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FACTORS FOR UNIT CONVERSIONS

Quantity	Equivalent Values
Mass	1 kg = 1000 g = 0.001 metric ton = 2.20462 lb_m = 35.27392 oz
	1 lb_m = 16 oz = 5 x 10^{-4} ton = 453.593 g = 0.453593 kg
Length	1 m = 100 cm = 1000 mm = 10^6 microns (μm) = 10^{10} angstroms (Å)
	= 39.3701 in = 3.28084 ft = 1.09361 yd = 0.000621371 mile
	1 ft = 12 in. = 1/3 yd = 0.3048 m = 30.48 cm
Volume	1 m^3 = 1000 liters = 10^6 cm^3 = 10^6 ml
	= 35.31467 ft^3 = 219.969 imperial gallons = 264.172 gal
	= 1056.69 qt
	1 ft^3 = 1728 in^3 = 7.48052 gal = 0.028317 m^3 = 28.3168 liters
	= 28,316.8 cm^3
Force	1 N = 1 kg·m/s^2 = 10^5 dynes = 10^5 g·cm/s^2 = 0.22481 lb_f
	1 lb_f = 32.174 lb_m·ft/s^2 = 4.4482 N = 4.4482 x 10^5 dynes
Pressure	1 atm = 1.01325 x 10^5 N/m^2 (Pa) = 101.325 kPa = 1.01325 bars
	= 1.01325 x 10^6 dynes/cm^2
	= 760 mm Hg at 0˚ C (torr) = 10.333 m H_2O at 4˚ C
	= 14.696 lb_f/in^2 (psi) = 33.9 ft H_2O at 4˚C
	100 kPa = 1 bar
Energy	1 J = 1 N·m = 10^7 ergs = 10^7 dyne·cm
	= 2.778 x 10^{-7} kW·h = 0.23901 cal
	= 0.7376 ft·lb_f = 9.47817 x 10^{-4} Btu
Power	1 W = 1 J/s = 0.23885 cal/s = 0.7376 ft·lb_f/s = 9.47817 x 10^{-4} Btu/s = 3.4121 Btu/h
	= 1.341 x 10^{-3} hp (horsepower)
Viscosity	1 Pa·s = 1 N·s/m^2 = 1 kg/m·s
	= 10 poise = 10 dynes·s/cm^2 = 10 g/cm·s
	= 10^3 cp (centipoise)
	= 0.67197 lb_m/ft·s = 2419.088 lb_m/ft·h
Density	1 kg/m^3 = 10^{-3} g/cm^3
	= 0.06243 lb_m/ft^3
	10^3 kg/m^3 = 1 g/cm^3 = 62.428 lb_m/ft^3
Volumetric Flow	1 m^3/s= 35.31467 ft^3/s=15,850.32 gal/min (gpm)
	1 gpm = 6.30902 x 10^{-5} m^3/s=2.228009 x 10^{-3} ft^3/s=3.7854 liter/min
	1 liter/min=0.26417 gpm

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Temperature	$T(^oC) = \frac{5}{9}\left[T(^oF) - 32\right]$	
	$T(^oF) = \frac{9}{5}T(^oC) + 32 = 1.8T(^oC) + 32$	
Absolute Temperature	T(K) = T(˚C) + 273.15	
	T(˚R) = T(˚F) + 459.67	
Temperature Interval (ΔT)	1 C˚ = 1 K = 1.8 F˚ = 1.8 R˚	
	1F˚ = 1 R˚ = (5/9) C˚ = (5/9) K	

USEFUL QUANTITIES

SG = ρ(20˚C)/ρ_{water} (4˚C)

ρ_{water}(4˚C) = 1000 kg/m^3 = 62.43 lb$_m$/ft^3 = 1.000 g/cm^3

ρ_{water}(25˚C) = 997.08 kg/m^3 = 62.25 lb$_m$/ft^3 = 0.99709 g/cm^3

g = 9.8066 m/s^2 = 980.66 cm/s^2 = 32.174 ft/s^2

μ_{water} (25˚C) = 8.937 x 10^{-4} Pa·s = 8.937 x 10^{-4} kg/m·s
= 0.8937 cp =0.8937 x 10^{-2} g/cm·s = 6.005 x 10^{-4} lb$_m$/ft·s

Composition of air:

N_2	78.03%
O_2	20.99%
Ar	0.94%
CO_2	0.03%
H_2, He, Ne, Kr, Xe	0.01%
	100.00%

M_{air} = 29 g/mol = 29 kg/kmol = 29 lb$_m$/lbmole

$\hat{C}_{p,water}$ (25˚C) = 4.182 kJ/kg K = 0.9989 cal/g˚C = 0.9997 Btu/lb$_m$˚F

R = 8.314 m^3·Pa/mol·K = 0.08314 liter·bar/mol·K = 0.08206 liter·atm/mol·K
= 62.36 liter·mm Hg/mol·K = 0.7302 ft^3·atm/lbmole·˚R
= 10.73 ft^3·psia/lbmole·˚R
= 8.314 J/mol·K
= 1.987 cal/mol·K = 1.987 Btu/lbmole·˚R

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The Equation of Continuity and the Equation of Motion in Cartesian, cylindrical, and spherical coordinates

CM3110 Fall 2011 Faith A. Morrison

Continuity Equation, Cartesian coordinates

$$\frac{\partial \rho}{\partial t} + \left(v_x \frac{\partial \rho}{\partial x} + v_y \frac{\partial \rho}{\partial y} + v_z \frac{\partial \rho}{\partial z} \right) + \rho \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) = 0$$

