CM3120: Module 2

Unsteady State Heat Transfer

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Module 2 Lecture V:

Low Biot Number Solutions

(Lumped Parameter Analysis)

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www.chem.mtu.edu/~fmorriso/cm3120/cm3120.html

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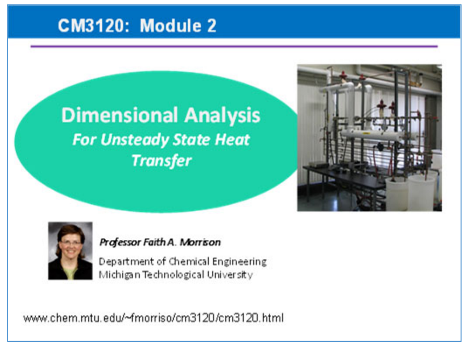
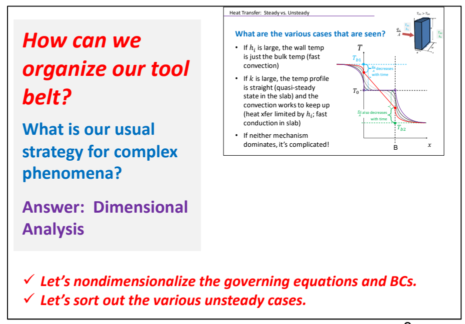
In the last lecture, we found that **Dimensional Analysis** helped us to organize our "tool belt" for engineering problem solving.



For **Unsteady Heat Transfer** problems, we added two dimensionless numbers, the **Biot number** (*bee oh*) Bi and the **Fourier number** Fo



Unsteady State Heat Transfer in a Body

Two Additional Dimensionless Numbers

$$\text{Bi} - \text{Biot Number} = \frac{hD_{char}}{k}$$

Quantifies the tradeoffs between the internal resistance D/k (due to conduction) and $1/h$, the resistance at the phase boundary (due to convection)

Dimensionless Numbers (momentum, energy, mass)

$\text{Re} - \text{Reynolds} = \frac{\rho V D}{\mu} = \frac{VD}{\nu}$

$\text{Fr} - \text{Froude} = \frac{V^2}{gD}$

$\text{Pe} - \text{Péclet}_h = \text{RePr} = \frac{\hat{C}_p \rho V D}{k} = \frac{VD}{\alpha}$

$\text{Pe} - \text{Péclet}_m = \text{ReSc} = \frac{VD}{D_{AB}}$

These numbers from the governing equations tell us about the relative importance of the terms they precede in the microscopic balances (***scenario*** *properties*).

$\text{Pr} - \text{Prandtl} = \frac{\hat{C}_p \mu}{k} = \frac{\nu}{\alpha}$

$\text{Sc} - \text{Schmidt} = \text{LePr} = \frac{\mu}{\rho D_{AB}} = \frac{\nu}{D_{AB}}$

$\text{Le} - \text{Lewis} = \frac{\alpha}{D_{AB}}$

These numbers compare the magnitudes of the diffusive transport coefficients ν, α, D_{AB} (***material*** *properties*).

$f - \text{Friction Factor} = \frac{\mathcal{F}_{drag}}{\left(\frac{1}{2}\rho V^2\right)A_c}$

$\text{Nu} - \text{Nusselt} = \frac{hD}{k}$

$\text{Sh} - \text{Sherwood} = \frac{k_m D}{D_{AB}}$

These numbers are defined to help us build transport data correlations based on the fewest number of grouped (dimensionless) variables (***scenario*** *properties*).

$$\text{Fo} - \text{Fourier Number} = \frac{\alpha t}{D_{char}^2}$$

Scales the time evolution of the temperature profile relative to the material's thermal properties, $\alpha = k/\rho \hat{C}_p$ (thermal diffusion time).

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We indicated that there are three ranges of Biot number to consider:

$\text{Bi} - \text{Biot Number} = \frac{hD}{k}$

$$\text{Bi} = \frac{D_{char}/k}{1/h}$$

At high Bi, <u>the surface temperature equals the bulk temperature</u>; heat transfer is limited by conduction in the body.

