CM3120: Module 2

Unsteady State Heat Transfer

I. Introduction
II. Unsteady Microscopic Energy Balance—(slash and burn)
III. Unsteady Macroscopic Energy Balance
IV. Dimensional Analysis (unsteady)—Biot number, Fourier number
V. Low Biot number solutions—Lumped parameter analysis
VI. Short Cut Solutions—(initial temperature T_0; finite h), Gurney and Lurie charts (as a function of position, $m = \frac{1}{\mathrm{Bi}}$, and Fo); Heissler charts (center point only, as a function of $m = 1/\mathrm{Bi}$, and Fo)
VII. Full Analytical Solutions (stretch)

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CM3120: Module 2

Module 2 Lecture IV
Dimensional Analysis
For Unsteady State Heat Transfer

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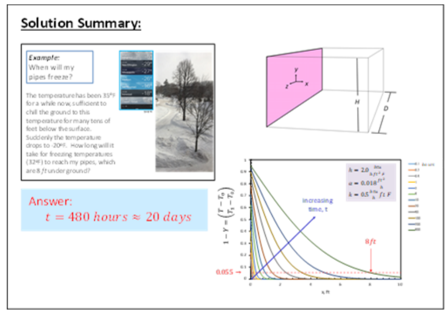
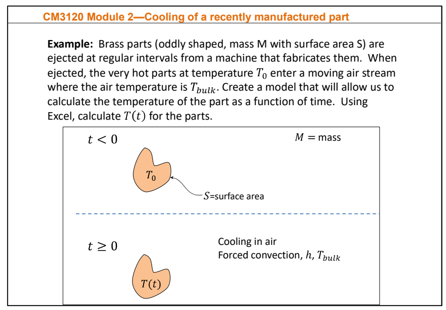
www.chem.mtu.edu/~fmorriso/cm3120/cm3120.html

 2

We used **microscopic** unsteady state **energy balance modeling to solve one practical problem.**



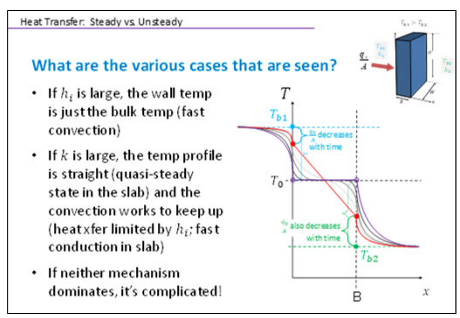
We used **macroscopic** unsteady state **energy balance modeling to solve another practical problem.**



How can we organize our tool belt?

What is our usual strategy for complex phenomena?

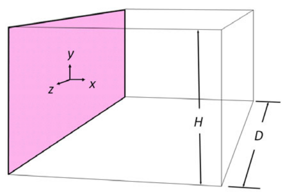
Answer: Dimensional Analysis



- ✓ ***Let's nondimensionalize the governing equations and BCs.***
- ✓ ***Let's sort out the various unsteady cases.***

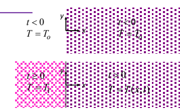
1D Heat Transfer: Unsteady State

Let's nondimensionalize the governing equations and BCs.
Let's sort out the various cases.



1D Heat Transfer: Unsteady State

Unsteady State Heat Conduction in a Semi-Infinite Slab



$$\frac{\partial T}{\partial t} = \frac{k}{\rho \hat{C}_p}\left(\frac{\partial^2 T}{\partial x^2}\right) = \alpha\left(\frac{\partial^2 T}{\partial x^2}\right)$$

thermal diffusivity $\alpha \equiv \frac{k}{\rho \hat{C}_p}$

Initial condition: $t = 0 \quad T = T_0 \quad \forall\ x$

Boundary conditions:

$x = 0 \quad \frac{q_x}{A} = -k\frac{dT}{dx} = h(T_1 - T) \quad t > 0$

$x = \infty \quad T = T_0 \quad \forall\ t$

(Review:
How did we do this before?)

