Exam 3
CM3120
Morrison
Spring 2019
Solution

1) a) Biot #: $Bi = \dfrac{h D_{characteristic}}{k}$

$= \dfrac{\text{heat transfer through surface}}{\text{conduction in solid}}$

Bi small: $\Rightarrow$ k large
conduction fast
xfer to surface is limiting

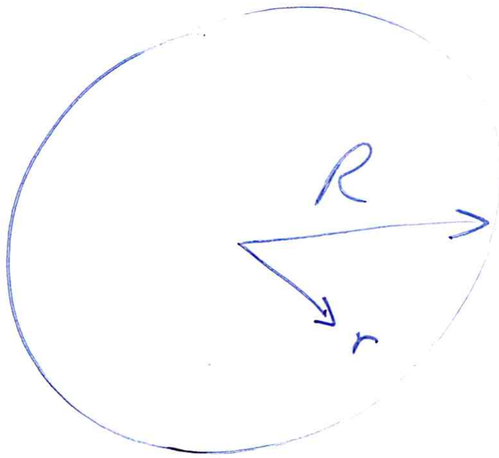
Bi large $\Rightarrow$ h large
conduction is limiting

Bi neither large nor small $\Rightarrow$ both are important.

1 b) Role/importance of diffusion/ mass xfer in Friday project?

Answer varies.

2)



$R = 1.106\ cm$
Aluminum
$T_0 = 20.0°C$

App H: $\alpha_{Al} = 9.16 \times 10^{-5} \frac{m^2}{s}$
$k_{Al} = 229 \frac{W}{mK}$

Suddenly

$T_{bath} = 30.0°C$
$h = 2300 \frac{W}{m^2 K}$

What is $T(r=0)$ after 20.0 seconds?

Use Heissler chart.

$$Fo = \frac{\alpha t}{R^2} = \frac{\left(9.16 \times 10^{-5} \frac{m^2}{s}\right)(20\ s)}{\left[(1.106\ cm)\left(\frac{m}{10^2\ cm}\right)\right]^2}$$

$$= 14.976 = \boxed{15 = Fo}$$

$$label = \frac{k}{hR} = \frac{\left(229 \frac{W}{\cancel{m}K}\right)\left(\frac{100 \cancel{cm}}{\cancel{m}}\right)}{\left(2300 \frac{W}{\cancel{m^2}K}\right)\left(1.106 \cancel{cm}\right)}$$

$$label = \frac{1}{Bi} = 9.00228 = \boxed{9}$$

From Heissler chart (next page)