Continuity Equation, cylindrical coordinates

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial(\rho r v_r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho v_\theta)}{\partial \theta} + \frac{\partial(\rho v_z)}{\partial z} = 0$$

Continuity Equation, spherical coordinates

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial(\rho r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\rho v_\theta \sin\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial(\rho v_\phi)}{\partial \phi} = 0$$

Equation of Motion for an incompressible fluid, 3 components in Cartesian coordinates

$$\begin{aligned}
\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) &= -\frac{\partial P}{\partial x} + \left(\frac{\partial \tilde{\tau}_{xx}}{\partial x} + \frac{\partial \tilde{\tau}_{yx}}{\partial y} + \frac{\partial \tilde{\tau}_{zx}}{\partial z}\right) + \rho g_x \\
\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) &= -\frac{\partial P}{\partial y} + \left(\frac{\partial \tilde{\tau}_{xy}}{\partial x} + \frac{\partial \tilde{\tau}_{yy}}{\partial y} + \frac{\partial \tilde{\tau}_{zy}}{\partial z}\right) + \rho g_y \\
\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) &= -\frac{\partial P}{\partial z} + \left(\frac{\partial \tilde{\tau}_{xz}}{\partial x} + \frac{\partial \tilde{\tau}_{yz}}{\partial y} + \frac{\partial \tilde{\tau}_{zz}}{\partial z}\right) + \rho g_z
\end{aligned}$$

Equation of Motion for an incompressible fluid, 3 components in cylindrical coordinates

$$\begin{aligned}
\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) &= -\frac{\partial P}{\partial r} + \left(\frac{1}{r}\frac{\partial(r\tilde{\tau}_{rr})}{\partial r} + \frac{1}{r}\frac{\partial \tilde{\tau}_{\theta r}}{\partial \theta} - \frac{\tilde{\tau}_{\theta\theta}}{r} + \frac{\partial \tilde{\tau}_{zr}}{\partial z}\right) + \rho g_r \\
\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\theta v_r}{r} + v_z\frac{\partial v_\theta}{\partial z}\right) &= -\frac{1}{r}\frac{\partial P}{\partial \theta} + \left(\frac{1}{r^2}\frac{\partial(r^2\tilde{\tau}_{r\theta})}{\partial r} + \frac{1}{r}\frac{\partial \tilde{\tau}_{\theta\theta}}{\partial \theta} + \frac{\partial \tilde{\tau}_{z\theta}}{\partial z} + \frac{\tilde{\tau}_{\theta r} - \tilde{\tau}_{r\theta}}{r}\right) + \rho g_\theta \\
\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) &= -\frac{\partial P}{\partial z} + \left(\frac{1}{r}\frac{\partial(r\tilde{\tau}_{rz})}{\partial r} + \frac{1}{r}\frac{\partial \tilde{\tau}_{\theta z}}{\partial \theta} + \frac{\partial \tilde{\tau}_{zz}}{\partial z}\right) + \rho g_z
\end{aligned}$$

Equation of Motion for an incompressible fluid, 3 components in spherical coordinates

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r}\right)$$

$$= -\frac{\partial P}{\partial r} + \left(\frac{1}{r^2}\frac{\partial(r^2\tilde{\tau}_{rr})}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\tilde{\tau}_{\theta r}\sin\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial \tilde{\tau}_{\phi r}}{\partial \phi} - \frac{\tilde{\tau}_{\theta\theta} + \tilde{\tau}_{\phi\phi}}{r}\right) + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta}{r} - \frac{v_\phi^2\cot\theta}{r}\right)$$

$$= -\frac{1}{r}\frac{\partial P}{\partial \theta} + \left(\frac{1}{r^3}\frac{\partial(r^3\tilde{\tau}_{r\theta})}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\tilde{\tau}_{\theta\theta}\sin\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial \tilde{\tau}_{\phi\theta}}{\partial \phi} + \frac{\tilde{\tau}_{\theta r} - \tilde{\tau}_{r\theta}}{r} - \frac{\tilde{\tau}_{\phi\phi}\cot\theta}{r}\right) + \rho g_\theta$$

$$\rho\left(\frac{\partial v_\phi}{\partial t} + v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r v_\phi}{r} + \frac{v_\phi v_\theta\cot\theta}{r}\right)$$

$$= -\frac{1}{r\sin\theta}\frac{\partial P}{\partial \phi} + \left(\frac{1}{r^3}\frac{\partial(r^3\tilde{\tau}_{r\phi})}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial\left(\tilde{\tau}_{\theta\phi}\sin\theta\right)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial \tilde{\tau}_{\phi\phi}}{\partial \phi} + \frac{\tilde{\tau}_{\phi r} - \tilde{\tau}_{r\phi}}{r} + \frac{\tilde{\tau}_{\phi\theta}\cot\theta}{r}\right) + \rho g_\phi$$

Equation of Motion for incompressible, Newtonian fluid (Navier-Stokes equation) 3 components in Cartesian coordinates

$$\rho\left(\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial P}{\partial x}+\mu\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right)+\rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial P}{\partial y}+\mu\left(\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}+\frac{\partial^2 v_y}{\partial z^2}\right)+\rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial P}{\partial z}+\mu\left(\frac{\partial^2 v_z}{\partial x^2}+\frac{\partial^2 v_z}{\partial y^2}+\frac{\partial^2 v_z}{\partial z^2}\right)+\rho g_z$$

Equation of Motion for incompressible, Newtonian fluid (Navier-Stokes equation), 3 components in cylindrical coordinates

