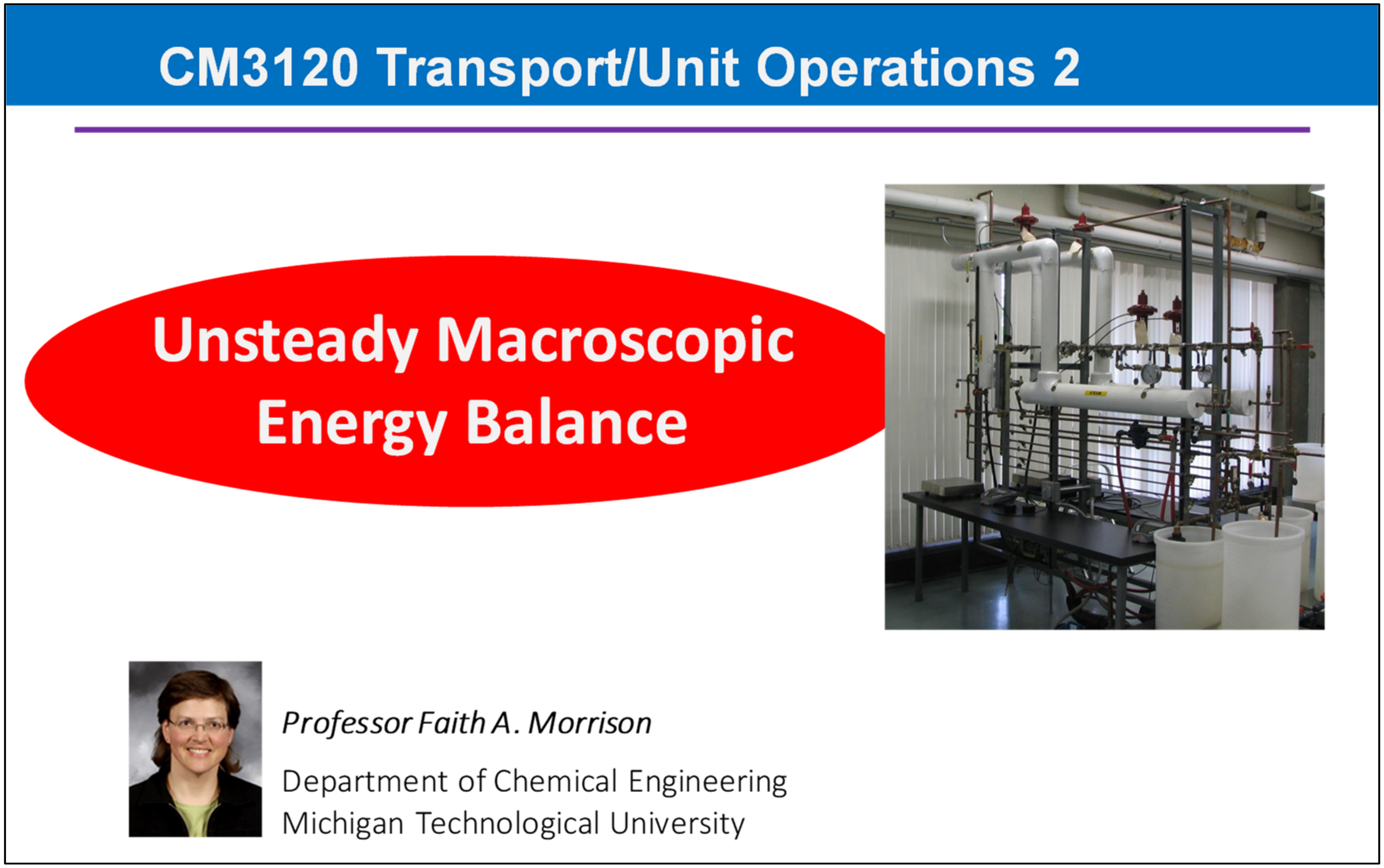
This topic is part of a more general subject:



CM3120 Transport/Unit Operations 2

Unsteady Macroscopic Energy Balance

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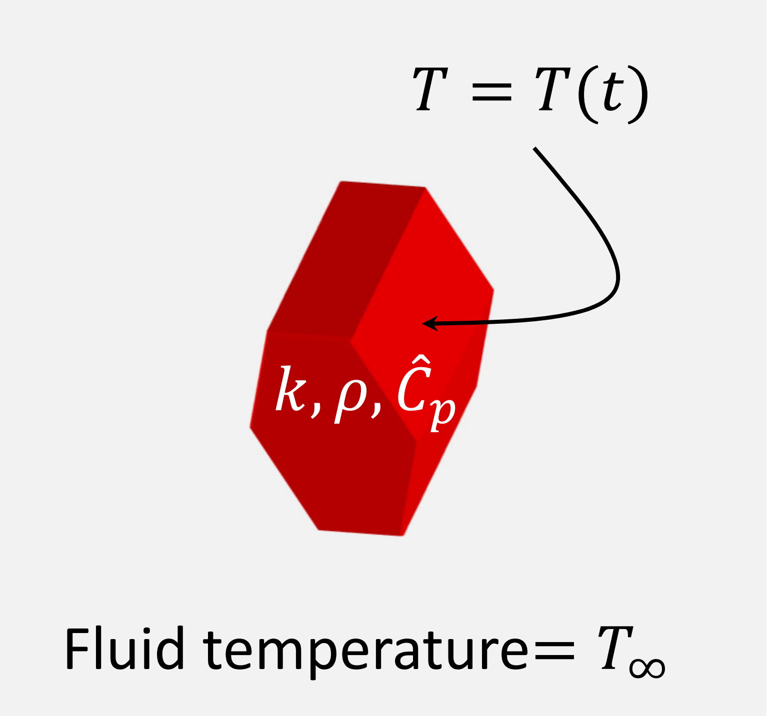
Unsteady State Heat Transfer: Low Biot Number

Example: Quench cooling of a manufactured part.

If a piece of steel with $T = T_0$ is dropped into a large, well stirred reservoir of fluid at bulk temperature T_∞, what is the temperature of the steel as a function of time?

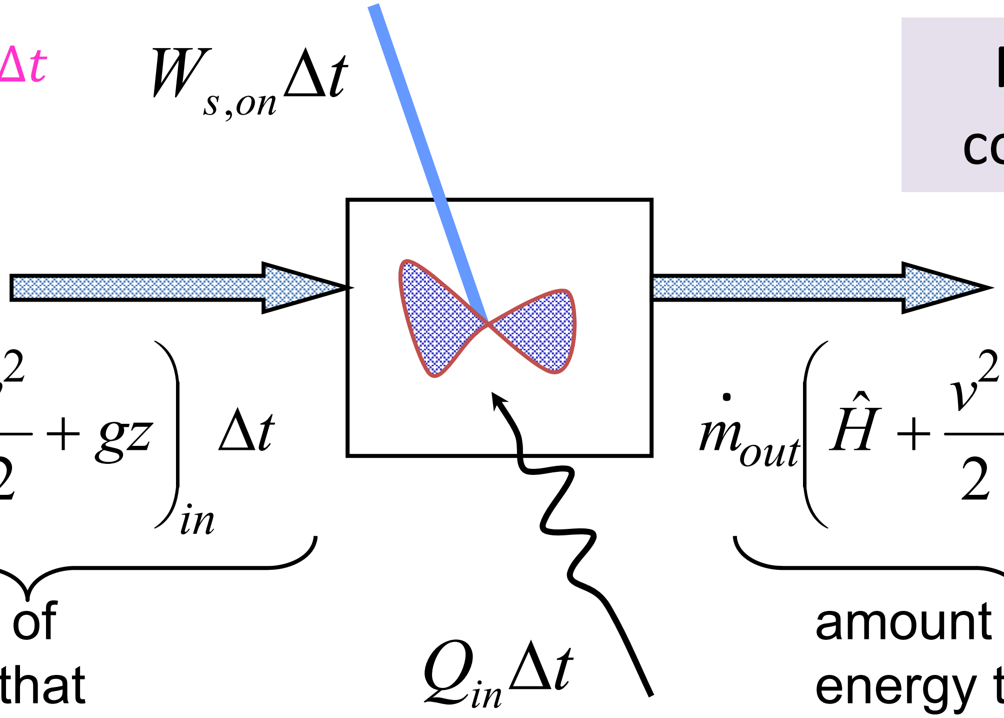
- k = large, which means that there is no internal resistance to heat transfer in the part
- Therefore, we are NOT calculating a temperature profile (internal T is uniform)
- $\Rightarrow$ **Use Unsteady, Macroscopic Energy Balance**