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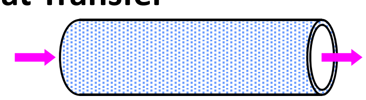
Method:

- Identify the governing equation(s)
- Choose "typical" values (*scale factors*)
- Use them to scale the equations

CM3110 REVIEW

We'll modify our solution for
Forced Convection Heat Transfer



Pipe flow

Dimensional Analysis

non-dimensional variables:

time: $t^* \equiv \frac{tV}{D}$

position: $r^* \equiv \frac{r}{D}$, $z^* \equiv \frac{z}{D}$

velocity: $v_z^* \equiv \frac{v_z}{V}$, $v_r^* \equiv \frac{v_r}{V}$, $v_\theta^* \equiv \frac{v_\theta}{V}$

driving force: $P^* \equiv \frac{P}{\rho V^2}$, $g_z^* \equiv \frac{g_z}{g}$

Energy

non-dimensional variables:

position: $r^* \equiv \frac{r}{D}$, $z^* \equiv \frac{z}{D}$

temperature: $T^* \equiv \frac{T - T_o}{(T_1 - T_o)}$

source: $S^* \equiv \frac{S}{S_0}$

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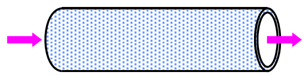
We'll modify our solution for

Forced Convective Heat Transfer

CM3110 REVIEW



Pipe flow

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driving force: $P^* \equiv \frac{P}{\rho V^2}$, $g_z^* \equiv \frac{g_z}{g}$

$t_{char} = \frac{D}{V}$ (forced convection)

Energy

non-dimensional variables:

position: $r^* \equiv \frac{r}{D}$, $z^* \equiv \frac{z}{D}$

temperature: $T^* \equiv \frac{T - T_o}{(T_1 - T_o)}$

source: $S^* \equiv \frac{S}{S_0}$

Slight problem: We need to nondimensionalize t for the unsteady case also, but there is no characteristic velocity in thermal conduction in a solid.

Choice:

For the unsteady case we'll choose a characteristic time based on the thermal diffusivity, α.

$$\frac{t}{t_{char}} = t^* \equiv \frac{\alpha t}{D_{char}^2}$$

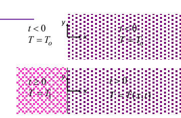
This dimensionless time is called Fourier number Fo.

$$t_{char} = \frac{D^2}{\alpha} = \frac{D^2 \rho \hat{C}_p}{k}$$

(thermal diffusion)

1D Heat Transfer: Unsteady State

Unsteady State Heat Conduction in a Semi-Infinite Slab



$$\frac{\partial T}{\partial t} = \frac{k}{\rho \hat{C}_p}\left(\frac{\partial^2 T}{\partial x^2}\right) = \alpha\left(\frac{\partial^2 T}{\partial x^2}\right)$$

thermal diffusivity $\alpha \equiv \frac{k}{\rho \hat{C}_p}$

Initial condition: $t = 0 \quad T = T_0 \quad \forall x$

Boundary conditions:

$x = 0 \quad \frac{q_x}{A} = -k\frac{dT}{dx} = h(T_1 - T) \quad t > 0$

$x = \infty \quad T = T_0 \quad \forall t$

thermal diffusivity $\alpha \equiv \frac{k}{\rho \hat{C}_p}$

(Appears in the energy balance)

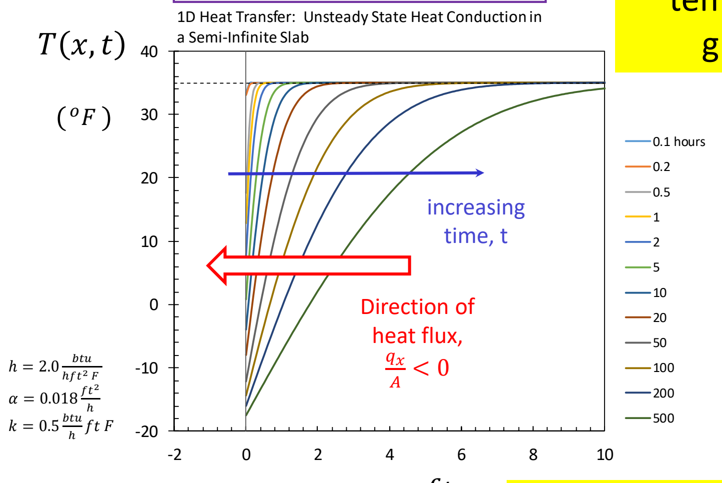
$\frac{D^2}{\alpha}$ = thermal diffusion time = $\frac{D^2 \rho \hat{C}_p}{k}$

1D Heat Transfer: Unsteady State

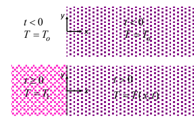
What is the meaning of time α/D^2?