$$\frac{T_\infty - T}{T_\infty - T_0} = 0.008$$

$$\frac{30 - T}{30 - 20} = 0.008$$

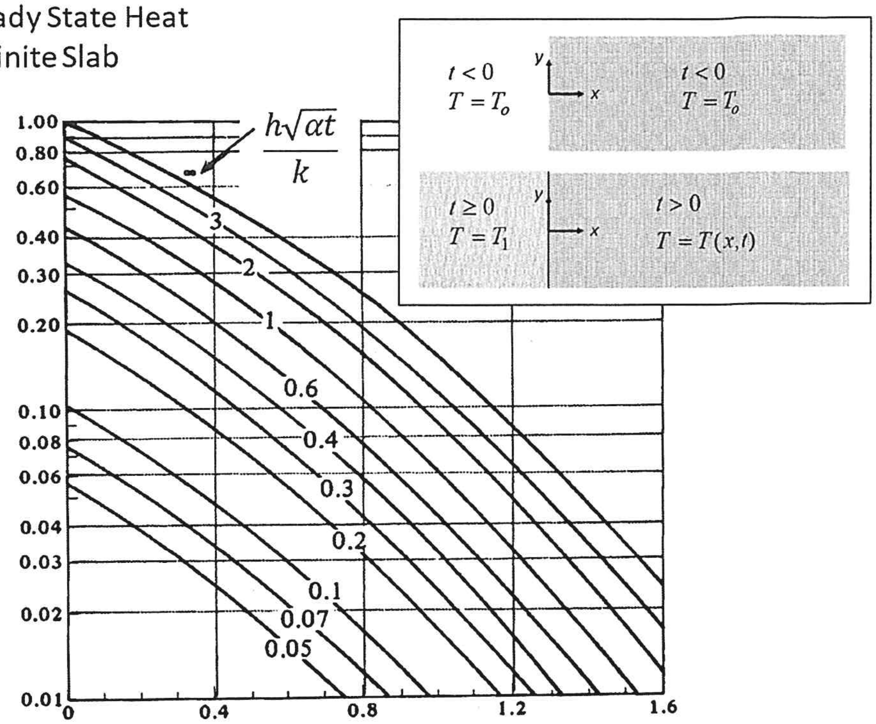
$$T = 29.92\,^\circ C$$

$$\boxed{T = 29.9\,^\circ C}$$

(S)

1D Heat Transfer: Unsteady State Heat Conduction in a Semi-Infinite Slab

$$1 - Y = \left(\frac{T - T_0}{T_1 - T_0}\right)$$



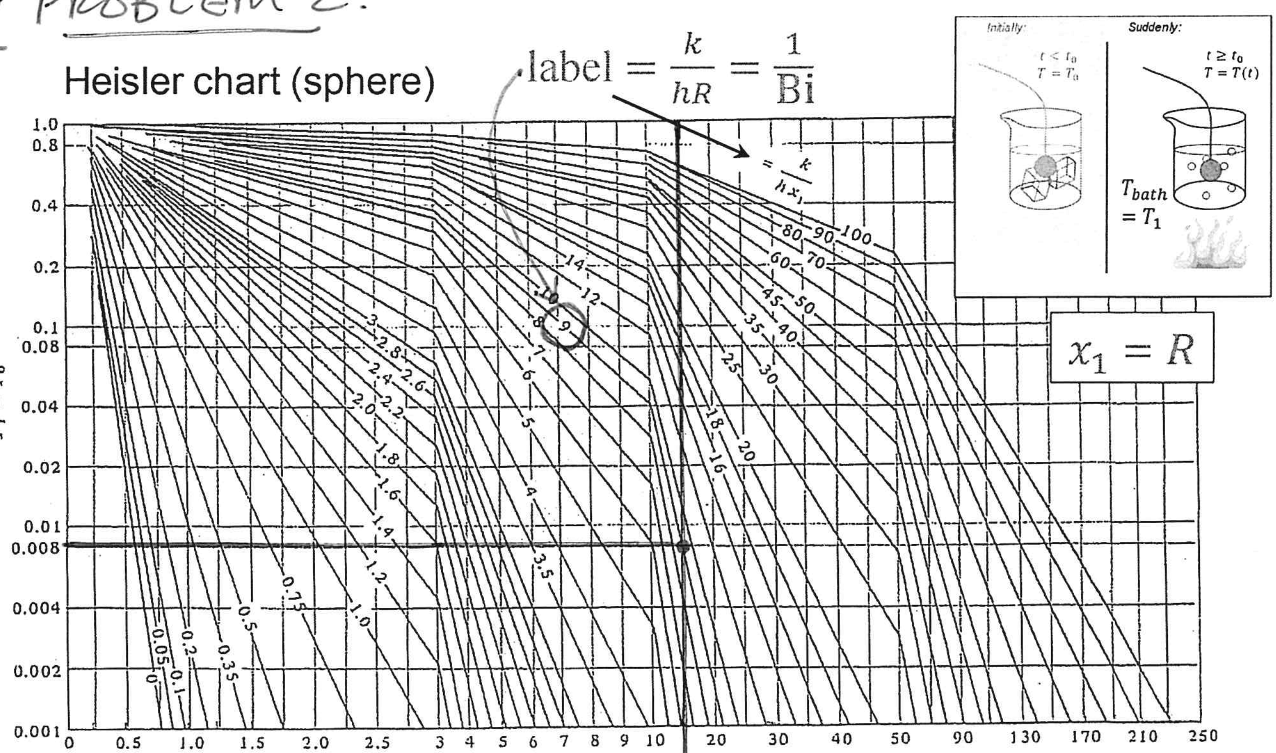
$$\zeta = \frac{x}{2\sqrt{\alpha\, t}}$$

Geankoplis 4th ed., Figure 5.3-3, page 364

SOLN PROBLEM 2:

Heisler chart (sphere)

$$\text{label} = \frac{k}{hR} = \frac{1}{\text{Bi}}$$



$X = \frac{\alpha t}{x_1^2}$ Fo = 15

$T_1 = 30$

$T_0 = 20$

FIGURE 5.3-10. Chart for determining the temperature at the center of a sphere for unsteady-state heat conduction. [From H. P. Heisler, Trans. A.S.M.E., 69, 227 (1947). With permission.] *From Geankpolis, 4th edition, page 374*

3) a) 1D rectangular diffusion
steady

(see sheet)

$$\frac{dN_{Az}}{dz} = 0$$

$$\Rightarrow \boxed{N_{Az} = \text{constant}}$$

b) cylindrical (see sheet)

$$0 = \cancel{-\frac{1}{r}} \frac{d}{dr}\left(rN_{Ar}\right)$$

$$\frac{d}{dr}\underbrace{\left(rN_{Ar}\right)}_{\equiv \Phi} = 0$$

$$\frac{d\Phi}{dr} = 0$$

$$rN_{Ar} = \Phi = C_1$$

$$\boxed{N_{Ar} = \frac{C_1}{r}}$$

varies as $1/r$

The Equation of Species Mass Balance in Terms of Combined Molar quantities in Cartesian, cylindrical, and spherical coordinates for binary mixtures of A and B.

The general case, where the combined molar flux with respect to molar velocity ($\underline{N}_A$), is given on page 1.

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In terms of total molar flux, $\underline{N}_A$

Microscopic species mass balance, in terms of molar flux; Gibbs notation

$$\frac{\partial c_A}{\partial t} = -\nabla \cdot \underline{N}_A + R_A$$

Microscopic species mass balance, in terms of combined molar flux; Cartesian coordinates

$$\cancel{\frac{\partial c_A}{\partial t}} = -\left(\cancel{\frac{\partial N_{A,x}}{\partial x}} + \cancel{\frac{\partial N_{A,y}}{\partial y}} + \frac{\partial N_{A,z}}{\partial z}\right) + \cancel{R_A}$$

steady

no homogeneous rxn

Microscopic species mass balance, in terms of combined molar flux; cylindrical coordinates

$$\cancel{\frac{\partial c_A}{\partial t}} = -\left(\frac{1}{r}\frac{\partial (rN_{A,r})}{\partial r} + \cancel{\frac{1}{r}\frac{\partial N_{A,\theta}}{\partial \theta}} + \cancel{\frac{\partial N_{A,z}}{\partial z}}\right) + \cancel{R_A}$$

steady   θ symmetry   Long   no homogeneous rxn

Microscopic species mass balance, in terms of combined molar flux; spherical coordinates

$$\cancel{\frac{\partial c_A}{\partial t}} = -\left(\frac{1}{r^2}\frac{\partial (r^2 N_{A,r})}{\partial r} + \cancel{\frac{1}{r\sin\theta}\frac{\partial (N_{A,\theta}\sin\theta)}{\partial \theta}} + \cancel{\frac{1}{r\sin\theta}\frac{\partial N_{A,z}}{\partial \phi}}\right) + \cancel{R_A}$$

no homogeneous rxn

steady   θ symmetry   φ symmetry

Fick's law of diffusion, Gibbs notation: $\underline{N}_A = x_A(\underline{N}_A + \underline{N}_B) - cD_{AB}\nabla x_A$

$$= c_A \underline{v}^* - cD_{AB}\nabla x_A$$

Fick's law of diffusion, Cartesian coordinates:

$$\begin{pmatrix} N_{A,x} \\ N_{A,y} \\ N_{A,z} \end{pmatrix}_{xyz} = \begin{pmatrix} x_A(N_{A,x} + N_{B,x}) - cD_{AB}\frac{\partial x_A}{\partial x} \\ x_A(N_{A,y} + N_{B,y}) - cD_{AB}\frac{\partial x_A}{\partial y} \\ x_A(N_{A,z} + N_{B,z}) - cD_{AB}\frac{\partial x_A}{\partial z} \end{pmatrix}_{xyz}$$