High Bi: low k, high h

At moderate Bi, heat transfer is affected by <u>both</u> conduction in the body and the rate of heat transfer to the surface.

Moderate Bi: nether process dominates

At low Bi, the temperature is <u>uniform in a finite body</u>; heat transfer is limited by rate of heat transfer to the surface (h).

Low Bi: high k, low h

We now explore these ranges

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$\text{Bi} - \text{Biot Number} = \frac{hD}{k}$

$$\text{Bi} = \frac{D_{char}/k}{1/h}$$

Quantifies the tradeoffs between the resistance to heat flow (due to conductivity, D/k) and the resistance to heat flow at the boundary ($1/h$)

At high Bi, <u>the surface temperature equals the bulk temperature</u>; heat tra... th...

High Bi: low k,

When the temperature is uniform in the body, we can do a macroscopic energy balance to solve many problems of interest. This is called a "**lumped parameter analysis**."

At... aff... bo... the surface.

dominates

$T = T(t)$

At low Bi, the temperature is <u>uniform in a finite body</u>; heat transfer is limited by rate of heat transfer to the surface (h).

Low Bi: high k, low h

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Unsteady State Heat Transfer: Ranges of Biot Number

Lumped parameter analysis

(negligible internal resistance)

Low Bi:
high k,
low h

At low Bi, the temperature is uniform in a finite body; heat transfer is limited by rate of heat transfer to the surface (h).

$$D_{char,LP} \equiv \frac{\text{volume}}{\text{area}} = \frac{V_{sys}}{A_{sys}}$$

(Because of irregular shapes…)

This is always the D_{char} we use for the Biot number in lumped parameter analysis. We use different D_{char} in other cases, however.

$$\text{Bi}_{LP} = \frac{hV_{sys}}{kA_{sys}}$$

The test for applicability of the lumped parameter analysis is:

$$\text{Bi}_{LP} = \frac{hV_{sys}}{kA_{sys}} < 0.1$$

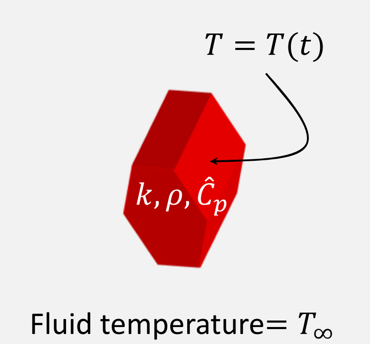
Unsteady State Heat Transfer: Low Biot Number

$$\text{Bi}_{LP} = \frac{hV_{sys}}{kA_{sys}}$$

Example: Quench cooling of a manufactured part.

If a piece of steel with $T = T_0$ is dropped into a large, well stirred reservoir of fluid at bulk temperature T_∞, what is the temperature of the steel as a function of time?

- k = large, which means that there is no internal resistance to heat transfer in the part
- Therefore, we are NOT calculating a temperature profile (internal T is uniform)
- ⇒ **Use Unsteady, Macroscopic Energy Balance**

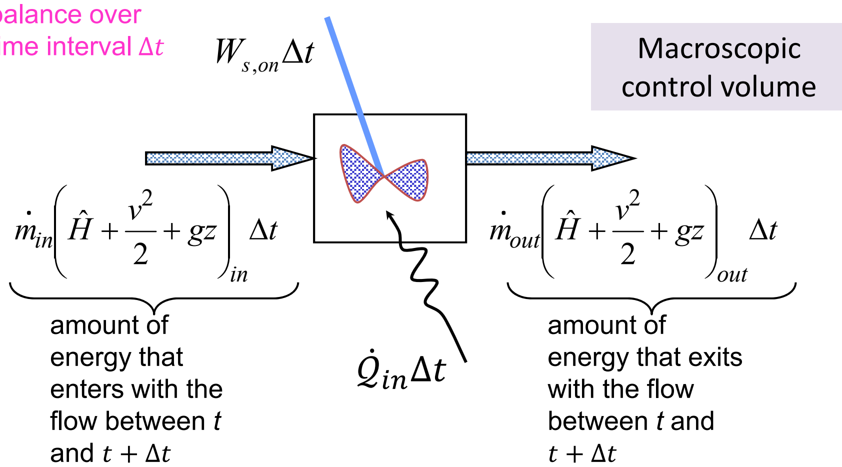


Unsteady State Heat Transfer: Low Biot Number

Unsteady Macroscopic Energy Balance

see Felder and Rousseau

balance over time interval Δt



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Unsteady State Heat Transfer: Low Biot Number

