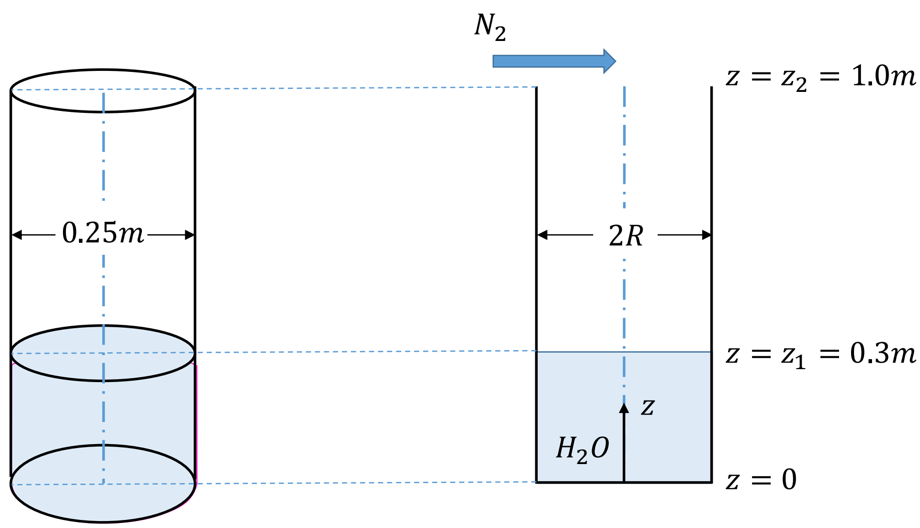
HW4

Michigan Technological University
Chemical Engineering

Module	2021 Problems	Topic
4	1	work with 1D evaporation; calculate rate; empty tank pseudo ss
4	2	**Micro SAMB: molar fluxes in EMCD**
4	3	**Micro SAMB: penetration model (homogeneous rxn)**
4	4	Describe diffusion limitations in reaction
4	5	UO: McCabe Thiele calculation
4	6	UO: calculate shaft work
4	7	calc kc for EMCD
4	8	equimolar counter diffusion; calculate molar flux from points
4	9	calc kc for film model of mass transfer (derive)
4	10	given functionality, identify the dimensionless number
4	11	devise mass xfer problem solvable with Heisler chart
4	12	*Macro SAMB*: dissolution of benzoic acid in packed bed
4	13	*Macro SAMB*: calculate mass xfer coef for air humidification
4	14	Describe difference between various coefficients
4	15	hydrogen sulfide stripping; overall mass xfer coeff
4	16	Height of an absorption column, HTU, NTU
4	17	Absorption column, minimum flow rate ratio

1. (Problem discussed during diffusion lecture 1 and parts a, b and c were in HW3, problem 13)

   Water (40^oC, $1.0\ atm$) slowly and steadily evaporates into nitrogen (40^oC, $1.0\ atm$) from the bottom of a cylindrical tank ($0.25m$ diameter) as shown in the figure below. A stream of dry nitrogen flows slowly past the open tank. The mole fraction of water in the gas at the top opening of the tank is 0.02. The geometry is as shown in the figure. What is water mole fraction as a function of vertical position? You may assume ideal gas properties. What is the rate of water evaporation?



   d. **(stretch)** How long will it take for the tank to empty? The initial height of the water is h_0. Answer: $t_{empty} = \frac{h_0 \rho_{A,l}\, x_{B_{ln}}}{M_A c D_{AB}(x_{A1} - x_{A2})}\left(z_2 - \frac{h_0}{2}\right)$, where $x_{B_{ln}} \equiv \frac{x_{B2} - x_{B1}}{\ln(x_{B2}/x_{B1})}$, c is moles mixture /volume mixture in the gas phase, $\rho_{A,l}$ is the density of liquid water. Hint: this is solved with a quasi steady state approach.
   e. **(stretch)** How fast is the interface moving during steady state evaporation? Answer: $1.6 \times 10^{-7} cm/s$
2. A distillation column is separating two components A and B at steady state. In the vapor phase the two components are moving in equimolar counter diffusion (Example 4 in class, Mod4, lecture I). What are the molar fluxes of A and B? What is the concentration distribution in the region of the equimolar counter diffusion? Answer: $N_{Az} = -N_{Bz} = \frac{D_{AB}}{RT(z_2 - z_1)}(p_{A1} - p_{A2})$
3. A gas absorption column is operating at steady state (Example 5). The gas stream is composed of component A and an inert carrier gas I. The liquid stream is chemical absorbent B. Component A diffuses across the gas-liquid interface until it reacts with B. What are the molar

