MORRISON

1.	/20
2.	/20
3.	/20
4.	/20
5.	/20

Exam 2

CM3120

Tuesday 11 February 2020

Name: Solution

Instructions:

i. Closed book, closed notes. One 8.5" by 11" study sheet allowed, two sided; you may use a calculator; you may not use the internet or a cell phone. All work on the exam must be your own.
ii. Write your solution on one side of the page only. Do not write on the back of any pages.
iii. Please be neat. Only neat answers will be granted partial credit.
iv. Significant figures always count.
v. Please box your final answers.

1. (20 points) To obtain heat transfer coefficients in turbulent fluid flow in pipes for real systems, we rely on data correlations from the literature (other people's data), For dimensionless heat transfer coefficient (Nusselt number) correlations for steady turbulent pipe flow, answer the following questions:
   a. What is the formula for Nusselt number? Identify each quantity that appears in the formula and give SI units (metric).
   b. For steady turbulent flow of liquids in pipes, what are the dimensionless numbers that appear in the correlation?
2. (20 points) Heat loss per area through the outside wall of a building is determined with an infrared sensor to be 140 W/m^2. The external surface temperature of the walls is 17^oC. The house is basically a cube (13.45m square, 12m tall), covered with asbestos board. On the day of the measurement, the outside air temperature is 5^oC (41^oF). Under these conditions, what is the heat transfer coefficient between the wall of the house and the environment?
3. (20 points) A cube of copper (length of side = 5.0cm) is subjected to annealing (heat treatment in a furnace) at 100.0^oC and subsequently placed in a 25^oC flowing air stream to cool. If the heat transfer coefficient in the air stream is 51 W/m^2K, what is the temperature of the cube after 30. seconds?
4. (20 points) A large, iron structure (approximate dimensions $3m \times 3m \times 3m$, iron) initially with a uniform temperature of 98^oC is exposed to outside air conditions (air temperature = 25^oC, heat transfer coefficient = $2.0 \times 10^1 W/m^2K$). What is the wall temperature after 15 minutes?
5. (20 points) A hollow sphere of inner radius R_1 and outer radius R_2 is exposed to an external fluid at constant bulk temperature T_{b2}. The inside surface temperature of the spherical shell is maintained at constant temperature T_{w1} by an unspecified system. What is the steady state heat flux as a function of position in the hollow spherical shell? Note that outer surface temperature of the spherical shell is not equal to the bulk fluid temperature at that location.

