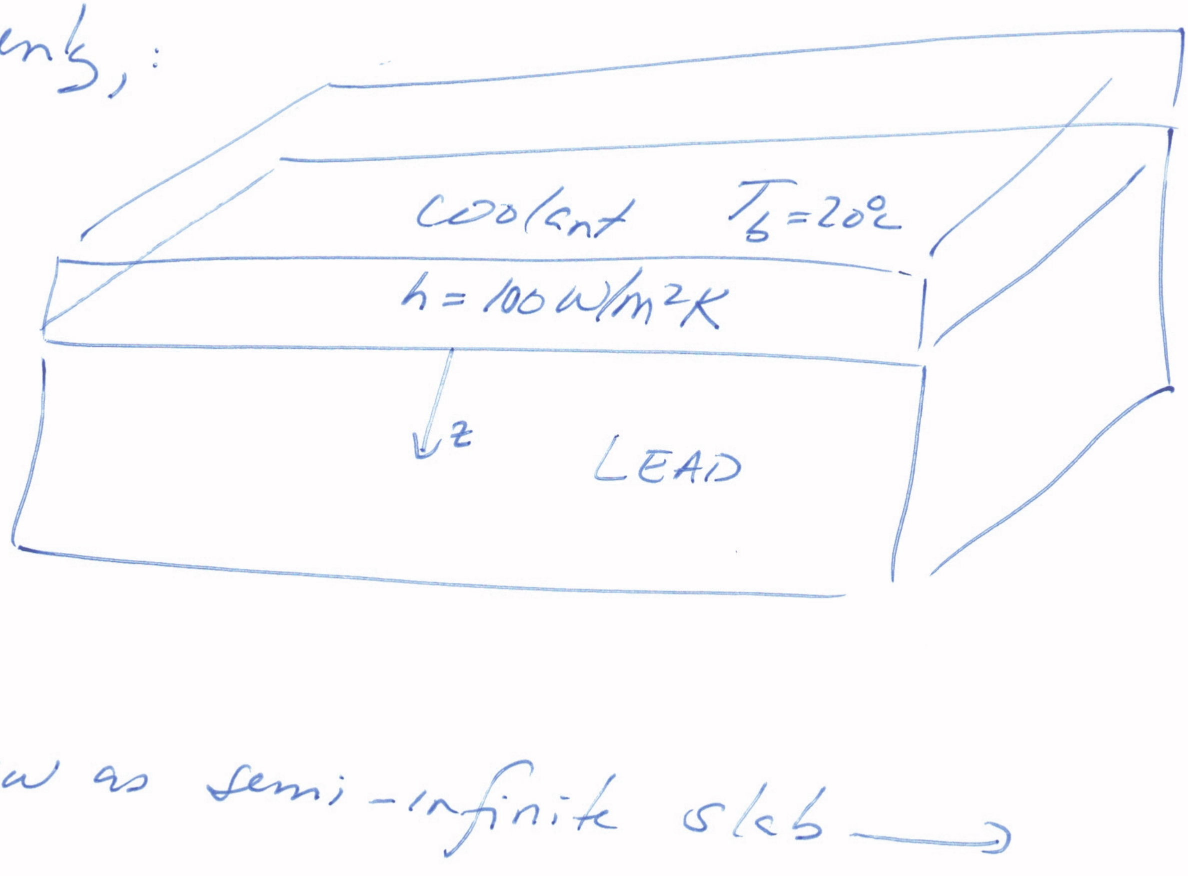
$$\boxed{\frac{\partial T}{\partial t} = \underbrace{\frac{k}{\rho \hat{C}_p}}_{\alpha} \frac{\partial^2 T}{\partial x^2}}$$

Reference: F. A. Morrison, "Web Appendix to *An Introduction to Fluid Mechanics*," Cambridge University Press, New York, 2013 (*pages.mtu.edu/~fmorriso/cm310/IFMWebAppendixDMicroEBalanceMorrison.pdf*). This worksheet is on the web at *pages.mtu.edu/~fmorriso/cm310/energy.pdf*.