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}-\frac{v_\theta^2}{r}+v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial P}{\partial r}+\mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r v_r)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}+\frac{\partial^2 v_r}{\partial z^2}\right)+\rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}\right) = -\frac{1}{r}\frac{\partial P}{\partial \theta}+\mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r v_\theta)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}+\frac{\partial^2 v_\theta}{\partial z^2}\right)+\rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial P}{\partial z}+\mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right)+\rho g_z$$

Equation of Motion for incompressible, Newtonian fluid (Navier-Stokes equation), 3 components in spherical coordinates

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi}-\frac{v_\theta^2+v_\phi^2}{r}\right)$$
$$= -\frac{\partial P}{\partial r}+\mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 v_r\right)\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial v_r}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_r}{\partial \phi^2}\right.$$
$$\left.-\frac{2}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(v_\theta\sin\theta\right)-\frac{2}{r^2\sin\theta}\frac{\partial v_\phi}{\partial \phi}\right)+\rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi}+\frac{v_r v_\theta}{r}-\frac{v_\phi^2\cot\theta}{r}\right)$$
$$= -\frac{1}{r}\frac{\partial P}{\partial \theta}+\mu\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\theta}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(v_\theta\sin\theta\right)\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\theta}{\partial \phi^2}\right.$$
$$\left.+\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}-\frac{2\cot\theta}{r^2\sin\theta}\frac{\partial v_\phi}{\partial \phi}\right)+\rho g_\theta$$

$$\rho\left(\frac{\partial v_\phi}{\partial t}+v_r\frac{\partial v_\phi}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}+\frac{v_r v_\phi}{r}+\frac{v_\phi v_\theta\cot\theta}{r}\right)$$
$$= -\frac{1}{r\sin\theta}\frac{\partial P}{\partial \phi}+\mu\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\phi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(v_\phi\sin\theta\right)\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\phi}{\partial \phi^2}\right.$$
$$\left.+\frac{2}{r^2\sin\theta}\frac{\partial v_r}{\partial \phi}+\frac{2\cot\theta}{r^2\sin\theta}\frac{\partial v_\theta}{\partial \phi}\right)+\rho g_\phi$$

Note: the r-component of the Navier-Stokes equation in spherical coordinates may be simplified by adding $0 = \frac{2}{r}\nabla\cdot\underline{v}$ to the component shown above. This term is zero due to the continuity equation (mass conservation). See Bird et. al.

References:

1. R. B. Bird, W. E. Stewart, and E. N. Lightfoot, *Transport Phenomena*, 2^{nd} edition, Wiley: NY, 2002.

2. R. B. Bird, R. C. Armstrong, and O. Hassager, *Dynamics of Polymeric Fluids: Volume 1 Fluid Mechanics*, Wiley: NY, 1987.

The **Equation of Energy** in Cartesian, cylindrical, and spherical coordinates for Newtonian fluids of constant density, with source term S. Source could be electrical energy due to current flow, chemical energy, etc. Two cases are presented: the general case where thermal conductivity may be a function of temperature (vector flux $\underline{\tilde{q}} = \underline{q}/A$ appears in the equations); and the more usual case, where thermal conductivity is constant.

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Microscopic energy balance, in terms of flux; Gibbs notation

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+\underline{v}\cdot\nabla T\right)=-\nabla\cdot\underline{\tilde{q}}+S$$

Microscopic energy balance, in terms of flux; Cartesian coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_x\frac{\partial T}{\partial x}+v_y\frac{\partial T}{\partial y}+v_z\frac{\partial T}{\partial z}\right)=-\left(\frac{\partial\tilde{q}_x}{\partial x}+\frac{\partial\tilde{q}_y}{\partial y}+\frac{\partial\tilde{q}_z}{\partial z}\right)+S$$

Microscopic energy balance, in terms of flux; cylindrical coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial\theta}+v_z\frac{\partial T}{\partial z}\right)=-\left(\frac{1}{r}\frac{\partial(r\tilde{q}_r)}{\partial r}+\frac{1}{r}\frac{\partial\tilde{q}_\theta}{\partial\theta}+\frac{\partial\tilde{q}_z}{\partial z}\right)+S$$

Microscopic energy balance, in terms of flux; spherical coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial\theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial T}{\partial\phi}\right)=-\left(\frac{1}{r^2}\frac{\partial(r^2\tilde{q}_r)}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial(\tilde{q}_\theta sin\theta)}{\partial\theta}+\frac{1}{r\sin\theta}\frac{\partial\tilde{q}_\phi}{\partial\phi}\right)+S$$

Fourier's law of heat conduction, Gibbs notation: $\underline{\tilde{q}} = -k\nabla T$

Fourier's law of heat conduction, Cartesian coordinates: $$\begin{pmatrix}\tilde{q}_x\\ \tilde{q}_y\\ \tilde{q}_z\end{pmatrix}_{xyz}=\begin{pmatrix}-k\frac{\partial T}{\partial x}\\ -k\frac{\partial T}{\partial y}\\ -k\frac{\partial T}{\partial z}\end{pmatrix}_{xyz}$$

Fourier's law of heat conduction, cylindrical coordinates: $$\begin{pmatrix}\tilde{q}_r\\ \tilde{q}_\theta\\ \tilde{q}_z\end{pmatrix}_{xyz}=\begin{pmatrix}-k\frac{\partial T}{\partial r}\\ -\frac{k}{r}\frac{\partial T}{\partial\theta}\\ -k\frac{\partial T}{\partial z}\end{pmatrix}_{r\theta z}$$