Example: When will my pipes freeze?

1D Heat Transfer: Unsteady State Heat Conduction in a Semi-Infinite Slab



Energy is *diffusing* **down** the temperature gradient



thermal diffusivity

$$\alpha \equiv \frac{k}{\rho \hat{C}_p}$$

$$\frac{\alpha}{D^2} = \text{thermal diffusion rate} = \frac{k}{D^2 \rho \hat{C}_p}$$

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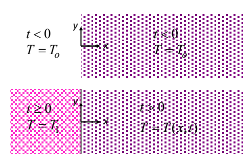
Dimensional Analysis, Unsteady State Convection

Non-dimensionalize (eqns, BCs)

$$\frac{\partial T}{\partial t} = \alpha \left(\frac{\partial^2 T}{\partial x^2} \right)$$

$$\frac{q_x}{A} = -k \frac{\partial T}{\partial x} = h(T_1 - T)$$



non-dimensional variables:

position:

$$x^* \equiv \frac{x}{D}$$

temperature:

$$Y = T^* \equiv \frac{(T_1 - T)}{(T_1 - T_0)}$$

time:

$$t^* \equiv \frac{\alpha t}{D^2}$$

This dimensionless time is called **Fourier number** Fo.

$$\text{Fo} - \text{Fourier Number} = \frac{\alpha t}{D^2}$$

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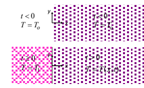
1D Heat Transfer: Unsteady State

Dimensional Analysis, Unsteady State Convection

Non-dimensionalize (eqns, BCs)

$$\frac{\partial T}{\partial t} = \alpha\left(\frac{\partial^2 T}{\partial x^2}\right)$$

$$q_x = -k\frac{\partial T}{\partial x} = hA(T_1 - T)$$



non-dimensional variables:

position: $x^* \equiv \frac{x}{D}$

temperature: $Y \equiv \frac{(T_1 - T)}{(T_1 - T_0)}$

time: $t^* \equiv \frac{\alpha t}{D^2}$

This dimensionless time is called Fourier number Fo.

$\text{Fo} - \text{Fourier Number} = \frac{\alpha t}{D^2}$

Let's do it.

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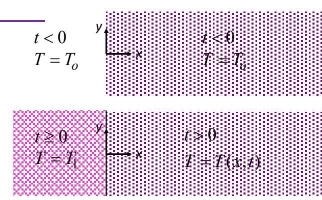
1D Heat Transfer: Unsteady State

Unsteady State Heat Conduction in a Semi-Infinite Slab



$$\frac{\partial Y}{\partial t^*} = \frac{\partial^2 Y}{\partial x^{*2}}$$

temperature: $Y \equiv \frac{(T_1 - T)}{(T_1 - T_0)}$

Initial condition: $t^* = 0 \quad Y = 1 \quad \forall\, x^*$

Boundary conditions: $x^* = \infty \quad Y = 1 \quad \forall\, t^*$

$$x^* = 0 \quad \frac{\partial Y}{\partial x^*} = \text{Bi}\, Y \quad t^* > 0$$

$$\text{Bi} \equiv \frac{hD}{k}$$

$\text{Bi} - \text{Biot Number} = \frac{hD}{k}$

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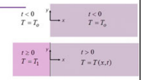
In dimensionless form, we see that this problem reduces to

$$Y = Y\left(\frac{x}{D}, \text{Fo}, \text{Bi}\right)$$

1D Heat Transfer: Unsteady State

Unsteady State Heat Conduction in a Semi-Infinite Slab



$$\frac{\partial T}{\partial t} = \frac{k}{\rho \hat{C}_p}\left(\frac{\partial^2 T}{\partial x^2}\right) = \alpha\left(\frac{\partial^2 T}{\partial x^2}\right)$$

thermal diffusivity $\alpha \equiv \frac{k}{\rho \hat{C}_p}$

Initial condition: $t=0 \quad T=T_0 \quad \forall\, x$

Boundary conditions:

$$x=0 \quad \frac{q_x}{A} = -k\frac{dT}{dx} = h(T_1 - T) \quad t>0$$

$$x=\infty \quad T=T_0 \quad \forall\, t$$

Dimensionless quantities:

$Y = \frac{(T_1 - T)}{(T_1 - T_0)}$ — **Y** (dimensionless temperature interval)

$t^* = \text{Fo} = \frac{\alpha t}{D^2}$ — **Fourier number** (dimensionless time based on thermal diffusivity)

$x^* = \frac{x}{D}$ — $\boldsymbol{x^*}$ (dimensionless position)

$\text{Bi} = \frac{hD}{k}$ — **Biot number** (pronounced BEE-OH) Ratio of internal heat transfer resistance to resistance at the boundary. This is a *transport* issue.