CM3120
2020
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EXAM 2

1. a) $Nu = \frac{hD}{k}$

h = heat transfer coefficient

$[=] \frac{W}{m^2K}$

D = pipe diameter

$[=]$ m

k = fluid thermal conductivity

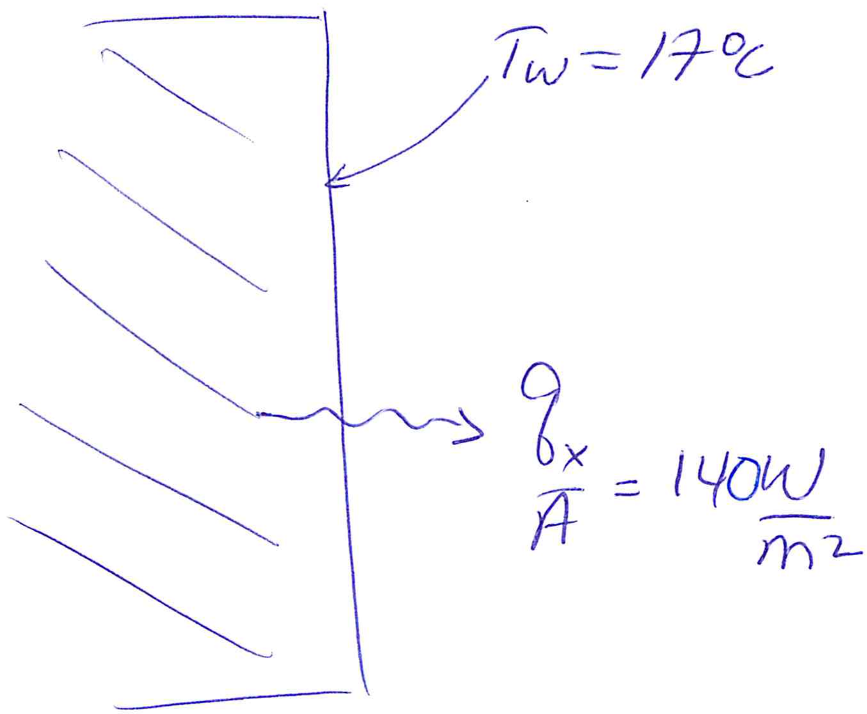
$[=] \frac{W}{mK}$

b) $Pr = \frac{C_p \mu}{k}$ ← (all for the fluid)

$Re = \frac{\rho V D}{\mu}$

ρ, μ for fluid
D pipe diameter
V average velocity

2.)



Newton's law of cooling:

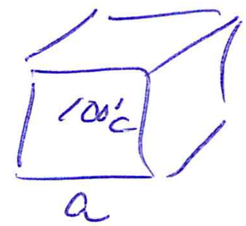
$$\frac{q_x}{A} = h\,(T_w - T_b)$$

$$h = \frac{q_x / A}{T_w - T_b} = \frac{\left(140 \frac{W}{m^2}\right)}{(17 - 5)\,°C}$$

$$= 11.6667 \frac{W}{m^2 K} = \boxed{12 \frac{W}{m^2 K}}$$

2 SIG FIGS (or 1 sig figs)

3) initially



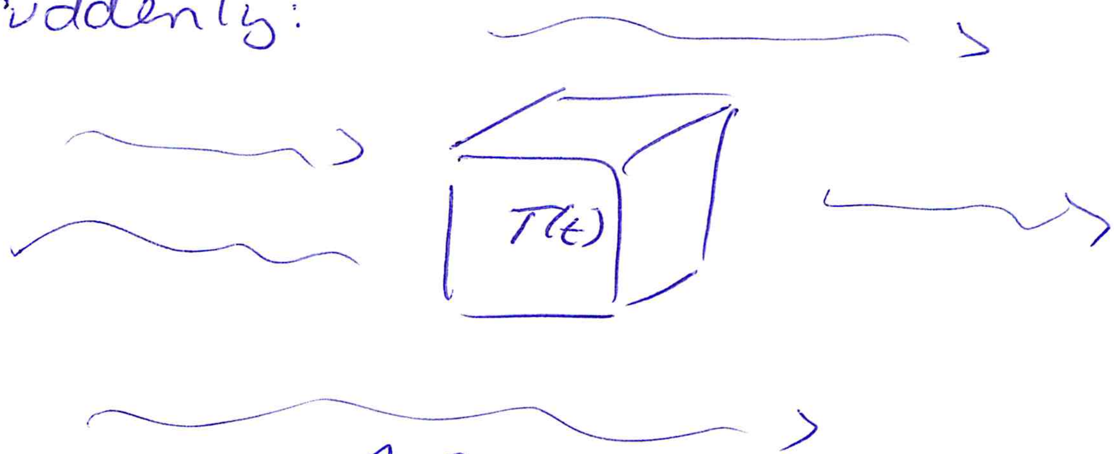
100°C

copper
$k = 377\ W/mK$

$a = 5\ cm$

$\boxed{a = 0.05\ m}$

suddenly:



AIR
25°C
$h = 51\ \frac{W}{m^2K}$

Try lumped parameter:

$Bi < 0.1$?