Fourier's law of heat conduction, spherical coordinates: $$\begin{pmatrix}\tilde{q}_r\\ \tilde{q}_\theta\\ \tilde{q}_\phi\end{pmatrix}_{xyz}=\begin{pmatrix}-k\frac{\partial T}{\partial r}\\ -\frac{k}{r}\frac{\partial T}{\partial\theta}\\ -\frac{k}{r\sin\theta}\frac{\partial T}{\partial\phi}\end{pmatrix}_{r\theta\phi}$$

The **Equation of Energy** for systems with **constant k**

Microscopic energy balance, constant thermal conductivity; Gibbs notation

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+\underline{v}\cdot\nabla T\right)=k\nabla^2 T+S$$

Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_x\frac{\partial T}{\partial x}+v_y\frac{\partial T}{\partial y}+v_z\frac{\partial T}{\partial z}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+S$$

Microscopic energy balance, constant thermal conductivity; cylindrical coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}+v_z\frac{\partial T}{\partial z}\right)=k\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}\right)+S$$

Microscopic energy balance, constant thermal conductivity; spherical coordinates

$$\begin{aligned}\rho\hat{C}_p&\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial T}{\partial \phi}\right)\\&=k\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \phi^2}\right)+S\end{aligned}$$

Reference: F. A. Morrison, "Web Appendix to *An Introduction to Fluid Mechanics*," Cambridge University Press, New York, 2013. On the web at www.chem.mtu.edu/~fmorriso/IFM_WebAppendixCD2013.pdf

Energy Balance Notes

CM2110/CM3110
Professor Faith A. Morrison
December 4, 2008

References

1. (FR) R. M. Felder, and R. W. Rousseau, *Elementary Principles of Chemical Processe,* 2nd Edition (Wiley, NY: 1986).
2. (G) C. J. Geankoplis, *Transport Processes and Unit Operations*, 3rd Edition (Prentice Hall: Englewood Cliffs, NJ, 1993).

1. Closed System (note: $\Delta = \Sigma_{final} - \Sigma_{initial}$)
    - $\Delta E_k + \Delta E_p + \Delta U = Q_{in} + W_{on}$ (FR)
    - Is it adiabatic? (if yes, Q_{in}=0)
    - Are there moving parts, e.g. do the walls move? (if no, W_{on}=0)
    - Is the *system* moving? (if no, ΔE_k=0)
    - Is there a change in elevation of the *system*? (if no, $\Delta E_p = 0$)
    - Does T, phase, or chemical composition change? (if no to all, ΔU=0)

2. Open System (the fluid is the system) (note: $\Delta = \Sigma_{out} - \Sigma_{in}$)
    1. Is it a Mechanical Energy Balance (MEB) problem?
       (turbulent, α=1; laminar, α=0.5; *F*= total frictional loss)
        - $\frac{\Delta P}{\rho} + \frac{1}{2\alpha}\Delta v^2 + g\,\Delta z + F = \frac{W_{s,on\,fluid}}{m}$ (FR)
        - $\frac{\Delta P}{\rho} + \frac{1}{2\alpha}\Delta v^2 + g\,\Delta z + F = -W_{s,by\,fluid}$ (G)

       The mechanical energy balance is only valid for systems for which the following is true:

        i. single-input, single output
        ii. small or zero Q_{in}
        iii. incompressible fluid (ρ = constant)
        iv. small or zero ΔT

    2. Is it a regular open system balance?
        - $\Delta E_k + \Delta E_p + \Delta H = Q_{in} + W_{s,on}$ (FR)
        - Is it adiabatic? (if yes, Q_{in}=0)
        - Are there moving parts, e.g. pump, turbine, mixing shaft? (if no, $W_{s,on}$=0)

- Does the average velocity of the fluid change between the input and the output? (if no, ΔE_k=0); remember $\langle v \rangle = v_{av} = v = \frac{v(\frac{m^3}{s})}{A(m^2)}$, where ($\frac{m^3}{s}$) is volumetric flow rate.
- Is there a change in elevation of the system between the input and the output? (if no, $\Delta E_p = 0$)
- Does T, phase, chemical composition, or **P change**? (if no to all, ΔH=0)

Calculating Internal Energy

1. Constant T, **P changes only**
    (a) real gases => look it up in a table (e.g. **steam**, Tables B4, B5, B6)
    (b) ideal gases => $\Delta\hat{u}$=0
    (c) liquids, solids => $\Delta\hat{u}$=0
2. Constant P, **T changes only**
    (a) real gases => look it up in a table (e.g. **steam**),
    or, if *V* is constant, $\Delta\hat{u} = \int_{T_1}^{T_2} \hat{C}_v\,(T)\,dT$
    (b) ideal gases => $\Delta\hat{u} = \int_{T_1}^{T_2} \hat{C}_v\,(T)\,dT$
    also, $\hat{C}_p = \hat{C}_v + R$
    (c) liquids, solids $\Delta\hat{u} = \int_{T_1}^{T_2} \hat{C}_v\,(T)\,dT$
    also $\hat{C}_p \approx \hat{C}_v$
3. Constant T, P, **phase changes**
    (a) real gases => look it up in a table (e.g. **steam**)
    (b) liquid to vapor => $\Delta\hat{u} = \Delta\hat{H}_{vap}(T) - P\Delta\hat{V}_{vap} \approx \Delta\hat{H}_{vap} - RT$
    (c) solid to vapor => $\Delta\hat{u} = \Delta\hat{H}_{sub}(T) - P\Delta\hat{V}_{sub} \approx \Delta\hat{H}_{sub} - RT$
    (d) solid to liquid => $\Delta\hat{u} = \Delta\hat{H}_{melt}(T) - P\Delta\hat{V}_{melt} \approx \Delta\hat{H}_{melt}$
4. Constant T, P, **mixing occurs**
    (a) gases => $\Delta\hat{u}$ =0
    (b) similar liquids => $\Delta\hat{u}$ =0
    (c) dissimilar liquids/solids => $\Delta\hat{u} = \Delta\hat{H}_{solution}$, Table 8.5-1, FR page 380
    Note: be careful with units, $\Delta\hat{H}_{solution}$ [=] $\frac{kJ}{mole\ solute}$
5. Constant T, P, **reaction occurs:** $\Delta\hat{u} = \Delta\hat{H}_{rxn}$