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Because we can solve this problem analytically, **we can confirm** that the dimensional analysis is correct:

Solution:

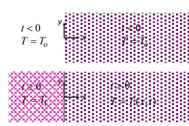
this:

Unsteady State Heat Conduction in a Semi-Infinite Slab

Solution:

$$\frac{T - T_0}{T_1 - T_0} = \text{erfc}\,\zeta - e^{\beta(2\zeta+\beta)}\,\text{erfc}(\zeta + \beta)$$

$$\beta \equiv \frac{h\sqrt{\alpha t}}{k} \qquad \zeta \equiv \frac{x}{2\sqrt{\alpha t}}$$



$$Y \equiv \frac{(T_1 - T)}{(T_1 - T_0)}$$

$$1 - Y = \frac{(T - T_0)}{(T_1 - T_0)}$$

+

$$\text{Bi} - \text{Biot Number} = \frac{hD}{k}$$

$$\text{Fo} - \text{Fourier Number} = \frac{\alpha t}{D^2}$$

=

$$1 - Y = \text{erfc}\left(\frac{x}{D}\frac{1}{2}\frac{1}{\sqrt{\text{Fo}}}\right) - e^{\text{Bi}\left(\frac{x}{D}\right) + \text{Bi}^2\text{Fo}}\,\text{erfc}\left(\sqrt{\text{Fo}}\left(\text{Bi} + \frac{x}{D}\frac{1}{\text{Fo}}\right)\right)$$

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Unsteady State Heat Transfer in a Body

Two Additional Dimensionless Numbers

$$\text{Bi} - \text{Biot Number} = \frac{hD_{\text{char}}}{k}$$

Quantifies the tradeoffs between the internal resistance D/k (due to conduction) and $1/h$, the resistance at the phase boundary (due to convection)

Dimensionless Numbers

momentum
energy
mass

$\text{Re} - \text{Reynolds} = \frac{\rho V D}{\mu} = \frac{VD}{\nu}$

$\text{Fr} - \text{Froude} = \frac{V^2}{gD}$

$\text{Pe} - \text{Péclet}_h = \text{RePr} = \frac{\hat{C}_p \rho V D}{k} = \frac{VD}{\alpha}$

$\text{Pe} - \text{Péclet}_m = \text{ReSc} = \frac{VD}{D_{AB}}$

These numbers from the governing equations tell us about the relative importance of the terms they precede in the microscopic balances (***scenario*** *properties*).

$\text{Pr} - \text{Prandtl} = \frac{\hat{C}_p \mu}{k} = \frac{\nu}{\alpha}$

$\text{Sc} - \text{Schmidt} = \text{LePr} = \frac{\mu}{\rho D_{AB}} = \frac{\nu}{D_{AB}}$

$\text{Le} - \text{Lewis} = \frac{\alpha}{D_{AB}}$

These numbers compare the magnitudes of the diffusive transport coefficients ν, α, D_{AB} (***material*** *properties*).

$f - \text{Friction Factor} = \frac{\mathcal{F}_{drag}}{\left(\frac{1}{2}\rho V^2\right) A_c}$

$\text{Nu} - \text{Nusselt} = \frac{hD}{k}$

$\text{Sh} - \text{Sherwood} = \frac{k_m D}{D_{AB}}$

These numbers are defined to help us build transport data correlations based on the fewest number of grouped (dimensionless) variables (***scenario*** *properties*).

$$\text{Fo} - \text{Fourier Number} = \frac{\alpha t}{D_{\text{char}}^2}$$

Scales the time evolution of the temperature profile relative to the material's thermal properties, $\alpha = k/\rho \hat{C}_p$ (thermal diffusion time).