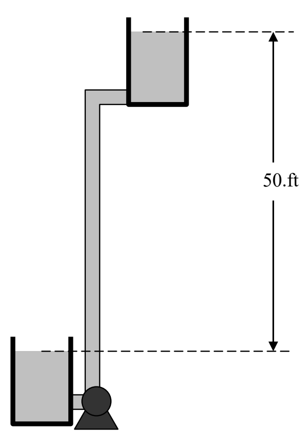
fluxes of A and B? What is the concentration distribution in the region in which A diffuses into liquid B? Set the problem up including boundary conditions. The answer can be found in the literature (see p506 in Welty et al.). Confirm that the answer given is a solution to the differential equation you find. Answers: $c_A = c_{A0}\cosh\left(z\sqrt{k/D_{AB}}\right) - \frac{c_{A0}\sinh\left(z\sqrt{k_1/D_{AB}}\right)}{\tanh\left(\delta\sqrt{k_1/D_{AB}}\right)}$,

$$N_{A,z} = -\mathcal{D}_{AB}\frac{dc_A}{dz} = -\mathcal{D}_{AB}\left[c_{A0}\sqrt{\frac{k_1}{\mathcal{D}_{AB}}}\sinh\left(z\sqrt{\frac{k_1}{\mathcal{D}_{AB}}}\right) - \frac{c_{A0}\sqrt{\frac{k_1}{\mathcal{D}_{AB}}}\cosh\left(z\sqrt{\frac{k_1}{\mathcal{D}_{AB}}}\right)}{\tanh\left(\delta\sqrt{\frac{k_1}{\mathcal{D}_{AB}}}\right)}\right]$$

, where δ is the penetration depth.

4. An irreversible, instantaneous chemical reaction $2A \rightarrow B$) takes place at a catalyst surface in a reactor (see Example 3, lecture VI, module 3). How might mass transfer affect the observed rate of reaction? Use the solution to the example in your discussion.

5. A distillation column with a total condenser and a partial reboiler is separating an ethanol-water mixture. Feed is 20.0 mole% ethanol with quality $q = 1.13$. Feed rate is $1.00 \times 10^3\ kmol/h$, and the feed and column temperature and pressure are $80^oF = 26.7C$ and 1.0 atm. Distillate is 80.0 mole% ethanol, and bottoms is 2.0 mole% ethanol. External reflux ratio is 5/3. Reflux is a saturated liquid, and constant molal overflow may be assumed. Find the optimum feed plate location and the total number of equilibrium stages required to produce this separation. Use the McCabe Thiele method and count off your trays beginning at the bottom of the column. The equilibrium data are given at the end of this homework; we recommend that you use Excel or some other appropriate software for the calculations and plotting.
6. What is the shaft work $W_{s,on}$ done by a pump moving liquid water ($25^oC, 1.00\ atm$) at 10. lb_m/s in the apparatus shown below? All piping is 4-inch schedule 40 pipe, and you may neglect the friction in the apparatus. The water flows from one open tank to another open tank. Please give your answer in $ft \cdot lb_f/s$. Answer: $5.0 \times 10^2 ft\ lb_f/s$.