Unsteady Macroscopic Energy Balance

see Felder and Rousseau or Himmelblau

balance over time interval Δt



$W_{s,on}\Delta t$

Macroscopic control volume

$$\underbrace{\dot{m}_{in}\left(\hat{H}+\frac{v^2}{2}+gz\right)_{in}\Delta t}$$

amount of energy that enters with the flow between t and $t+\Delta t$

$Q_{in}\Delta t$

$$\underbrace{\dot{m}_{out}\left(\hat{H}+\frac{v^2}{2}+gz\right)_{out}\Delta t}$$

amount of energy that exits with the flow between t and $t+\Delta t$

Unsteady Macroscopic Energy Balance

$$accumulation = input - output$$

$$\frac{d}{dt}\left(U_{sys} + E_{k,sys} + E_{p,sys}\right) = -\Delta H - \Delta E_k - \Delta E_p + Q_{in} + W_{s,on}$$

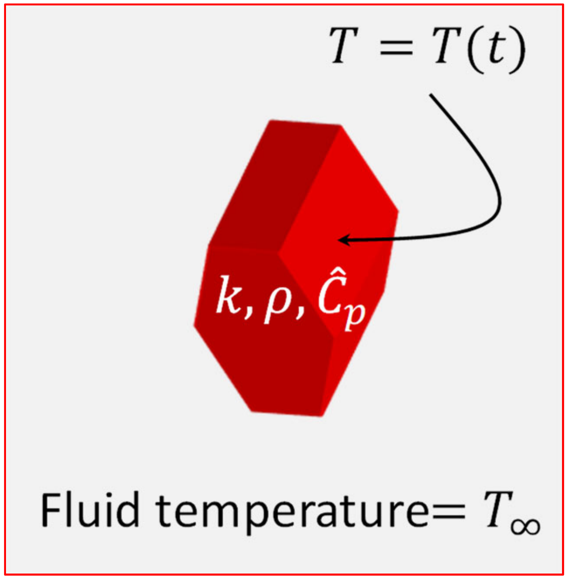
Background:
pages.mtu.edu/~fmorriso/cm310/IFMWeb
AppendixDMicroEBalanceMorrison.pdf

Unsteady Macroscopic Energy Balance

$$accumulation = input - output$$

$$\frac{d}{dt}\left(U_{sys} + E_{k,sys} + E_{p,sys}\right) = -\Delta H - \Delta E_k - \Delta E_p + Q_{in} + W_{s,on}$$

How do we apply this balance to our current problem?

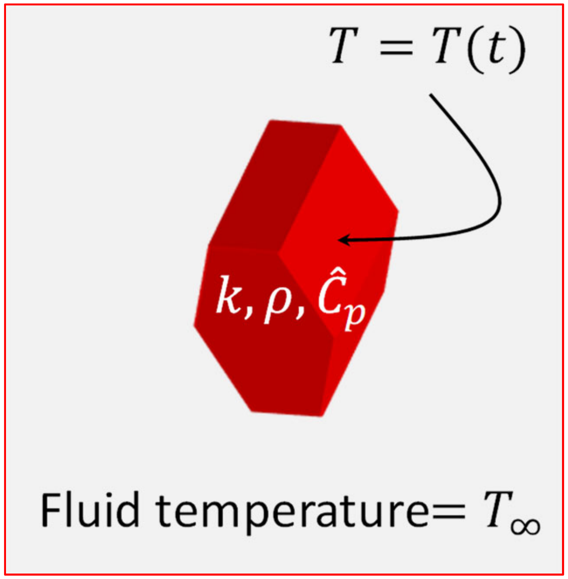


Unsteady Macroscopic Energy Balance

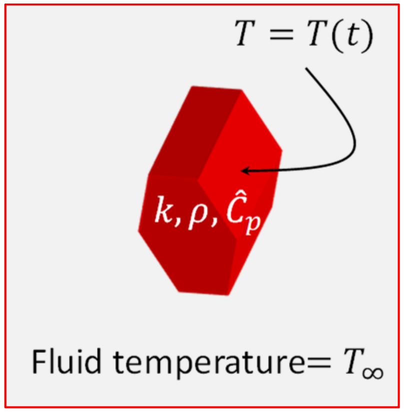
$$accumulation = input - output$$

$$\frac{d}{dt}\left(U_{sys} + E_{k,sys} + E_{p,sys}\right) = -\Delta H - \Delta E_k - \Delta E_p + Q_{in} + W_{s,on}$$

You try.