Fick's law of diffusion, cylindrical coordinates:

$$\begin{pmatrix} N_{A,r} \\ N_{A,\theta} \\ N_{A,z} \end{pmatrix}_{r\theta z} = \begin{pmatrix} x_A(N_{A,r} + N_{B,r}) - cD_{AB}\frac{\partial x_A}{\partial r} \\ x_A(N_{A,\theta} + N_{B,\theta}) - \frac{cD_{AB}}{r}\frac{\partial x_A}{\partial \theta} \\ x_A(N_{A,z} + N_{B,z}) - cD_{AB}\frac{\partial x_A}{\partial z} \end{pmatrix}_{r\theta z}$$

Fick's law of diffusion, spherical coordinates:

$$\begin{pmatrix} N_{A,r} \\ N_{A,\theta} \\ N_{A,\phi} \end{pmatrix}_{r\theta\phi} = \begin{pmatrix} x_A(N_{A,r} + N_{B,r}) - cD_{AB}\frac{\partial x_A}{\partial r} \\ x_A(N_{A,\theta} + N_{B,\theta}) - \frac{cD_{AB}}{r}\frac{\partial x_A}{\partial \theta} \\ x_A(N_{A,\phi} + N_{B,\phi}) - \frac{cD_{AB}}{r\sin\theta}\frac{\partial x_A}{\partial \phi} \end{pmatrix}_{r\theta\phi}$$

c) spherical (see sheet)

$$0 = -\cancel{\frac{1}{r^2}}\frac{d}{dr}(r^2 N_{Ar})$$

$$\frac{d}{dr}\underbrace{(r^2 N_{Ar})}_{\equiv \Phi} = 0$$

$$\frac{d\Phi}{dr} = 0$$

$$r^2 N_{Ar} = \Phi = c_1$$

$$\boxed{N_{Ar} = \frac{c_1}{r^2}}$$

varies as $\frac{1}{r^2}$

4.)

$\longrightarrow z$

Water (A) (dilute) $\longrightarrow$ in AIR (B)

298 K

1.000 atm

$$j_{Az} = 5.00 \times 10^{-9} \frac{g\,(A)}{cm^2\,s}$$

what is $\dfrac{dw_A}{dz}$?

<u>Fick's Law</u>:

1D diffusion $\Rightarrow \partial \rightarrow d$

$$j_{Az} = -\rho\, D_{AB} \frac{\partial w_A}{\partial z}$$

$$j_{Az} = -\rho\, D_{AB} \frac{dw_A}{dz}$$

know

from ideal gas

from Table

IDEAL GAS:

$$PV = nRT$$

$$M_{mix} \frac{P}{RT} = \frac{n}{V} M_{mix}$$

$$\frac{\cancel{\text{moles}}}{\text{volume}} \quad \frac{g_{mix}}{\cancel{\text{mol}}_{mix}}$$

$= \rho$ ← density of the mixture

$$\rho = \frac{MP}{RT}$$

$$\approx \frac{M_B P}{RT}$$

Since the soln is dilute, $M \cong M_{AIR} = M_B = 29 \frac{g}{mol}$

Need D_{AB} (water/AIR).

→ Table J.1:

$$\boxed{D_{AB} P = 0.260 \frac{cm^2\ atm}{s}}$$

combine: $j_{Az} = -\rho D_{AB} \dfrac{dw_A}{dz}$

$$j_{Az} = -\frac{M_B P}{RT} D_{AB} \frac{dw_A}{dz}$$

$$\left(-\frac{dw_A}{dz}\right) = \frac{RT \; j_{Az}}{M_B \; (P D_{AB})}$$

$$= \frac{\left(5.00 \times 10^{-9} \frac{g_A}{cm^2 \cancel{s}}\right)\left(0.08206 \frac{\cancel{\ell} \cancel{atm}}{\cancel{mol} \cancel{K}}\right)\left(298 \cancel{K}\right)}{\left(29 \frac{g_{mix}}{\cancel{mol}}\right)\left(0.260 \frac{\cancel{cm^2} \cancel{atm}}{\cancel{s}}\right)\left(\frac{\cancel{10^3} \cancel{\ell}}{10^{\cancel{6}\,3} cm^3}\right)}$$