Calculating Enthalpy

1. Constant T, **P changes only** (Note: Since T is constant, $\hat{U}$ does not change.)
    (a) real gases - look it up in a table (e.g. **steam**, Tables B4, B5, B6)
    (b) ideal gases

$$\hat{H} = \hat{U} + P\hat{V} \quad (1)$$
$$= \hat{U} + RT \quad (2)$$
$$(\hat{H}_2 - \hat{H}_1) = (\hat{U}_2 - \hat{U}_1) + R(T_2 - T_1) \quad (3)$$
$$\Delta\hat{H} = \Delta\hat{U} = 0 \quad (4)$$

    (c) liquids, solids

$$\hat{H} = \hat{U} + P\hat{V} \quad (5)$$
$$\Delta\hat{H} = \Delta(P\hat{V}) \quad (6)$$
$$\hat{V} \approx \text{constant wrt } P \quad (7)$$
$$\Delta\hat{H} = \hat{V}(\Delta P) \quad (8)$$

2. Constant P, **T changes only**
    (a) real gases => look it up in a table to be most accurate (e.g. **steam**),
    otherwise $\Delta\hat{H} = \int_{T_1}^{T_2} \hat{C}_p\,(T)\,dT$
    (b) ideal gases => $\Delta\hat{H} = \int_{T_1}^{T_2} \hat{C}_p\,(T)\,dT$
    (c) liquids, solids => $\Delta\hat{H} = \int_{T_1}^{T_2} \hat{C}_p\,(T)\,dT$
3. Constant T, P, **phase changes**
    (a) liquid to vapor => $\Delta\hat{H} = \Delta\hat{H}_{vap}(T)$
    Note: $\frac{d\ \ln P*}{d \ln 1/T} = \frac{\Delta\hat{H}_{vap}}{R}$ Clapeyron equation
    (b) solid to vapor => $\Delta\hat{H} = \Delta\hat{H}_{sub}(T)$
    (c) solid to liquid => $\Delta\hat{H} = \Delta\hat{H}_{melt}(T)$
4. Constant T, P, **mixing occurs**
    (a) gases => $\Delta\hat{H}$ =0
    (b) similar liquids => $\Delta\hat{H}$ =0
    (c) dissimilar liquids/solids => $\Delta\hat{H} = \Delta\hat{H}_{solution}$, Table 8.5-1, FR page 380
    Note: be careful with units, $\Delta\hat{H}_{solution}$ [=] $\frac{J}{mole\ solute}$
5. Constant T, P, **reaction occurs::** $\Delta\hat{u} = \Delta\hat{H}_{rxn}$

Welty, Rorrer, Foster 6th Ed Appendix (2015)

Gases

T (°F)	ρ (lbm/ft³)	c_p (Btu/lbm °F)	μ × 10⁵ (lbm/ft s)	ν × 10³ (ft²/s)	k (Btu/h ft °F)	α (ft²/h)	Pr	β × 10³ (1/°F)	gβρ²/μ² (1/°F · ft³)
				Air					
0	0.0862	0.240	1.09	0.126	0.0132	0.639	0.721	2.18	4.39 × 10⁶
30	0.0810	0.240	1.15	0.142	0.0139	0.714	0.716	2.04	3.28
60	0.0764	0.240	1.21	0.159	0.0146	0.798	0.711	1.92	2.48
80	0.0735	0.240	1.24	0.169	0.0152	0.855	0.708	1.85	2.09
100	0.0710	0.240	1.28	0.181	0.0156	0.919	0.703	1.79	1.76
150	0.0651	0.241	1.36	0.209	0.0167	1.06	0.698	1.64	1.22
200	0.0602	0.241	1.45	0.241	0.0179	1.24	0.694	1.52	0.840
250	0.0559	0.242	1.53	0.274	0.0191	1.42	0.690	1.41	0.607
300	0.0523	0.243	1.60	0.306	0.0203	1.60	0.686	1.32	0.454
400	0.0462	0.245	1.74	0.377	0.0225	2.00	0.681	1.16	0.264
500	0.0413	0.247	1.87	0.453	0.0246	2.41	0.680	1.04	0.163
600	0.0374	0.251	2.00	0.535	0.0270	2.88	0.680	0.944	79.4 × 10³
800	0.0315	0.257	2.24	0.711	0.0303	3.75	0.684	0.794	50.6
1000	0.0272	0.263	2.46	0.906	0.0337	4.72	0.689	0.685	27.0
1500	0.0203	0.277	2.92	1.44	0.0408	7.27	0.705	0.510	7.96