Unsteady Macroscopic Energy Balance



$$accumulation = input - output$$

$$\frac{d}{dt}\left(U_{sys} + \cancel{E_{k,sys}} + \cancel{E_{p,sys}}\right) = -\cancel{\Delta H} - \cancel{\Delta E_k} - \cancel{\Delta E_p} + Q_{in} + \cancel{W_{s,on}}$$

negligible

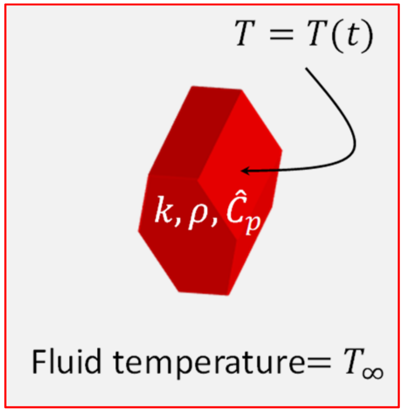
no flow

no shafts

For negligible changes in E_p and E_k, no flow, no phase change, no chemical rxn, and no shafts:

$$\boxed{\begin{gathered}\frac{dU_{sys}}{dt} = Q_{in} \\ \rho V_{sys}\hat{C}_v\frac{dT_{sys}}{dt} = \mathrm{Q}_{in}\end{gathered}}$$

$\hat{C}_v \approx \hat{C}_p$ for liquids, solids



Unsteady Macroscopic Energy Balance

How do we quantify the heat in Q_{in}?

$$\frac{d}{dt}\left(U_{sys}+\cancel{E_{k,sys}}+\cancel{E_{p,sys}}\right)=-\cancel{\Delta H}-\cancel{\Delta E_k}-\cancel{\Delta E_p}\boxed{+Q_{in}}+\cancel{W_{s,on}}$$

negligible

no flow

no shafts

For negligible changes in E_p and E_k, no flow, no phase change, no chemical rxn, and no shafts:

$$\frac{dU_{sys}}{dt}=Q_{in}$$

$$\rho V_{sys}\hat{C}_v\frac{dT_{sys}}{dt}=\mathrm{Q}_{in}$$

$\hat{C}_v \approx \hat{C}_p$ for liquids, solids

Unsteady Macroscopic Energy Balance

accumulation =
input − output

Q_{in} = Heat ***in***to the chosen macroscopic control volume

$$\frac{d}{dt}\left(U_{sys}+E_{k,sys}+E_{p,sys}\right)=-\Delta H-\Delta E_k-\Delta E_p+Q_{in}+W_{s,on}$$

$Q_{in}=\sum_i q_{in,i}$ comes from a variety of sources:

- Thermal conduction: $q_{in}=-kA\frac{dT}{dx}$
- Convection: $q_{in}=hA(T_b-T)$
- Radiation: $q_{in}=\varepsilon\sigma A\left(T_{surroundings}^4-T_{surface}^4\right)$
- Electric current: $q_{in}=I^2R_{elec}L$
- Chemical Reaction: $q_{in}=S_{rxn}V_{sys}$

$$S[=]\frac{\text{energy}}{\text{time volume}}$$

pages.mtu.edu/~fmorriso/cm310/IFMWebAppendixDMicroEBalanceMorrison.pdf
Incropera and DeWitt, 6th edition p18

Unsteady Macroscopic Energy Balance

accumulation = input − output

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Signs must match transfer from outside (bulk fluid) to inside (metal)

$$S [=] \frac{\text{energy}}{\text{time volume}}$$

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Unsteady Macroscopic Energy Balance

$$\frac{d}{dt}\left(U_{sys} + E_{k,sys} + E_{p,sys}\right) = -\Delta H - \Delta E_k - \Delta E_p + \bigcirc\!\!\!\!\!\!Q_{in} + W_{s,on}$$

$Q_{in} = \sum_i q_{in,i}$ comes from a variety of sources:

- **Thermal conduction:** $q_{in} = -kA\frac{dT}{dx}$

  e.g. device held by bracket; a solid phase that extends through boundaries of control volume

- **Convection:** $q_{in} = hA(T_b - T)$

  e.g. device dropped in stirred liquid; forced air stream flows past, natural convection occurs outside system; phase change at boundary