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Dimensional Analysis in Unsteady State Heat Transfer

Note Two Different Numbers

Warning!

with completely different purposes and meanings but confusingly similar definitions

$$\text{Bi} - \text{Biot Number} = \frac{hD}{k} = \frac{hD_{\text{body}}}{k_{\text{body}}}$$

Quantifies the tradeoffs between the internal resistance to heat flow D/k (due to conduction) and the external resistance to heat flow at the boundary $1/h$ (due to convection) for a body in contact with a moving fluid.

$$\text{Nu} - \text{Nusselt Number} = \frac{hD}{k} = \frac{hD_{flow}}{k_{fluid}}$$

Dimensionless heat transfer coefficient in convection. Quantifies the physics in the moving fluid and how this results in a resistance to heat transfer, captured in the heat transfer coefficient.

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Dimensional Analysis in Heat Transfer

Note also:

$t_{char,1} = \frac{D}{V}$ (forced convection)

$t_{char,2} = \frac{D^2}{\alpha} = \frac{D^2 \rho \hat{C}_p}{k}$ (thermal diffusion)

$$\frac{convective\ rate}{diffusive\ rate} = \frac{1/t_{char,1}}{1/t_{char,2}}$$

$$= \frac{V/D}{k/D^2 \rho \hat{C}_p} = \frac{\rho V D \hat{C}_p}{k} = \text{Pe}$$

Dimensionless Numbers

momentum
energy
mass

Re – Reynolds $= \frac{\rho V D}{\mu} = \frac{VD}{\nu}$

Fr – Froude $= \frac{V^2}{gD}$

Pe – Péclet$_h$ = RePr $= \frac{\hat{C}_p \rho V D}{k} = \frac{VD}{\alpha}$

Pe – Péclet$_m$ = ReSc $= \frac{VD}{D_{AB}}$

These numbers from the governing equations tell us about the relative importance of the terms they precede in the microscopic balances (***scenario*** *properties*).

Pr – Prandtl $= \frac{\hat{C}_p \mu}{k} = \frac{\nu}{\alpha}$

Sc – Schmidt = LePr $= \frac{\mu}{\rho D_{AB}} = \frac{\nu}{D_{AB}}$

Le – Lewis $= \frac{\alpha}{D_{AB}}$

These numbers compare the magnitudes of the diffusive transport coefficients ν, α, D_{AB} (***material*** *properties*).

f – Friction Factor $= \frac{\mathcal{F}_{drag}}{\left(\frac{1}{2}\rho V^2\right) A_c}$

Nu – Nusselt $= \frac{hD}{k}$

Sh – Sherwood $= \frac{k_m D}{D_{AB}}$

These numbers are defined to help us build transport data correlations based on the fewest number of grouped (dimensionless) variables (***scenario*** *properties*).

Pe – Péclet$_h$ = RePr
Pr – Prandtl

The Peclet number is the ratio of convective and diffusive heat transport rates

Non-dimensional Energy Equation

energy

$$\left(\frac{\partial T^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* T^*\right) = \frac{1}{\text{RePr}} \left(\nabla^{*2} T^*\right) + S^*$$

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Bi – Biot Number $= \frac{hD}{k}$

$$\text{Bi} = \frac{\mathcal{R}_k}{\mathcal{R}_h} = \frac{D/k}{1/h}$$

Quantifies the tradeoffs between the resistance to heat flow (due to conductivity, D/k) and the resistance to heat flow at the boundary ($1/h$)

At high Bi, the surface temperature equals the bulk temperature; heat transfer is limited by conduction in the body.

High Bi:
low k,
high h

At moderate Bi, heat transfer is affected by both conduction in the body and the rate of heat transfer to the surface.

Moderate Bi:
nether process dominates

At low Bi, the temperature is uniform in a finite body; heat transfer is limited by rate of heat transfer to the surface (h).

Low Bi:
high k,
low h

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$\text{Bi} - \text{Biot Number} = \frac{hD}{k}$

Quantifies the tradeoffs between the resistance to heat flow (due to conductivity, D/k) and the resistance to heat flow at the boundary ($1/h$)

$$\text{Bi} = \frac{D_{char}/k}{1/h}$$

$$D_{char} = \frac{V_{body}}{A_{body}}$$

At high Bi, the surface temperature equals the bulk temperature; heat tran... the ...

High Bi:

At m... affe... bod... the surface.

dominates

When the temperature is uniform in the body, we can do a macroscopic energy balance to solve many problems of interest. This is called a "lumped parameter analysis."

$T = T(t)$

At low Bi, the temperature is uniform in a finite body; heat transfer is limited by rate of heat transfer to the surface (h).