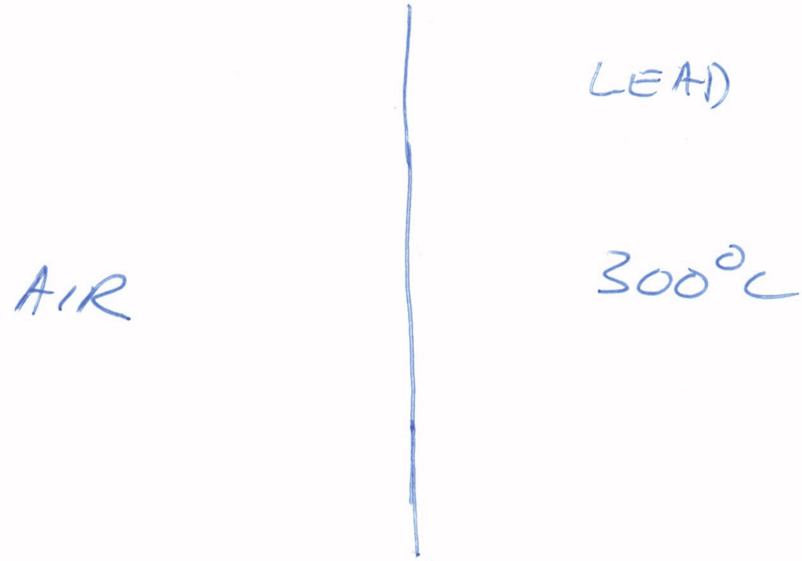
4)



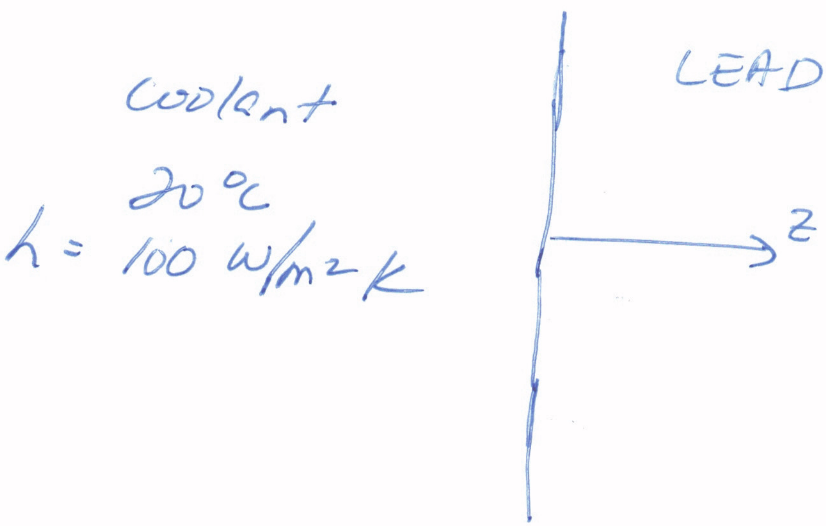
Suddenly:



Redraw as semi-infinite slab →



"Suddenly"



What is:

$$T(t,z) = T(3\,\text{min}, 5.5\,\text{cm})$$

Use semi infinite slab →

7

$$\frac{h\sqrt{\alpha t}}{k} \qquad \zeta = \frac{x}{2\sqrt{\alpha t}}$$

physical properties:

$$\boxed{\alpha = 2.1\times10^{-5}\,\frac{m^2}{s}} = \frac{k}{\rho C_p} = \frac{31\,\frac{\cancel{W}}{m\cancel{K}}\,\frac{J/s}{\cancel{W}}}{\left(11370\,\frac{\cancel{kg}}{m^3}\right)\left(0.130\times10^{3}\,\frac{J}{\cancel{kg}\cancel{K}}\right)}$$

$$\boxed{k = 31\,\frac{W}{mK}}$$

$$\beta = \frac{h\sqrt{\alpha t}}{k} = \frac{\left(\frac{100\cancel{W}}{m^2K}\right)\left(2.1\times10^{-5}\,\frac{m^{\cancel{2}}}{\cancel{s}}\cdot 3\times60\cancel{s}\right)^{\frac{1}{2}}}{31\,\frac{\cancel{W}}{\cancel{m}\cancel{K}}}$$

$$\boxed{\beta = 0.20}$$

$$\zeta = \frac{x}{2\sqrt{\alpha t}} = \frac{5.5\times10^{-2}\,\cancel{m}}{2\sqrt{2.1\times10^{-5}\,\frac{m^{\cancel{2}}}{\cancel{s}}\cdot180\cancel{s}}}$$

$$\boxed{\zeta = 0.45}$$

at $z = 5.5$ cm, $t = 180$ s:

8

$$\frac{T-T_0}{T_1-T_0} = 0.08 = \left(\frac{T-300}{20-300}\right)$$

$$\boxed{T = 278\ ^\circ C}$$

or

$$\boxed{280^\circ C}$$

OR If you wrote the full soln in your shee[t]

$$\frac{T-T_0}{T_1-T_0} = \operatorname{erfc}\zeta - e^{\beta(2\zeta+\beta)} \operatorname{erfc}(\zeta + \beta$$

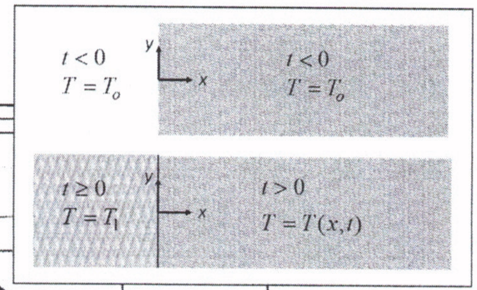
from calculator (Excel):

$$278\ ^\circ C$$

$$= \boxed{280\ ^\circ C}$$

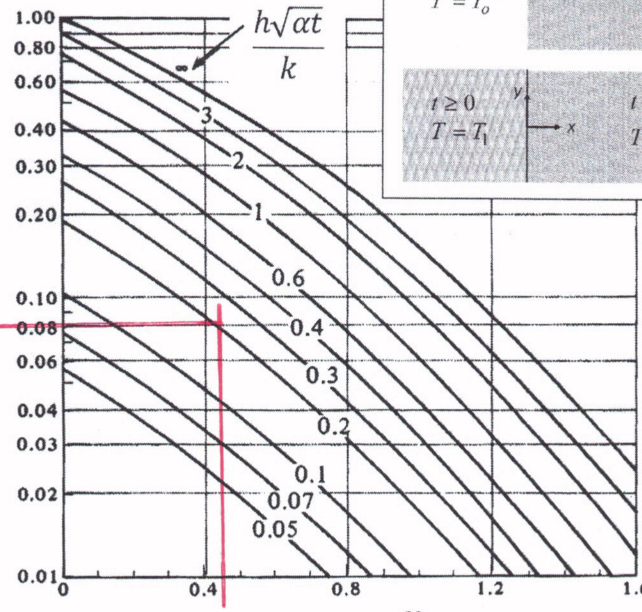
Problem 4

1D Heat Transfer: Unsteady State Heat Conduction in a Semi-Infinite Slab



$$1 - Y = \left(\frac{T - T_0}{T_1 - T_0}\right)$$

0.08



Geankoplis 4th ed.,
Figure 5.3-3, page 364

0.45 $\zeta = \frac{x}{2\sqrt{\alpha t}}$

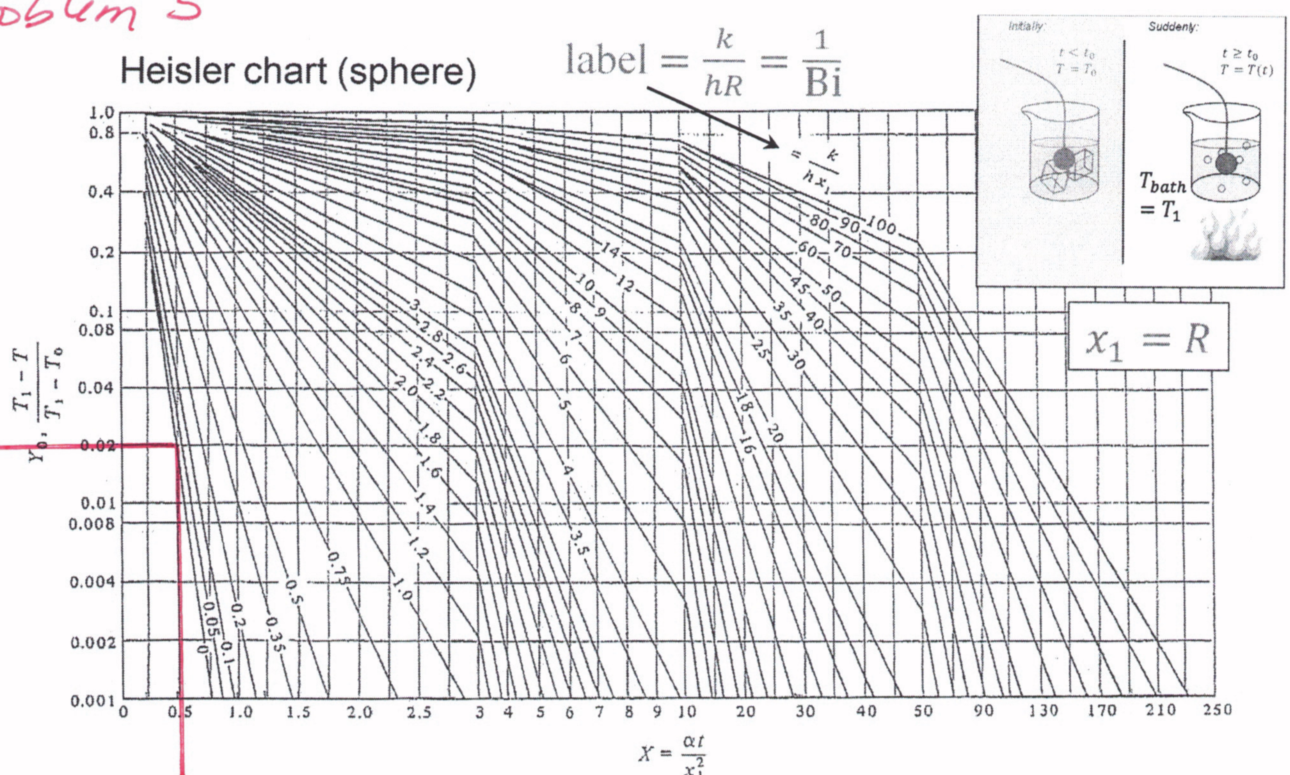
Problem 5

Heisler chart (sphere)

$$\text{label} = \frac{k}{hR} = \frac{1}{\text{Bi}}$$

$x_1 = R$

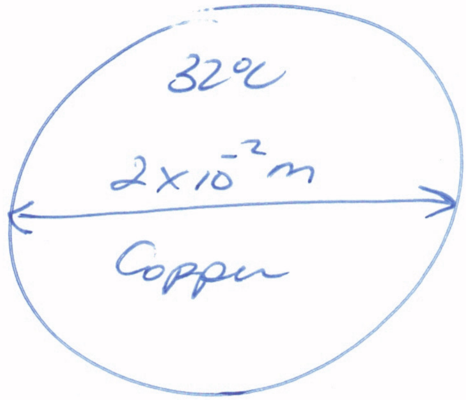
0.02



Fo = 0.5

FIGURE 5.3-10. *Chart for determining the temperature at the center of a sphere for unsteady-state heat conduction. [From H. P. Heisler, Trans. A.S.M.E., 69, 227 (1947). With permission.]* *From Geankpolis, 4th edition, page 374*

5.



Suddenly,

Wall temperature specified

$\Rightarrow h \to \infty$

Looks like Heisler chart w/ $h \to \infty$

$$\frac{\partial T}{\partial t} = \alpha \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right)$$

$T = T_0$ at $t = 0$ $\forall r$

$$Y_0 = \frac{T_1 - T_{center}}{T_1 - T_0}$$

$$= \frac{82 - 81}{82 - 32} = 0.02$$

$$m = \frac{k}{hR} = 0$$ $h \to \infty$ when $T_0 = T_{wall}$

From Heisler Chart:

$$Y = 0.02,\ m = \frac{k}{hR} = 0 \Rightarrow \boxed{\frac{\alpha t}{R^2} = 0.5}$$

$$t = \frac{(0.5)(1 \times 10^{-2}\,m)^2}{\underbrace{1.11 \times 10^{-4}\ \frac{m^2}{s}}_{= \alpha_{copper}}}$$

$$\boxed{t = 0.45\ s = \boxed{0.5\ s}}$$