T (K)	ρ (kg/m³)	c_p × 10⁻³ (J/kg × K)	μ × 10⁵ (Pa × s)	ν × 10⁵ (m²/s)	k × 10² (W/m × K)	α × 10⁵ (m²/s)	Pr	gβρ²/μ² (1/K · m³)
				Air				
250	1.4133	1.0054	1.5991	1.1315	2.2269	1.5672	0.722	4.638 × 10⁸
260	1.3587	1.0054	1.6503	1.2146	2.3080	1.6896	0.719	2.573
280	1.2614	1.0057	1.7503	1.3876	2.4671	1.9448	0.713	1.815
300	1.1769	1.0063	1.8464	1.5689	2.6240	2.2156	0.708	1.327
320	1.1032	1.0073	1.9391	1.7577	2.7785	2.5003	0.703	0.9942
340	1.0382	1.0085	2.0300	1.9553	2.9282	2.7967	0.699	0.7502
360	0.9805	1.0100	2.1175	2.1596	3.0779	3.1080	0.695	0.5828
400	0.8822	1.0142	2.2857	2.5909	3.3651	3.7610	0.689	0.3656
440	0.8021	1.0197	2.4453	3.0486	3.6427	4.4537	0.684	0.2394
480	0.7351	1.0263	2.5963	3.5319	3.9107	5.1836	0.681	0.1627
520	0.6786	1.0339	2.7422	4.0410	4.1690	5.9421	0.680	0.1156
580	0.6084	1.0468	2.9515	4.8512	4.5407	7.1297	0.680	7.193 × 10⁶
700	0.5040	1.0751	3.3325	6.6121	5.2360	9.6632	0.684	3.210
800	0.4411	1.0988	3.6242	8.2163	5.7743	11.9136	0.689	1.804
1000	0.3529	1.1421	4.1527	11.1767	6.7544	16.7583	0.702	0.803

T (°F)	ρ (lbm/ft³)	c_p (Btu/lbm °F)	μ × 10⁵ (lbm/ft s)	ν × 10³ (ft²/s)	k (Btu/h ft °F)	α (ft²/h)	Pr	β × 10³ (1/°F)	gβρ²/μ² (1/°F · ft³)
					Steam				
212	0.0372	0.493	0.870	0.234	0.0145	0.794	1.06	1.49	0.873 × 10⁶
250	0.0350	0.483	0.890	0.254	0.0155	0.920	0.994	1.41	0.698
300	0.0327	0.476	0.960	0.294	0.0171	1.10	0.963	1.32	0.493
400	0.0289	0.472	1.09	0.377	0.0200	1.47	0.924	1.16	0.262
500	0.0259	0.477	1.23	0.474	0.0228	1.85	0.922	1.04	0.148
600	0.0234	0.483	1.37	0.585	0.0258	2.29	0.920	0.944	88.9 × 10³
800	0.0197	0.498	1.63	0.828	0.0321	3.27	0.912	0.794	37.8
1000	0.0170	0.517	1.90	1.12	0.0390	4.44	0.911	0.685	17.2
1500	0.0126	0.564	2.57	2.05	0.0580	8.17	0.906	0.510	3.97

T (K)	ρ (kg/m³)	c_p × 10⁻³ (J/kg · K)	μ × 10⁵ (Pa · s)	ν × 10⁵ (m²/s)	k × 10² (W/m · K)	α × 10⁵ (m²/s)	Pr	gβρ²/μ² (1/K · m³)
					Steam			
380	0.5860	2.0592	12.70	2.1672	2.4520	2.0320	1.067	5.5210 × 10⁷
400	0.5549	2.0098	13.42	2.4185	2.6010	2.3322	1.037	4.1951
450	0.4911	1.9771	15.23	3.1012	2.9877	3.0771	1.008	2.2558
500	0.4410	1.9817	17.03	3.8617	3.3903	3.8794	0.995	1.3139
550	0.4004	2.0006	18.84	4.7053	3.8008	4.7448	0.992	0.8069
600	0.3667	2.0264	20.64	5.6286	4.2161	5.6738	0.992	0.5154
650	0.3383	2.0555	22.45	6.6361	4.6361	6.6670	0.995	0.3415
700	0.3140	2.0869	24.25	7.7229	5.0593	7.7207	1.000	0.2277
750	0.2930	2.1192	26.06	8.8942	5.4841	8.8321	1.007	0.1651
800	0.2746	2.1529	27.86	10.1457	5.9089	9.9950	1.015	0.1183

T (°F)	ρ (lbm/ft³)	c_p (Btu/lbm °F)	μ × 10⁵ (lbm/ft s)	ν × 10³ (ft²/s)	k (Btu/h ft °F)	α (ft²/h)	Pr	β × 10³ (1/°F)	gβρ²/μ² (1/°F · ft³)
					Nitrogen				
0	0.0837	0.249	1.06	0.127	0.0132	0.633	0.719	2.18	4.38 × 10⁶
30	0.0786	0.249	1.12	0.142	0.0139	0.710	0.719	2.04	3.29
60	0.0740	0.249	1.17	0.158	0.0146	0.800	0.716	1.92	2.51
80	0.0711	0.249	1.20	0.169	0.0151	0.853	0.712	1.85	2.10
100	0.0685	0.249	1.23	0.180	0.0154	0.915	0.708	1.79	1.79
150	0.0630	0.249	1.32	0.209	0.0168	1.07	0.702	1.64	1.22
200	0.0580	0.249	1.39	0.240	0.0174	1.25	0.690	1.52	0.854
250	0.0540	0.249	1.47	0.271	0.0192	1.42	0.687	1.41	0.616
300	0.0502	0.250	1.53	0.305	0.0202	1.62	0.685	1.32	0.457
400	0.0443	0.250	1.67	0.377	0.0212	2.02	0.684	1.16	0.263
500	0.0397	0.253	1.80	0.453	0.0244	2.43	0.683	1.04	0.163
600	0.0363	0.256	1.93	0.532	0.0252	2.81	0.686	0.944	0.108
800	0.0304	0.262	2.16	0.710	0.0291	3.71	0.691	0.794	0.0507
1000	0.0263	0.269	2.37	0.901	0.0336	4.64	0.700	0.685	0.0272
1500	0.0195	0.283	2.82	1.45	0.0423	7.14	0.732	0.510	0.00785