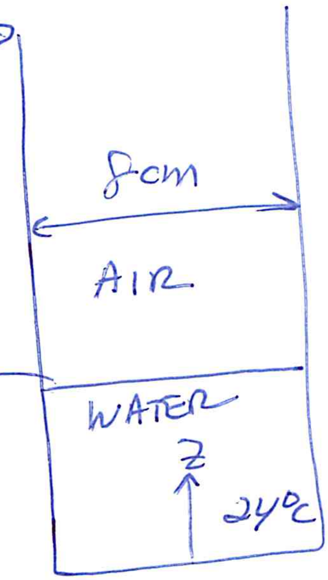
$$= 1.622 \times 10^{-5} \frac{g_A}{g_{mix}} \frac{1}{cm}$$

$$\boxed{\frac{dw_A}{dz} = -1.62 \times 10^{-5} \frac{g_A / g_{mix}}{cm}}$$

//

5.)

$z = z_2$
$x_A = x_{A2}$
$= 0.011$



$z = z_1$
$x_{A_1} = x_A = x_A^*$

$2R = 8$ cm
$R = 4$ cm

Steady
1D diffusion
Stagnant B
no homogeneous reaction

assume saturation near water surface

★ Use $\underline{N}_A$ for species A mass bal

$$x_A^* = \frac{P^*(24°C)}{P} = \frac{2.985}{101.325} = 0.02946$$

(TABLE A.2-9) GEANKOPLIS

<u>SPECIES</u> <u>A</u> <u>MASS</u> <u>BAL</u>

$$\frac{dN_{Az}}{dz} = 0$$

$$\boxed{N_{Az} = C_1}$$

B

The Equation of Species Mass Balance in Terms of Combined Molar quantities in Cartesian, cylindrical, and spherical coordinates for binary mixtures of A and B.

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Microscopic species mass balance, in terms of combined molar flux; Cartesian coordinates

$$\frac{\partial c_A}{\partial t} = -\left(\frac{\partial N_{A,x}}{\partial x} + \frac{\partial N_{A,y}}{\partial y} + \frac{\partial N_{A,z}}{\partial z}\right) + R_A$$

Microscopic species mass balance, in terms of combined molar flux; cylindrical coordinates

$$\frac{\partial c_A}{\partial t} = -\left(\frac{1}{r}\frac{\partial (rN_{A,r})}{\partial r} + \frac{1}{r}\frac{\partial N_{A,\theta}}{\partial \theta} + \frac{\partial N_{A,z}}{\partial z}\right) + R_A$$

Steady   wide   no homogeneous rxn

Microscopic species mass balance, in terms of combined molar flux; spherical coordinates

θ symmetry

$$\frac{\partial c_A}{\partial t} = -\left(\frac{1}{r^2}\frac{\partial (r^2 N_{A,r})}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial (N_{A,\theta}\sin\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial N_{A,z}}{\partial \phi}\right) + R_A$$

$$\Rightarrow \boxed{\frac{dN_{Az}}{dz} = 0}$$

Fick's law of diffusion, Gibbs notation: $\underline{N}_A = x_A(\underline{N}_A + \underline{N}_B) - cD_{AB}\nabla x_A$

STAGNANT

$$= c_A \underline{v}^* - cD_{AB}\nabla x_A$$

Fick's law of diffusion, Cartesian coordinates:

$$\begin{pmatrix} N_{A,x} \\ N_{A,y} \\ N_{A,z} \end{pmatrix}_{xyz} = \begin{pmatrix} x_A(N_{A,x} + N_{B,x}) - cD_{AB}\frac{\partial x_A}{\partial x} \\ x_A(N_{A,y} + N_{B,y}) - cD_{AB}\frac{\partial x_A}{\partial y} \\ x_A(N_{A,z} + N_{B,z}) - cD_{AB}\frac{\partial x_A}{\partial z} \end{pmatrix}_{xyz}$$

Fick's law of diffusion, cylindrical coordinates:

$$\begin{pmatrix} \cancel{N_{A,r}} \\ \cancel{N_{A,\theta}} \\ N_{A,z} \end{pmatrix}_{r\theta z} = \begin{pmatrix} \cancel{x_A(N_{A,r} + N_{B,r}) - cD_{AB}\frac{\partial x_A}{\partial r}} \\ \cancel{x_A(N_{A,\theta} + N_{B,\theta}) - \frac{cD_{AB}}{r}\frac{\partial x_A}{\partial \theta}} \\ \boxed{x_A(N_{A,z} + N_{B,z}) - cD_{AB}\frac{\partial x_A}{\partial z}} \end{pmatrix}_{r\theta z} \rightarrow$$