Low Bi: high k, low h

$\text{Bi} - \text{Biot Number} = \frac{hD}{k}$

Quantifies the tradeoffs between the resistance to heat flow (due to conductivity, D/k) and the resistance to heat flow at the boundary ($1/h$)

$$\text{Bi} = \frac{D/k}{1/h}$$

$T = T(x, y, z, t)$, **easy BC**

At high Bi, the surface temperature equals the bulk temperature; heat transfer is limited by conduction in the body.

High Bi: low k, high h

At moderate Bi, heat transfer is affected by... body and t... the surface...

Moderate Bi:

At low Bi, t... uniform in ... transfer is limited by rate of heat transfer to the surface (h).

high k, low h

When the wall temperature and the bulk temperature are equal, the microscopic energy balance is easier to carry out (temperature boundary conditions).

Bi – Biot Number $= \frac{hD}{k}$

Quantifies the tradeoffs between the resistance to heat flow (due to conductivity, D/k) and the resistance to heat flow at the boundary ($1/h$)

$$\text{Bi} = \frac{D/k}{1/h}$$

When both processes affect the outcomes, the full solution may be necessary. For uniform starting temperatures, the solutions are published.

the body.

high h

$T = T(x, y, z, t)$

hard BC

At moderate Bi, heat transfer is affected by <u>both</u> conduction in the body and the rate of heat transfer to the surface.

Moderate Bi: nether process dominates

At low Bi, the temperature is <u>uniform in a finite body</u>; heat transfer is limited by rate of heat transfer to the surface (h).

Low Bi: high k, low h

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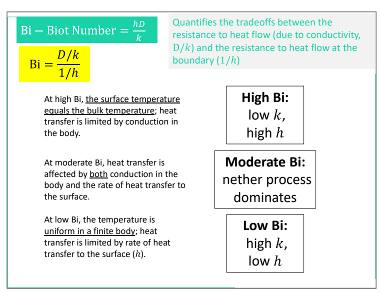
NEXT: Lecture V (Talk about the three Bi cases)



***But! Before dimensionless numbers get more out of hand*: Library of Dimensionless Numbers**

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Library of Dimensionless Numbers in Transport

Dimensionless numbers:

1. From the Equations of Change for
   a. Momentum
   b. Energy
   c. Species A Mass
2. Comparing Transport Coefficients (material properties)
3. Involving Engineering Quantities of Interest (scenario properties)
4. Unsteady Heat Transfer

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Dimensional Analysis

These numbers tell us about the relative importance of the terms they precede.

Dimensionless numbers from the
Equations of Change *(microscopic balances)*

momentum

Non-dimensional Navier-Stokes Equation

$$\left(\frac{\partial v_z^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* v_z^*\right) = -\frac{\partial P^*}{\partial z^*} + \frac{1}{\mathrm{Re}}(\nabla^{*2} v_z^*) + \frac{1}{\mathrm{Fr}} g^*$$

Re – Reynolds
Fr – Froude

energy

Non-dimensional Energy Equation

$$\left(\frac{\partial T^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* T^*\right) = \frac{1}{\mathrm{RePr}}(\nabla^{*2} T^*) + S^*$$

Pe – Péclet$_h$ = RePr
Pr – Prandtl

mass

Non-dimensional Continuity Equation (species A)

$$\left(\frac{\partial x_A^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* x_A^*\right) = \frac{1}{\mathrm{ReSc}}(\nabla^{*2} x_A^*)$$

Pe – Péclet$_m$ = ReSc
Sc – Schmidt

ref: BSL1, p581, 644

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Dimensionless Numbers

Dimensionless numbers from the
Equations of Change

$\text{Re} - \text{Reynolds} = \frac{\rho V D}{\mu} = \frac{VD}{\nu}$

$\text{Fr} - \text{Froude} = \frac{V^2}{gD}$

$\text{Pe} - \text{Péclet}_h = \text{RePr} = \frac{\hat{C}_p \rho V D}{k} = \frac{VD}{\alpha}$

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These numbers tell us about the **relative importance of the terms** they precede in the microscopic balances (*scenario properties*).

$\text{Pr} - \text{Prandtl} = \frac{\hat{C}_p \mu}{k} = \frac{\nu}{\alpha}$

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$\text{Le} - \text{Lewis} = \frac{\alpha}{D_{AB}}$

These numbers compare the **magnitudes of the diffusive transport coefficients** ν, α, D_{AB} (*material properties*).