Unsteady Macroscopic Energy Balance

$$accumulation = input - output$$

$$\frac{d}{dt}\left(U_{sys}+E_{k,sys}+E_{p,sys}\right) = -\Delta H - \Delta E_k - \Delta E_p + \dot{Q}_{in} + W_{s,on}$$

Background:
pages.mtu.edu/~fmorriso/cm310/IFMWeb
AppendixDMicroEBalanceMorrison.pdf
Felder and Rousseau, Chapter 11

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Unsteady State Heat Transfer: Low Biot Number

Unsteady Macroscopic Energy Balance

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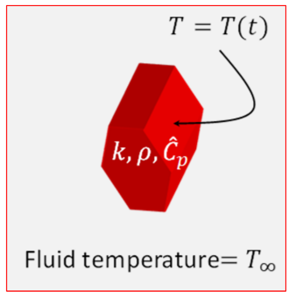

You try.

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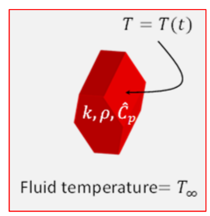
Unsteady State Heat Transfer: Low Biot Number



Unsteady Macroscopic Energy Balance

$$accumulation = input - output$$

$$\frac{d}{dt}\left(U_{sys} + E_{k,sys} + E_{p,sys}\right) = -\Delta H - \Delta E_k - \Delta E_p + \dot{Q}_{in} + W_{s,on}$$

negligible no flow no shafts

For negligible changes in E_p and E_k, no flow, no phase change, no chemical reaction, and no shafts:

$$\frac{dU_{sys}}{dt} = \dot{Q}_{in}$$

$$\rho V_{sys} \hat{C}_v \frac{dT_{sys}}{dt} = \dot{Q}_{in}$$

$\hat{C}_v \approx \hat{C}_p$ for liquids, solids

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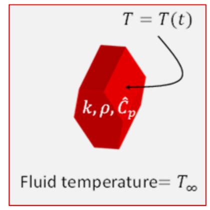
Unsteady State Heat Transfer: Low Biot Number

Unsteady Macroscopic Energy Balance



How do we quantify the heat in $\dot{Q}_{in}$?

$$\frac{d}{dt}\left(U_{sys} + E_{k,sys} + E_{p,sys}\right) = -\Delta H - \Delta E_k - \Delta E_p + \dot{Q}_{in} + W_{s,on}$$

negligible no flow no shafts

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Unsteady Macroscopic Energy Balance

accumulation = input − output

Q_{in} = Heat ***in***to the chosen macroscopic control volume

$$\frac{d}{dt}\left(U_{sys} + E_{k,sys} + E_{p,sys}\right) = -\Delta H - \Delta E_k - \Delta E_p + \dot{Q}_{in} + W_{s,on}$$

$\dot{Q}_{in} = \sum_i q_{in,i}$ comes from a variety of sources:

- Thermal conduction: $q_{in} = -kA\frac{dT}{dx}$
- Convection heat xfer: $|q_{in}| = |hA(T_b - T)|$
- Radiation: $q_{in} = \varepsilon\sigma A\left(T^4_{surroundings} - T^4_{surface}\right)$
- Electric current: $q_{in} = I^2 R_{elec} L$
- Chemical Reaction: $q_{in} = S_{rxn} V_{sys}$

$S[=]\frac{\text{energy}}{\text{time volume}}$

pages.mtu.edu/~fmorriso/cm310/IFMWebAppendixDMicroEBalanceMorrison.pdf
Incropera and DeWitt, 6th edition p18