Fick's law of diffusion, spherical coordinates:

$$\begin{pmatrix} N_{A,r} \\ N_{A,\theta} \\ N_{A,\phi} \end{pmatrix}_{r\theta\phi} = \begin{pmatrix} x_A(N_{A,r} + N_{B,r}) - cD_{AB}\frac{\partial x_A}{\partial r} \\ x_A(N_{A,\theta} + N_{B,\theta}) - \frac{cD_{AB}}{r}\frac{\partial x_A}{\partial \theta} \\ x_A(N_{A,\phi} + N_{B,\phi}) - \frac{cD_{AB}}{r\sin\theta}\frac{\partial x_A}{\partial \phi} \end{pmatrix}_{r\theta\phi}$$

Fick's Law:

$$N_{Az} = \underbrace{x_A N_{Az}}\ - c D_{AB} \frac{dx_A}{dz}$$

$$\underbrace{N_{Az}}_{=C_1} (1 - x_A) = - c D_{AB} \frac{dx_A}{dz}$$

$$\int \left(\frac{-C_1}{c D_{AB}} \right) dz = (-1) \int \left(\frac{-dx_A}{1 - x_A} \right)$$

$\int \frac{du}{u} = \ln u + C_2$

$$\left(- \frac{C_1}{c D_{AB}} \right) z = - \ln (1 - x_A) + C_2$$

BC: $z = z_1 \quad x_A = x_{A1} = \frac{P^*(24°C)}{P}$

$z = z_2 \quad x_A = x_{A2} = 0.011$

$$\left(-\frac{c_1}{cD_{AB}}\right) z_1 = -\ln(1 - X_{A1}) + C_2$$

$$\left(\frac{-c_1}{cD_{AB}}\right) z_2 = -\ln(1 - X_{A2}) + C_2$$

subtract

$$\left(-\frac{c_1}{cD_{AB}}\right)(z_1 - z_2) = -\ln(1 - X_{A1}) + \ln(1 - X_{A2})$$

$$= \ln\left(\frac{1 - X_{A2}}{1 - X_{A1}}\right)$$

solve for $c_1 = N_{Az}$

$$\boxed{N_{Az} = C_1 = \frac{cD_{AB}}{(z_2 - z_1)} \ln\left(\frac{1 - X_{A2}}{1 - X_{A1}}\right)}$$

$$z_2 - z_1 = 10.0 \text{ cm}$$

$$c = \frac{\text{moles mix}}{\text{volume mix}}$$

IDEAL GAS: $PV = nRT$

$$\frac{P}{RT} = \frac{n}{V} = c$$

D_{AB} (air/water) From Table J.1:

$$D_{AB} P = 0.260 \ \frac{\text{cm}^2 \text{ atm}}{\text{s}}$$

combine:

$$N_{Az} = \frac{c D_{AB}}{(z_2 - z_1)} \ln\left(\frac{1 - x_{A2}}{1 - x_{A1}}\right)$$

$$= \frac{P D_{AB}}{RT (z_2 - z_1)} \ln\left(\frac{1 - x_{A2}}{1 - x_{A1}}\right)$$

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$$\ln\left(\frac{1-X_{A2}}{1-X_{A1}}\right) = \ln\left(\frac{\overbrace{1-0.011}^{0.9890}}{\underbrace{1-\underbrace{\frac{2.985}{101.325}}_{0.02946}}_{0.97054}}\right)$$

$$= 0.01884$$

$$N_{Az} = \frac{\left(0.260\ \frac{cm^2\ \cancel{atm}}{s}\right)(0.01884)}{\left(0.08206\ \frac{\cancel{l}\ \cancel{atm}}{mol\ \cancel{K}}\right)(297\ \cancel{K})(10\ cm)\left(\frac{10^3\ cm^3}{\cancel{1}\ \cancel{l}}\right)}$$

$$N_{Az} = 2.01001 \times 10^{-8}\ \frac{mol}{cm^2\ s}$$

$$\boxed{N_{Az} = 2.0 \times 10^{-8}\ \frac{mol}{cm^2 s}}$$