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Dimensionless Numbers

Dimensionless numbers from the
Equations of Change

$\text{Re} - \text{Reynolds} = \frac{\rho V D}{\mu} = \frac{VD}{\nu}$

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These numbers tell us about the **relative importance of the terms** they precede in the microscopic balances (*scenario properties*).

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These numbers compare the **magnitudes of the diffusive transport coefficients** ν, α, D_{AB} (*material properties*).

Transport coefficients

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Dimensional Analysis

$\mathrm{Nu}_{AB} =$

These numbers are defined to help us build **transport data correlations** based on the fewest number of grouped (dimensionless) variables (*scenario property*).

Dimensionless numbers from the **Engineering Quantities of Interest**

momentum — Dimensionless Force on the Wall (Drag)

$$f = \frac{1}{\pi}\frac{D}{L}\frac{1}{\mathrm{Re}}\int_0^{\frac{L}{D}}\int_0^{2\pi}\left(\frac{\partial v_z^*}{\partial r^*}\right)\Bigg|_{r^*=\frac{1}{2}} d\theta dz^*$$

(Fanning)
f – Friction Factor
$\frac{L}{D}$ – Aspect Ratio

$$f = \frac{\mathcal{F}_{drag}}{\left(\frac{1}{2}\rho V^2\right) A_c}$$

energy — Newton's Law of Cooling

$$Nu = \frac{1}{2\pi L/D}\int_0^{2\pi}\int_0^{L/D} -\frac{\partial T^*}{\partial r^*}\bigg|_{r^*=1/2} dz^* d\theta$$

Nu – Nusselt
$\frac{L}{D}$ – Aspect Ratio

$$\mathrm{Nu} = \frac{hD}{k}$$

mass xfer — Dimensionless Mass Transfer Coefficient

$$Sh = \frac{1}{2\pi}\frac{D}{L}\int_0^{\frac{L}{D}}\int_0^{2\pi}\left(-\frac{\partial x_A^*}{\partial r^*}\right)\Bigg|_{r^*=\frac{1}{2}} d\theta dz^*$$

Nu_{AB} = Sh – Sherwood
$\frac{L}{D}$ – Aspect Ratio

$$\mathrm{Sh} = \frac{k_m D}{D_{AB}} = \mathrm{Nu}_{AB}$$

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Dimensionless Numbers

momentum
energy
mass species A

Re – Reynolds $= \frac{\rho V D}{\mu} = \frac{VD}{\nu}$

Fr – Froude $= \frac{V^2}{gD}$

Pe – Péclet$_h$ = RePr $= \frac{\hat{C}_p \rho V D}{k} = \frac{VD}{\alpha}$

Pe – Péclet$_m$ = ReSc $= \frac{VD}{D_{AB}}$

These numbers from the governing equations tell us about the relative importance of the terms they precede in the microscopic balances (***scenario*** *properties*).

Pr – Prandtl $= \frac{\hat{C}_p \mu}{k} = \frac{\nu}{\alpha}$

Sc – Schmidt = LePr $= \frac{\mu}{\rho D_{AB}} = \frac{\nu}{D_{AB}}$

Le – Lewis $= \frac{\alpha}{D_{AB}}$

These numbers compare the magnitudes of the diffusive transport coefficients ν, α, D_{AB} (***material*** *properties*).

f – Friction Factor $= \frac{\mathcal{F}_{drag}}{\left(\frac{1}{2}\rho V^2\right) A_c}$

Nu – Nusselt $= \frac{hD}{k}$

Sh – Sherwood $= \frac{k_m D}{D_{AB}} = \mathrm{Nu}_{AB}$

These numbers are defined to help us build transport data correlations based on the fewest number of grouped (dimensionless) variables (***scenario*** *properties*).

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Dimensionless Numbers (Unsteady Heat Transfer)

Bi – Biot $= \frac{hD_{char}}{k}$	Ratio of internal resistance to external resistance to heat transfer
Fo – Fourier $= \frac{\alpha D}{V^2}$	Dimensionless time
Pe – Péclet$_h$ $= \frac{\hat{C}_p \rho V D}{k} = \frac{VD}{\alpha} = \frac{t_{convection}}{t_{thermal\,diffusion}}$	Ratio of convective timescale to thermal diffusion time scale

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