7. Equimolar counter diffusion (EMCD) is a scenario that we can solve for the composition distribution using the film model. Draw a sketch of EMCD with the film model applied. Derive $x_A(z)$ for steady EMCD. What is the mass-transfer coefficient for EMCD? Answer: D_{AB}/δ.
8. Water (species A, gas, dilute) and air (species B) form a binary gas mixture in which steady equimolar counter diffusion is occurring (one dimensional, ideal gas, $298K$). At one point in the diffusion space, the concentration of A is $0.0050\ kmol/m^3$, and the concentration of B is $0.0360\ kmol/m^3$. A distance $\Delta z = 1.0 \times 10^{-2} m$ away from this first point, the concentration of A is $0.0045\ kmol/m^3$, and the concentration of B is $0.0365\ kmol/m^3$. What is the molar flux N_{Az} in this region? Please give your answer in $kmol/m^2s$. Answer: $1.3 \times 10^{-7}\ mol/cm^2s$.
9. What is k_c for the film model of mass transfer? Derive your answer from the predictions of the film model. What are the units of k_c? Answer: $k_c = \frac{\mathcal{D}_{AB}}{\delta} \frac{\ln\left(\frac{1-x_{A1}}{1-x_{A2}}\right)}{(x_{A1}-x_{A2})} = \frac{P\mathcal{D}_{AB}}{\delta\left(p_{B_{lm}}\right)}$.
10. Please identify the following dimensionless numbers and indicate where they appear:
    a. Determines the importance of mass diffusion relative to mass convection.
    b. Determines the importance of inertial forces relative to viscous forces.
    c. Compares the magnitude of a material's momentum diffusion and its mass diffusion.
    d. Dimensionless mass transfer coefficient.
    e. Dimensionless drag on the wall of a pipe.
    f. Dimensionless heat transfer coefficient.
11. What mass-transfer problem could we solve using the Heissler chart for temperature at the center of a rectangular slab? Explain how you know this would be appropriate.
12. Two-centimeter diameter spheres of benzoic acid (soluble in water) are packed into a bed as shown in the figure in Example 8 (lecture IV) (length $= 1.0 \times 10^2 cm$). The spheres have 23 cm^2 of surface area per cm^3 volume of bed. What is the mass transfer coefficient when pure water flowing in ("superficial velocity"$= 5.0\ cm/s$) exits 62% saturated with benzoic acid? Answer: $k_c = 2.1 \times 10^{-3} cm/s$.
13. Bone dry air and liquid water (water volume $= 0.80\ liters$) are introduced into a closed container (cross sectional area $= 150\ cm^2$; total volume $= 19.2\ liters$). Both air and water are at 25^oC throughout this scenario. Three minutes after the air and water are placed in the closed container, the vapor is found to be 5.0% saturated with water vapor. What is the mass transfer coefficient for the water transferring from the liquid to the gas? How long will it take for the gas to become 90% saturated with water? Answer: $k_c = 3.4 \times 10^{-2} cm/s$, 2.3 $hours$.
14. Compare and contrast the following transfer coefficients. Provide a sketch to illustrate the applicability and assumptions that accompany each. Where would you get a value for each of these if you need it? When would you need it?
    g. Heat transfer coefficient h and overall heat transfer coefficient U
    h. Mass transfer coefficient k_c and overall mass transfer coefficient K_L

15. (WRF) A liquid stripping process (20^oC, $1.5\ atm$) is used to transfer hydrogen sulfide (H_2S = species A) dissolved in water into an air stream. At a particular elevation in the column, the mole fraction of H_2S in the gas phase is 0.010 and the mole fraction of H_2S in the liquid phase is 6.0×10^{-5}. The individual mass-transfer coefficients are $k_x = 0.30\ kmol/m^2\ s$ for the liquid film and $k_y = 4.5 \times\ 10^{-3}\ kmol/m^2\ s$ for the gas film (obtained from data correlations). The p_A versus c_A equilibrium relationship is $p_A^*(atm) = H\ c_A^*\big((mol\ H_2S)/m^3\big)$, where $H = 8.8\ m^3atm/kmol$.
    d. Plot the equilibrium relationship and the operating point (Excel is suggested)
    e. Calculate the flux of hydrogen sulfide
    f. Calculate the overall mass transfer coefficients
    g. Calculate the interface composition

    The liquid density may be estimated as water density at $20^oC, \rho_{water} = 992.3\ kg/m^3$.
    Answers: Operating point $= 3.3 \frac{mol\ A}{m^3}, 0.015\ atm$; $K_G = K_p = 5.1 \times 10^{-4} \frac{kmol}{m^2\ atm\ s}$; $N_A = -7.2 \times 10^{-6} \frac{kmol}{m^2 s}$ and $K_L = K_c = 4.5 \times 10^{-3} m/s$.

16. (Cussler) A packed tower uses an organic amine to absorb carbon dioxide. The entering gas, which contains 1.26 mol% carbon dioxide, is to leave with only 0.04 mol% carbon dioxide. The amine enters pure, without CO_2. If the amine left in equilibrium with the entering gas (which it does not), it would contain 0.80 mole% CO_2. The gas flow is 2.3 mol/s, the liquid flow is $4.8\ mol/s$, the tower's diameter is $4.0 \times 10^1 cm$, and the overall mass transfer coefficient times the area per volume $K_y a$ is $5.0 \times 10^{-5} \frac{mol}{cm^3\ s}$. Determine the height of the tower, the NTU and the HTU. Answer: $B = 3.2\ m$, HTU$37cm$, NTU$= 8.8$.