$$Bi_{LP} = \frac{h\ V/A}{k}$$

$$Bi_{LP} = \frac{\left(51 \frac{\cancel{W}}{\cancel{m^2 K}}\right)(0.05\cancel{m})^3}{6(0.05\cancel{m})^2\left(377 \frac{\cancel{W}}{\cancel{mK}}\right)}$$

$$Bi_{LP} = 0.001127 < 0.1$$ $\boxed{\text{Yes}}$

LUMPED PARAMETER

Soln:

$$\frac{T_\infty - T}{T_\infty - T_0} = e^{-\left(\frac{hA}{\rho c_p V}\right)t} = e^{-Bi\,Fo}$$

$$Fo = \frac{\alpha t}{D_{char}^2} = \frac{\alpha t}{(V/A)^2}$$

$$\alpha_{copper} = 1.11 \times 10^{-4} \frac{m^2}{s}$$

$$Fo = \frac{\left(1.11 \times 10^{-4} \frac{\cancel{m^2}}{\cancel{s}}\right)(30\cancel{s})}{(\cancel{0.00833\,m})^2}$$

$$\frac{V}{A} = \frac{(0.05m)^3}{6(0.05)^2} = 0.00833\,m$$

$$\boxed{Fo = 47.95}$$

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$$\frac{T_\infty - T}{T_\infty - T_o} = e^{-Bi\,Fo}$$

$$\frac{25^\circ C - T}{(25-100)^\circ C} = e^{-(0.001127)(47.95)}$$

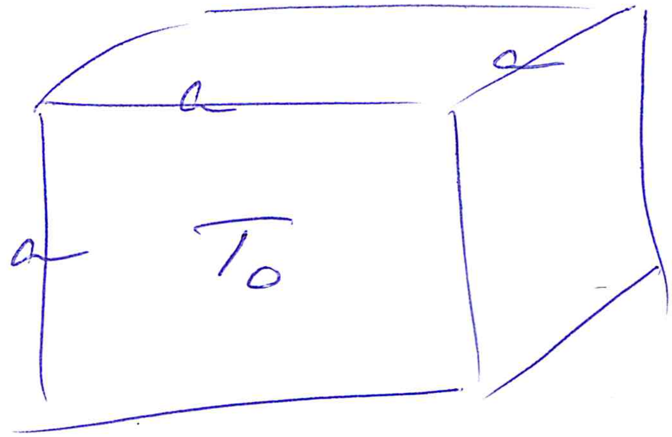
$$= 0.947392$$

$$25^\circ C - T = (-75^\circ C)(0.947392)$$

$$= -71.05$$

$$\boxed{T = 96\,^\circ C}$$

4.

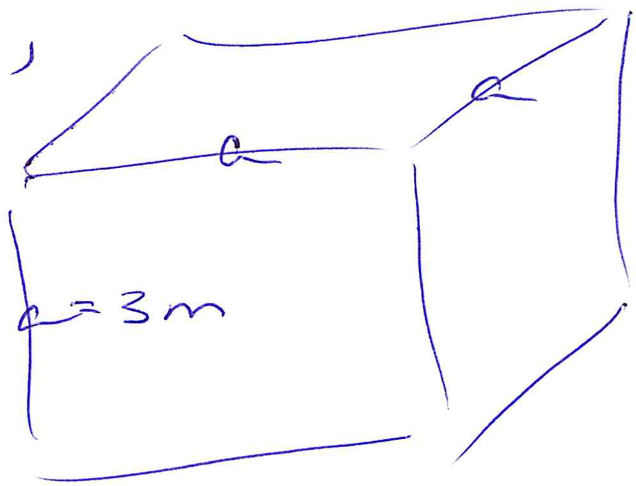


$a = 3\,m$ $\qquad T_o = 98^oC$

iron

$k = 52\ W/mK$

Suddenly,



$T_b = 25^oC$

$h = 20\ \frac{W}{m^2K}$

What is $T(15min)$?

$$\frac{V}{A} = \frac{a^3}{6\cancel{a^2}} = \frac{a}{6}$$

Check Biot #:

LUMPED PARAM: $Bi = \frac{h\,V/A}{k}$

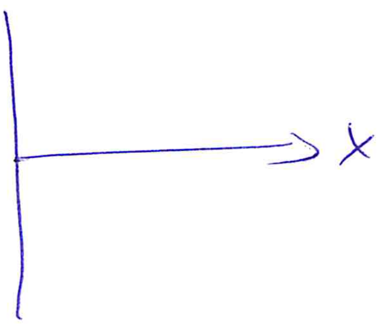
$$Bi = \frac{\left(\frac{20\cancel{W}}{\cancel{m^2K}}\right)\left(\frac{3\cancel{m}}{6}\right)}{52\ \cancel{W/mK}} = 0.1923$$