T (°F)	ρ (lb_m/ft^3)	c_p ($Btu/lb_m\,°F$)	$\mu \times 10^7$ ($lb_m/ft\,s$)	$\nu \times 10^3$ (ft^2/s)	k (Btu/h ft °F)	α (ft^2/h)	Pr	$\beta \times 10^3$ (1/°F)	$g\beta\rho^2/\mu^2$ ($1/°F \cdot ft^3$)
				Helium					
0	0.0119	1.24	122	1.03	0.0784	5.30	0.698	2.18	66,800
30	0.0112	1.24	127	1.14	0.0818	5.89	0.699	2.04	51,100
60	0.0106	1.24	132	1.25	0.0852	6.46	0.700	1.92	40,000
80	0.0102	1.24	135	1.32	0.0872	6.88	0.701	1.85	33,900
100	0.00980	1.24	138	1.41	0.0892	7.37	0.701	1.79	29,000
150	0.00900	1.24	146	1.63	0.0937	8.36	0.703	1.64	20,100
200	0.00829	1.24	155	1.87	0.0977	9.48	0.705	1.52	14,000
250	0.00772	1.24	162	2.09	0.102	10.7	0.707	1.41	10,400
300	0.00722	1.24	170	2.36	0.106	11.8	0.709	1.32	7,650
400	0.00637	1.24	185	2.91	0.114	14.4	0.714	1.16	4,410
500	0.00572	1.24	198	3.46	0.122	17.1	0.719	1.04	2,800
600	0.00517	1.24	209	4.04	0.130	20.6	0.720	0.994	1,850
800	0.00439	1.24	232	5.28	0.145	27.6	0.722	0.794	915
1000	0.00376	1.24	255	6.78	0.159	35.5	0.725	0.685	480
1500	0.00280	1.24	309	11.1	0.189	59.7	0.730	0.510	135

T (°F)	ρ (lb_m/ft^3)	c_p ($Btu/lb_m\,°F$)	$\mu \times 10^5$ ($lb_m/ft\,s$)	$\nu \times 10^3$ (ft^2/s)	k (Btu/h ft °F)	α (ft^2/h)	Pr	$\beta \times 10^3$ (1/°F)	$g\beta\rho^2/\mu^2$ ($1/°F \cdot ft^3$)
				Sulfur dioxide					
0	0.195	0.142	0.700	3.59	0.00460	0.166	0.778	2.03	50.6×10^6
100	0.161	0.149	0.890	5.52	0.00560	0.233	0.854	1.79	19.0
200	0.136	0.157	1.05	7.74	0.00670	0.313	0.883	1.52	8.25
300	0.118	0.164	1.20	10.2	0.00790	0.407	0.898	1.32	4.12
400	0.104	0.170	1.35	13.0	0.00920	0.520	0.898	1.16	2.24
500	0.0935	0.176	1.50	16.0	0.00990	0.601	0.958	1.04	1.30
600	0.0846	0.180	1.65	19.5	0.0108	0.711	0.987	0.994	0.795

Liquids

T (°F)	ρ (lb_m/ft^3)	c_p ($Btu/lb_m\,°F$)	$\mu \times 10^3$ ($lb_m/ft\,s$)	$\nu \times 10^5$ (ft^2/s)	k (Btu/h ft °F)	$\alpha \times 10^3$ (ft^2/h)	Pr	$\beta \times 10^4$ (1/°F)	$g\beta\rho^2/\mu^2 \times 10^{-6}$ ($1/°F \cdot ft^3$)
				Water					
32	62.4	1.01	1.20	1.93	0.319	5.06	13.7	−0.350	
60	62.3	1.00	0.760	1.22	0.340	5.45	8.07	0.800	17.2
80	62.2	0.999	0.578	0.929	0.353	5.67	5.89	1.30	48.3
100	62.1	0.999	0.458	0.736	0.364	5.87	4.51	1.80	107
150	61.3	1.00	0.290	0.474	0.383	6.26	2.72	2.80	403
200	60.1	1.01	0.206	0.342	0.392	6.46	1.91	3.70	1,010
250	58.9	1.02	0.160	0.272	0.395	6.60	1.49	4.70	2,045
300	57.3	1.03	0.130	0.227	0.395	6.70	1.22	5.60	3,510
400	53.6	1.08	0.0930	0.174	0.382	6.58	0.950	7.80	8,350
500	49.0	1.19	0.0700	0.143	0.349	5.98	0.859	11.0	17,350
600	42.4	1.51	0.0579	0.137	0.293	4.58	1.07	17.5	30,300

T (K)	ρ (kg/m^3)	c_p (J/kg × K)	$\mu \times 10^6$ (Pa × s)	$\nu \times 10^6$ (m^2/s)	k (W/m × K)	$\alpha \times 10^6$ (m^2/s)	Pr	$g\beta\rho^2/\mu^2 \times 10^{-9}$ ($1/K \cdot m^3$)
				Water				
273	999.3	4226	1794	1.795	0.558	0.132	13.6	
293	998.2	4182	993	0.995	0.597	0.143	6.96	2.035
313	992.2	4175	658	0.663	0.633	0.153	4.33	8.833
333	983.2	4181	472	0.480	0.658	0.160	3.00	22.75
353	971.8	4194	352	0.362	0.673	0.165	2.57	46.68
373	958.4	4211	278	0.290	0.682	0.169	1.72	85.09
473	862.8	4501	139	0.161	0.665	0.171	0.94	517.2
573	712.5	5694	92.2	0.129	0.564	0.139	0.93	1766.0