17. (WRF) Ammonia (NH_3) will be absorbed from an air mixture at $293K$ and $1.0\ atm$ in a countercurrent packed tower, using water as the absorption solvent. The total inlet gas molar flow rate (air plus NH_3) is $5.03 \times 10^{-4}\ kmol/s$ and the ammonia composition in the inlet gas is 0.0352% mole fraction. Ammonia-free water (species B) at a mass flow rate of $L_S = 9.46 \times 10^{-3} kg/s$ will be used as the absorption solvent. If the ammonia concentration in the outlet gas is reduced to a mole fraction of 0.0129, determine the ratio of the operating solvent molar flow rate to the minimum molar solvent flow rate. Equilibrium data for the NH_3-water-air system at the tower's temperature and pressure are given by $Y_A = mX_A$, where $m = 0.82$ and Y_A is the molar ratio of ammonia to air in the gas and X_A is the molar ratio of ammonia to water in the liquid.

    $$X_A = \frac{moles\ A}{moles\ B} = \frac{x_A}{(1-x_A)} \quad \text{(mixture of A and liquid B)}$$
    $$Y_A = \frac{moles\ A}{moles\ I} = \frac{x_A}{(1-x_A)} \quad \text{(mixture of A and gas I)}$$

    These problems are often carried out on a solute-free basis. See the notes for some help with this problem. Answer: ratio$= 2.1$

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References:

- Christie J. Geankoplis, *Transport Processes and Unit Operations*, 4th Edition, Prentice Hall, New York (2003)
- Phillip C. Wankat, *Separation Process Engineering*, 4th edition, Prentice Hall, New York (2017)
- E. L. Cussler, Diffusion: Mass Transfer in Fluid Systems, 3rd Edition, Cambridge University Press, New York (2009).
- (WRF) James R. Welty, Gregory L. Rorrer, and David G. Foster, "*Fundamentals of Momentum, Heat, and Mass Transfer*," 6th edition (Wiley, New York, 2015).

Notes and **Stretch**:

1. See Example 1 and HW3.
   c. **Stretch 1:** How long will it take for the tank to empty? The initial height of the water is h_0. Answer: $t_{empty} = \frac{h_0}{\beta}\left(z_2 - \frac{h_0}{2}\right)$, where $x_{B_{ln}} \equiv \frac{x_{B2}-x_{B1}}{\ln(x_{B2}/x_{B1})}$, c is moles mix/volume mix in the gas phase, $\rho_{A,l}$ is the density of liquid water, $\beta = \frac{M_A c D_{AB}(x_{A1}-x_{A2})}{\rho_{A,l} x_{B_{ln}}}$, and M_A is the molecular weight of A. Hint: this is solved with a quasi-steady state approach.
   d. **Stretch 2:** How fast is the interface moving at steady state? Answer: $1.6 \times 10^{-7}\ cm/s$
2. See Example 4 notes and WRF page 499. You may assume ideal gas. Answer for concentration distribution: $\frac{(x_A - x_{A1})}{(x_{A1}-x_{A2})} = \frac{(z-z_1)}{(z_1-z_2)}$.
3. See Example 10 notes and WRF page 505.
4. See Example 3.
5. Wankat, 2017 is a good source for more information on the McCable Thiele method, along with my YouTube video on the subject (DrMorrisonMTU).
6. I hope all these MEB problems are "a piece of cake." Please go to DrMorrisonMTU on YouTube for videos on MEB and unit conversions and g_c.
7. See Example 4.
8. Start with the definition of equimolar counterdiffusion.
9. Note that k_c is defined by this equation: $|N_A| = k_c|c_{A,bulk} - c_{A,i}|$ and may be written in terms of pressures for an ideal gas.
10. Practice being clear in your answers. A long answer is not always better. What are the "sibling" dimensionless numbers for those indicated? Please indicate the "sibling" relationship.
11. Both the microscopic balance and the boundary conditions must match to make use of the Heissler chart.
12. See Example 8.
13. See Example 7.
14. "Compare and contrast" means find the things that the two items have in common and then find the things that make them different.