NO

☆ Cannot use lumped parameter

Use semi-infinite slab:

$T_b = 25°C = T_1$



$T_0 = 98°C$

$$\frac{T - T_0}{T_1 - T_0} = \text{see figure 5.3-3}$$

$$\beta = \frac{h\sqrt{\alpha t}}{k}$$

$$= \frac{\left(20 \frac{W}{m^2 K}\right)}{\left(52 \frac{W}{mK}\right)} \sqrt{\left(2.3 \times 10^{-5} \frac{m^2}{s}\right)(15)(60\,s)}$$

$$\boxed{\beta = 0.055}$$

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$$\zeta = \frac{x^{\leftarrow \text{at wall}}}{2\sqrt{\alpha t}}$$

$$\boxed{\zeta = 0}$$

Read from Fig 5.3-3 (see next pg)

$$\frac{T - T_0}{T_1 - T_0} = 0.06 \text{ (from graph)}$$

$$\frac{T - 98°C}{(25 - 98)°C} = 0.06$$

$$\boxed{T = 94°C}$$

no more than 2 sig figs

graph follows

(9)

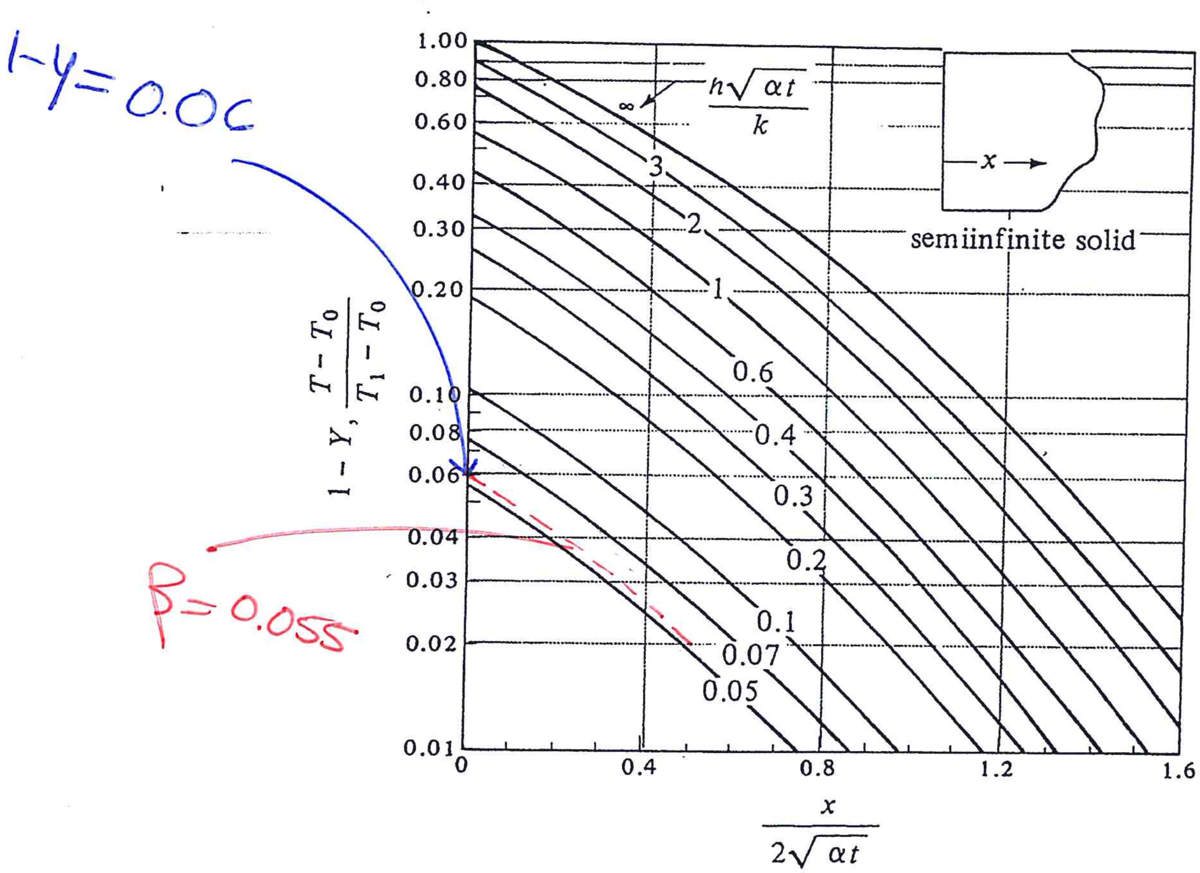
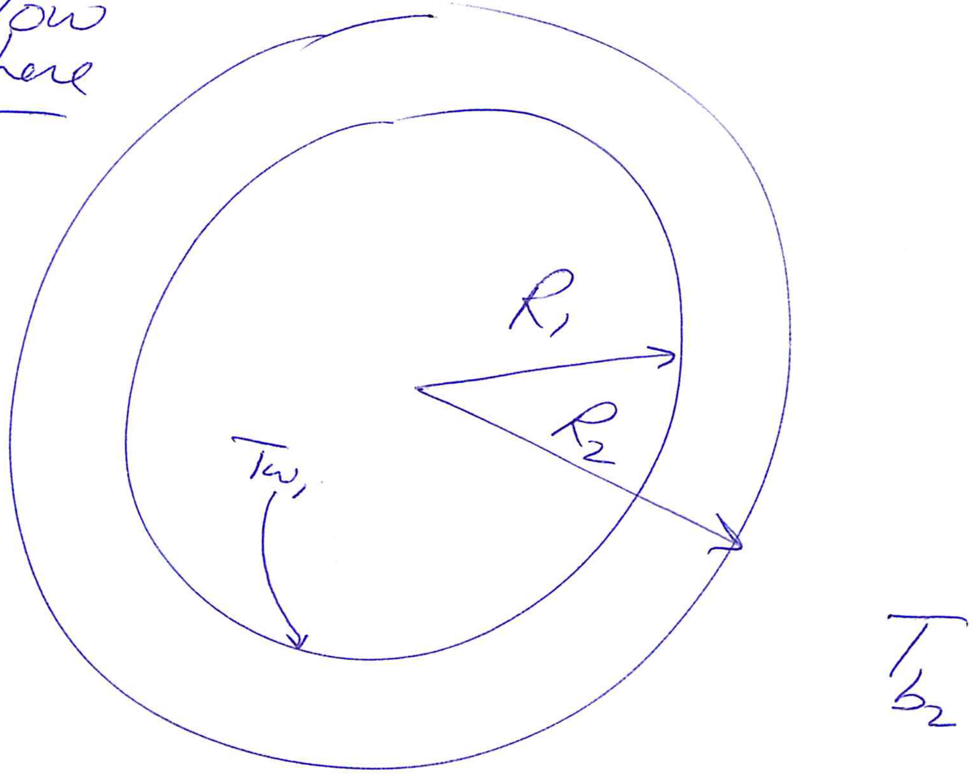


FIGURE 5.3-3. *Unsteady-state heat conducted in a semi-infinite solid with surface convection. Calculated from Eq. (5.3-7)(S1).*

$$\boxed{\frac{T - T_0}{T_1 - T_0} = 0.06}$$

Geankoplis, 4th edition

5. Hollow sphere



What is $\frac{q_r}{A}(r)$?

Must perform micro Energy balance to obtain a Temp or flux distribution.