T (°F)	ρ (lb_m/ft^3)	c_p ($Btu/lb_m\,°F$)	$\mu \times 10^5$ ($lb_m/ft\,s$)	$\nu \times 10^5$ (ft^2/s)	k (Btu/h ft °F)	$\alpha \times 10^3$ (ft^2/h)	Pr	$\beta \times 10^3$ (1/°F)	$g\beta\rho^2/\mu^2 \times 10^{-6}$ ($1/°F \cdot ft^3$)
				Aniline					
60	64.0	0.480	305	4.77	0.101	3.29	52.3		
80	63.5	0.485	240	3.78	0.100	3.25	41.8		
100	63.0	0.490	180	2.86	0.100	3.24	31.8	0.45	17.7
150	61.6	0.503	100	1.62	0.0980	3.16	18.4		
200	60.2	0.515	62	1.03	0.0962	3.10	12.0		
250	58.9	0.527	42	0.714	0.0947	3.05	8.44		
300	57.5	0.540	30	0.522	0.0931	2.99	6.28		

T (°F)	ρ (lb_m/ft^3)	c_p ($Btu/lb_m\,°F$)	$\mu \times 10^5$ ($lb_m/ft\,s$)	$\nu \times 10^5$ (ft^2/s)	k (Btu/h ft °F)	$\alpha \times 10^3$ (ft^2/h)	Pr	$\beta \times 10^3$ (1/°F)	$g\beta\rho^2/\mu^2 \times 10^{-7}$ ($1/°F \cdot ft^3$)
				Ammonia					
−60	43.9	1.07	20.6	0.471	0.316	6.74	2.52	0.94	132
−30	42.7	1.07	18.2	0.426	0.317	6.93	2.22	1.02	265
0	41.3	1.08	16.9	0.409	0.315	7.06	2.08	1.1	467
30	40.0	1.11	16.2	0.402	0.312	7.05	2.05	1.19	757
60	38.5	1.14	15.0	0.391	0.304	6.92	2.03	1.3	1130
80	37.5	1.16	14.2	0.379	0.296	6.79	2.01	1.4	1650
100	36.4	1.19	13.5	0.368	0.287	6.62	2.00	1.5	2200
120	35.3	1.22	12.6	0.356	0.275	6.43	2.00	1.68	3180

T (°F)	ρ (lb_m/ft^3)	c_p ($Btu/lb_m\,°F$)	$\mu \times 10^5$ ($lb_m/ft\,s$)	$\nu \times 10^5$ (ft^2/s)	k (Btu/h ft °F)	$\alpha \times 10^3$ (ft^2/h)	Pr	$\beta \times 10^3$ (1/°F)	$g\beta\rho^2/\mu^2 \times 10^{-6}$ ($1/°F \cdot ft^3$)
				Freon-12					
−40	94.5	0.202	125	1.32	0.0650	3.40	14.0	9.10	168
−30	93.5	0.204	123	1.32	0.0640	3.35	14.1	9.60	179
0	90.9	0.212	116	1.28	0.0578	3.00	15.4	11.4	225
30	87.4	0.221	108	1.24	0.0564	2.92	15.3	13.1	277
60	84.0	0.230	99.6	1.19	0.0528	2.74	15.6	14.9	341
80	81.3	0.238	94.0	1.16	0.0504	2.60	16.0	16.0	384
100	78.7	0.246	88.4	1.12	0.0480	2.48	16.3	17.2	439
150	71.0	0.271	74.8	1.05	0.0420	2.18	17.4	19.5	625

T (°F)	ρ (lb_m/ft^3)	c_p ($Btu/lb_m\,°F$)	$\mu \times 10^5$ ($lb_m/ft\,s$)	$\nu \times 10^5$ (ft^2/s)	k (Btu/h ft °F)	$\alpha \times 10^3$ (ft^2/h)	Pr	$\beta \times 10^3$ (1/°F)	$g\beta\rho^2/\mu^2 \times 10^{-6}$ ($1/°F \cdot ft^3$)
				n-Butyl Alcohol					
60	50.5	0.55	225	4.46	0.100	3.59	44.6		
80	50.0	0.58	180	3.60	0.099	3.41	38.0	0.25	6.23
100	49.6	0.61	130	2.62	0.098	3.25	29.1	0.43	2.02
150	48.5	0.68	68	1.41	0.098	2.97	17.1		

T (°F)	ρ (lb_m/ft^3)	c_p ($Btu/lb_m\,°F$)	$\mu \times 10^5$ ($lb_m/ft\,s$)	$\nu \times 10^5$ (ft^2/s)	k (Btu/h ft °F)	$\alpha \times 10^3$ (ft^2/h)	Pr $\times 10^{-2}$	$\beta \times 10^4$ (1/°F)	$g\beta\rho^2/\mu^2 \times 10^{-6}$ ($1/°F \cdot ft^3$)
				Benzene					
60	55.2	0.395	44.5	0.806	0.0856	3.93	7.39		
80	54.6	0.410	38	0.695	0.0836	3.73	6.70	7.5	498
100	53.6	0.420	33	0.615	0.0814	3.61	6.13	7.2	609
150	51.8	0.450	24.5	0.473	0.0762	3.27	5.21	6.8	980
200	49.9	0.480	19.4	0.390	0.0711	2.97	4.73		