- **Radiation:** $q_{in} = \varepsilon\sigma A\left(T_{surroundings}^4 - T_{surface}^4\right)$

  e.g. device at high temp. exposed to a gas/vacuum; hot enough to produce nat. conv.=possibly hot enough for radiation

S-B constant: $\sigma = 5.676 \times 10^{-8}\,\frac{W}{m^2K^4}$

- **Electric current:** $q_{in} = I^2 R_{elec} L$

  e.g. if electric current is flowing <u>within</u> the device/control volume/ system

- **Chemical Reaction:** $q_{in} = S_{rxn} V_{sys}$

  e.g. if a homogeneous reaction is taking place <u>throughout</u> the device/ control volume/system

Unsteady Macroscopic Energy Balance

$$\mathit{accumulation} = \mathit{input} - \mathit{output}$$

Q_{in} = Heat ***in***to the chosen macroscopic control volume

$$\frac{d}{dt}\left(U_{sys} + E_{k,sys} + E_{p,sys}\right) = -\Delta H - \Delta E_k - \Delta E_p + Q_{in} + W_{s,on}$$

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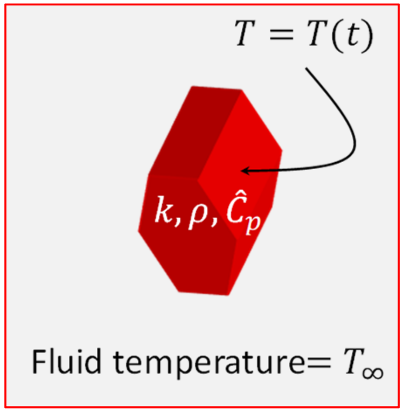
Unsteady State Heat Transfer: Low Biot Number



Unsteady Macroscopic Energy Balance Applied to cooling steel part:

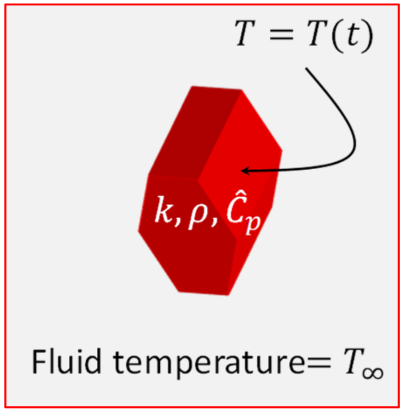
$$\underbrace{\rho V_{sys}\hat{C}_p \frac{dT_{sys}}{dt}} = \mathrm{Q}_{in}$$

The temperature changes in the part are due to the heat loss

The heat loss depends on the heat-transfer coefficient from the part to the environment

$$\boxed{\mathrm{Q}_{in} = Ah(T_\infty - T)}$$

$\hat{C}_v \approx \hat{C}_p$ for liquids, solids



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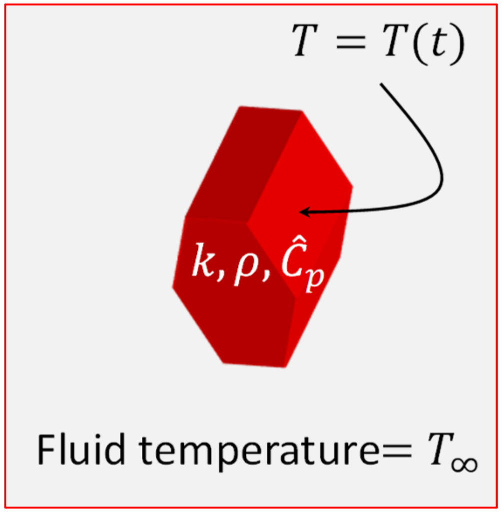
The heat loss depends on the heat-transfer coefficient from the part to the environment

$$\boxed{\mathrm{Q}_{in} = Ah(T_\infty - T)}$$

You solve.

Unsteady State Heat Transfer: Low Biot Number



Unsteady Macroscopic Energy Balance Applied to cooling steel part:

$$\frac{(T_\infty - T)}{(T_\infty - T_0)} = e^{-\left(\frac{hA}{\rho \hat{C}_p V}\right) t}$$

$$V_{sys} = V$$

$$\ln\left(\frac{(T_\infty - T)}{(T_\infty - T_0)}\right) = -\left(\frac{hA}{\rho \hat{C}_p V}\right) t$$