- $\underline{v} = 0$
- Steady
- θ symmetry
- ϕ symmetry
- no rxn, no electric current

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The **Equation of Energy** for systems with **constant k**

Microscopic energy balance, constant thermal conductivity; Gibbs notation

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+\underline{v}\cdot\nabla T\right)=k\nabla^2 T+S_e$$

Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_x\frac{\partial T}{\partial x}+v_y\frac{\partial T}{\partial y}+v_z\frac{\partial T}{\partial z}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+S_e$$

Microscopic energy balance, constant thermal conductivity; cylindrical coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}+v_z\frac{\partial T}{\partial z}\right)=k\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}\right)+S_e$$

$v=0$

Microscopic energy balance, constant thermal conductivity; spherical coordinates

ϕ symmetry

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial T}{\partial \phi}\right)$$
$$=k\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \phi^2}\right)+S_e$$

steady

θ symmetry

no electric current; no chem rxn

$$\boxed{0=k\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)\right)}$$

Reference: F. A. Morrison, "Web Appendix to *An Introduction to Fluid Mechanics*," Cambridge University Press, New York, 2013 (*pages.mtu.edu/~fmorriso/cm310/IFMWebAppendixDMicroEBalanceMorrison.pdf*). This worksheet is on the web at *pages.mtu.edu/~fmorriso/cm310/energy.pdf*.

$$0 = \frac{k}{r^2}\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right)$$

multiply both sides by $\frac{r^2}{k}$

$$0 = \frac{d}{dr}\Big(\underbrace{r^2 \frac{dT}{dr}}_{\equiv \Phi}\Big)$$

$$\frac{d\Phi}{dr} = 0$$

integrate:

$$\Phi = C_1$$

$$\uparrow$$

$$\Phi = r^2 \frac{dT}{dr} = C_1$$

$$\frac{dT}{dr} = C_1 \frac{1}{r^2}$$

integrate:

$$\boxed{T = C_1 \frac{-1}{r} + C_2}$$

B

Boundary Conditions

assume
hotter inside
than out

$r = R_1 \quad T = T_{w1}$

$r = R_2$ Newton's Law of Cooling:

$$-k\frac{dT}{dr} = h\left(T - T_{b_2}\right)$$

substitute + solve for C_1 & C_2:

$$T = \frac{-C_1}{r} + C_2$$

$$\frac{dT}{dr} = \frac{C_1}{r^2}$$

BC1 $\boxed{T_{w1} = -\frac{C_1}{R_1} + C_2}$

BC2 $\boxed{-k\frac{C_1}{R_2^2} = h\left(\frac{-C_1}{R_2} + C_2 - T_{b_2}\right)}$

(note: we actually only need c_1 because we are asked for $\frac{q_r}{A}$:

need c_1

Fourier's Law $\frac{q_r}{A} = -k \frac{dT}{dr} = \boxed{-k c_1 \frac{1}{r^2} = \frac{q_r}{A}}$

SOLVING FOR c_1:

$$C_2 = T_{w_1} + \frac{c_1}{R_1}$$

substitute into 2nd BC

$$-k \frac{c_1}{R_2^2} = h \left(\frac{-c_1}{R_2} + T_{w_1} + \frac{c_1}{R_1} - T_{b_2} \right)$$

$$c_1 \left(-\frac{k}{R_2^2} + \frac{h}{R_2} - \frac{h}{R_1} \right) = h \left(T_{w_1} - T_{b_2} \right)$$

$$c_1 \left(-\frac{k}{h R_2^2} + \frac{1}{R_2} - \frac{1}{R_1} \right) = \left(T_{w_1} - T_{b_2} \right)$$

$$C_1 = \frac{(T_{b2} - T_{w1})}{\frac{k}{hR_2^2} + \left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

Therefore the flux is

$$\frac{q_r}{A} = -k C_1 \frac{1}{r^2}$$

$$= \frac{(T_{w1} - T_{b2})\, k \frac{1}{r^2}}{\frac{k}{hR_2^2} + \left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

$$\boxed{\frac{q_r}{A} = \frac{(T_{w1} - T_{b2}) \frac{1}{r^2}}{\frac{1}{hR_2^2} + \frac{1}{k}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}}$$

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