Dimensional Analysis

These numbers tell us about the relative importance of the terms they precede in the governing equations.

Dimensionless numbers from the
Equations of Change *(microscopic balances)*

momentum

Non-dimensional Navier-Stokes Equation

$$\left(\frac{\partial v_z^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* v_z^*\right) = -\frac{\partial P^*}{\partial z^*} + \frac{1}{\mathrm{Re}}\left(\nabla^{*2} v_z^*\right) + \frac{1}{\mathrm{Fr}} g^*$$

Re – Reynolds
Fr – Froude

energy

Non-dimensional Energy Equation

$$\left(\frac{\partial T^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* T^*\right) = \frac{1}{\mathrm{RePr}}\left(\nabla^{*2} T^*\right) + S^*$$

Pe – Péclet$_h$ = RePr
Pr – Prandtl

mass

Non-dimensional Continuity Equation (species A)

$$\left(\frac{\partial x_A^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* x_A^*\right) = \frac{1}{\mathrm{ReSc}}\left(\nabla^{*2} x_A^*\right)$$

Pe – Péclet$_m$ = ReSc
Sc – Schmidt

Oops! This is dimensionless $\underline{v}$, NOT molar average velocity; sorry!

ref: BSL1, p581, 644

Dimensionless Numbers

Dimensionless numbers from the
Equations of Change

$$\text{Re} - \text{Reynolds} = \frac{\rho V D}{\mu} = \frac{VD}{\nu}$$

$$\text{Fr} - \text{Froude} = \frac{V^2}{gD}$$

$$\text{Pe} - \text{Péclet}_h = \text{RePr} = \frac{\hat{C}_p \rho V D}{k} = \frac{VD}{\alpha}$$

$$\text{Pe} - \text{Péclet}_m = \text{ReSc} = \frac{VD}{\mathcal{D}_{AB}}$$

These numbers tell us about the **<u>relative importance of the terms</u>** they precede in the microscopic balances (*scenario properties*).

$$\text{Pr} - \text{Prandtl} = \frac{\hat{C}_p \mu}{k} = \frac{\nu}{\alpha}$$

$$\text{Sc} - \text{Schmidt} = \text{LePr} = \frac{\mu}{\rho \mathcal{D}_{AB}} = \frac{\nu}{\mathcal{D}_{AB}}$$

$$\text{Le} - \text{Lewis} = \frac{\alpha}{\mathcal{D}_{AB}}$$

These numbers compare the **<u>magnitudes of the diffusive transport coefficients</u>** $\nu, \alpha, \mathcal{D}_{AB}$ (*material properties*).

Transport coefficients

Dimensional Analysis

These numbers are defined to help us build **transport data correlations** based on the fewest number of grouped (dimensionless) variables (*scenario property*).

Dimensionless numbers from the
Engineering Quantities of Interest

momentum

Dimensionless Force on the Wall (Drag)

$$f = \frac{1}{\pi}\frac{D}{L}\frac{1}{\mathrm{Re}}\int_0^{\frac{L}{D}}\int_0^{2\pi}\left(\frac{\partial v_z^*}{\partial r^*}\right)\Bigg|_{r^*=\frac{1}{2}} d\theta dz^*$$

(Fanning)

f — Friction Factor

$\frac{L}{D}$ — Aspect Ratio

$$f = \frac{\mathcal{F}_{drag}}{\left(\frac{1}{2}\rho V^2\right) A_c}$$

energy

Newton's Law of Cooling

$$Nu = \frac{1}{2\pi L/D}\int_0^{2\pi}\int_0^{L/D} -\frac{\partial T^*}{\partial r^*}\Bigg|_{r^*=1/2} dz^* d\theta$$

Nu — Nusselt

$\frac{L}{D}$ — Aspect Ratio

$$\mathrm{Nu} = \frac{hD}{k}$$

$$\mathrm{St}_h = \frac{h}{\rho V \hat{C}_p} = \frac{\mathrm{Nu}}{\mathrm{RePr}}$$

mass xfer

Dimensionless Mass Transfer Coefficient

$$Sh = \frac{1}{2\pi}\frac{D}{L}\int_0^{\frac{L}{D}}\int_0^{2\pi}\left(-\frac{\partial x_A^*}{\partial r^*}\right)\Bigg|_{r^*=\frac{1}{2}} d\theta dz^*$$

Sh — Sherwood

$\frac{L}{D}$ — Aspect Ratio

$$\mathrm{Sh} = \frac{k_c D}{\mathcal{D}_{AB}}$$

$$\mathrm{St}_m = \frac{k_c}{V} = \frac{\mathrm{Sh}}{\mathrm{ReSc}}$$

St — Stanton

momentum
energy
mass

Dimensionless Numbers

$\text{Re} - \text{Reynolds} = \frac{\rho V D}{\mu} = \frac{VD}{\nu}$

$\text{Fr} - \text{Froude} = \frac{V^2}{gD}$

$\text{Pe} - \text{Péclet}_h = \text{RePr} = \frac{\hat{C}_p \rho V D}{k} = \frac{VD}{\alpha}$

$\text{Pe} - \text{Péclet}_m = \text{ReSc} = \frac{VD}{\mathcal{D}_{AB}}$

These numbers from the governing equations tell us about the relative importance of the terms they precede in the microscopic balances (***scenario*** *properties*).

$\text{Pr} - \text{Prandtl} = \frac{\hat{C}_p \mu}{k} = \frac{\nu}{\alpha}$

$\text{Sc} - \text{Schmidt} = \text{LePr} = \frac{\mu}{\rho \mathcal{D}_{AB}} = \frac{\nu}{\mathcal{D}_{AB}}$

$\text{Le} - \text{Lewis} = \frac{\alpha}{\mathcal{D}_{AB}}$

These numbers compare the magnitudes of the diffusive transport coefficients ν, α, D_{AB} (***material*** *properties*).

$f - \text{Friction Factor} = \frac{\mathcal{F}_{drag}}{\left(\frac{1}{2}\rho V^2\right) A_c}$

$\text{Nu} - \text{Nusselt} = \frac{hD}{k}$

$\text{Sh} - \text{Sherwood} = \frac{k_c D}{\mathcal{D}_{AB}}$

These numbers are defined to help us build transport data correlations based on the fewest number of grouped (dimensionless) variables (***scenario*** *properties*).

$\text{St}_h = \text{Nu}/\text{Pe}_h, \text{St}_m = \text{Sh}/\text{Pe}_m - \text{Stanton}$