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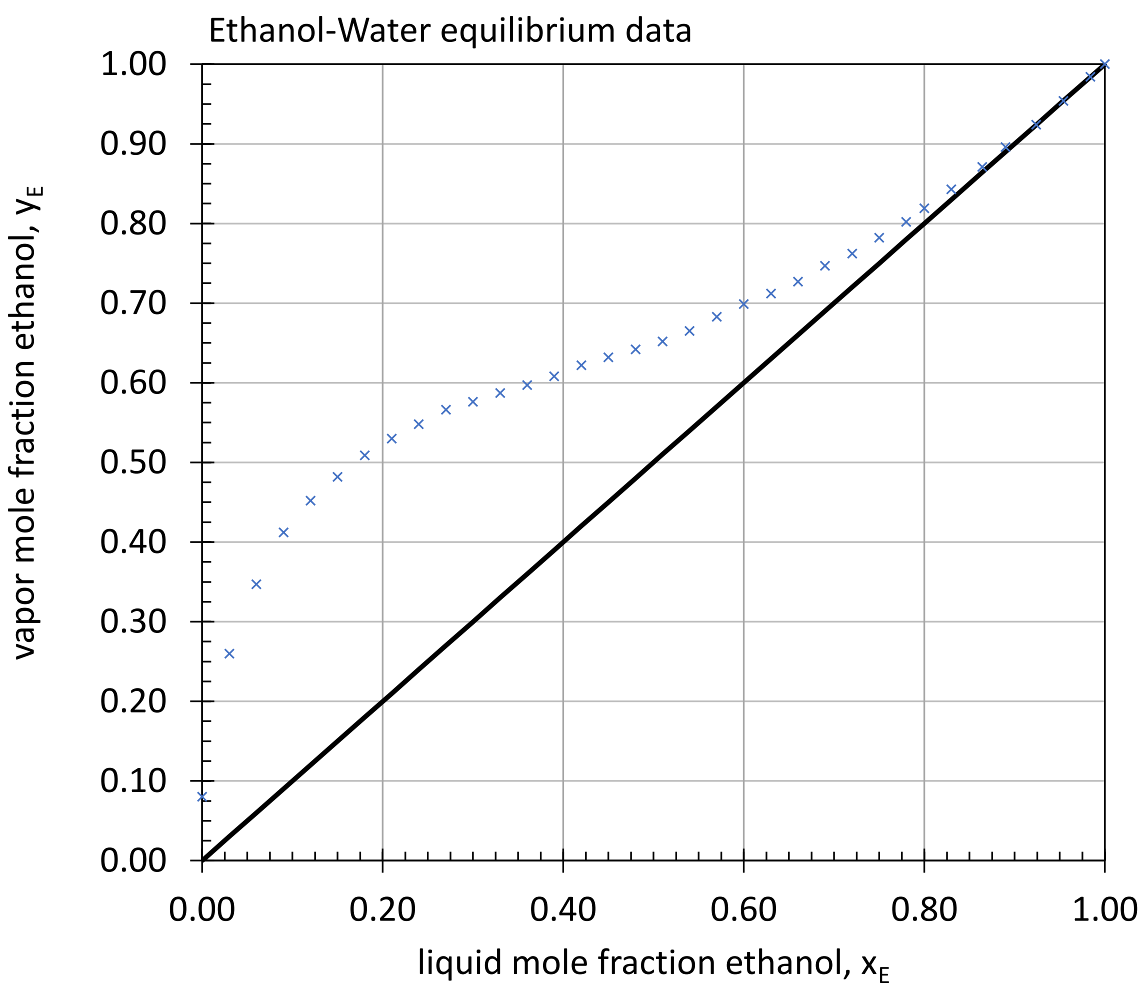
15. See Example 6.
16. See Cussler, chapter 10.
17. A good strategy is to change the provided information to be on a solute-free basis. The solute-free gas flow rate G_s is the (constant) molar flow rate of air. The solute-free liquid flow rate is the (constant) molar flow rate of water. Convert all the mole fractions to molar ratios, performing the appropriate balances to obtain missing values. The operating line is a balance cutting through the bottom streams and an arbitrary place within the column characterized by molar ratios of X_A, Y_A. The minimum liquid flow rate L_s (for the same gas flow rate G_s is determined as discussed in class.



Equilibrium data for problem 5 (digitized from Wankat, 2017, p21, table and plot):

n	x_E	yE	n	x_E	yE
1	0.000	0.080	18	0.510	0.652
2	0.030	0.260	19	0.540	0.665
3	0.060	0.347	20	0.570	0.683
4	0.090	0.412	21	0.600	0.699
5	0.120	0.452	22	0.630	0.712
6	0.150	0.482	23	0.660	0.727
7	0.180	0.509	24	0.690	0.747
8	0.210	0.530	25	0.720	0.762
9	0.240	0.548	26	0.750	0.782
10	0.270	0.566	27	0.780	0.802
11	0.300	0.576	28	0.800	0.819
12	0.330	0.587	29	0.830	0.843
13	0.360	0.597	30	0.864	0.871
14	0.390	0.608	31	0.890	0.896
15	0.420	0.622	32	0.924	0.924
16	0.450	0.632	33	0.954	0.954
17	0.480	0.642	34	0.984	0.984
			35	1.000	1.000