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Unsteady Macroscopic Energy Balance

accumulation = input − output

Q_{in} = Heat ***in***to the chosen macroscopic control volume

$$\frac{d}{dt}\left(U_{sys}+E_{k,sys}+E_{p,sys}\right)=-\Delta H-\Delta E_k-\Delta E_p+\dot{Q}_{in}+W_{s,on}$$

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- Chemical Reaction: $q_{in}=S_{rxn}V_{sys}$

Signs must match transfer from outside (bulk fluid) to inside (metal)

$S[=]\frac{\text{energy}}{\text{time volume}}$

pages.mtu.edu/~fmorriso/cm310/IFMWebAppendixDMicroEBalanceMorrison.pdf
Incopera and DeWitt, 6th edition p18

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Unsteady Macroscopic Energy Balance

$$\frac{d}{dt}\left(U_{sys}+E_{k,sys}+E_{p,sys}\right)=-\Delta H-\Delta E_k-\Delta E_p+\dot{Q}_{in}+W_{s,on}$$

$\dot{Q}_{in}=\sum_i q_{in,i}$ comes from a variety of sources:

- **Thermal conduction: $q_{in}=-kA\frac{dT}{dx}$**
  e.g. device held by bracket; a solid phase that extends through boundaries of control volume
- **Convection heat xfer: $|q_{in}|=|hA(T_b-T)|$**
  e.g. device dropped in stirred liquid; forced air stream flows past, natural convection occurs outside system; phase change at boundary
- **Radiation: $q_{in}=\varepsilon\sigma A\left(T^4_{surroundings}-T^4_{surface}\right)$**
  e.g. device at high temp. exposed to a gas/vacuum; hot enough to produce nat. conv.=possibly hot enough for radiation
- **Electric current: $q_{in}=I^2R_{elec}L$**
  e.g. if electric current is flowing within the device/control volume/ system
- **Chemical Reaction: $q_{in}=S_{rxn}V_{sys}$**
  e.g. if a homogeneous reaction is taking place throughout the device/ control volume/system

S-B constant: $\sigma=5.676\times10^{-8}\frac{W}{m^2K^4}$

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Unsteady Macroscopic Energy Balance

accumulation = input − output

$\dot{Q}_{in}$ = Heat ***in***to the chosen macroscopic control volume

$$\frac{d}{dt}\left(U_{sys}+E_{k,sys}+E_{p,sys}\right)=-\Delta H-\Delta E_k-\Delta E_p+\dot{Q}_{in}+W_{s,on}$$

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- X Radiation: $q_{in}=\varepsilon\sigma A\left(T_{surroundings}^4-T_{surface}^4\right)$
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- X Chemical Reaction: $q_{in}=S_{rxn}V_{sys}$

$S_{rxn}[=]\frac{\text{energy}}{\text{time volume}}$

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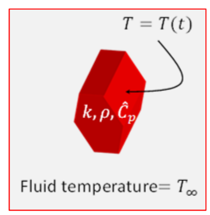
Unsteady State Heat Transfer: Low Biot Number



Unsteady Macroscopic Energy Balance Applied to cooling steel part:

$$\rho V_{sys}\hat{C}_p\frac{dT_{sys}}{dt}=\dot{Q}_{in}$$

The temperature changes in the part are due to the heat loss

The heat loss depends on the heat-transfer coefficient from the part to the environment

$$\dot{Q}_{in}=Ah(T_\infty-T)$$

$\hat{C}_v\approx\hat{C}_p$ for liquids, solids

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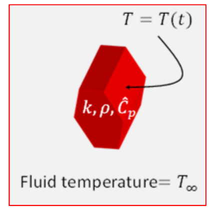
Unsteady State Heat Transfer: Low Biot Number



Unsteady Macroscopic Energy Balance Applied to cooling steel part:

$$\rho V_{sys} \hat{C}_p \frac{dT_{sys}}{dt} = \dot{Q}_{in}$$

The temperature changes in the part are due to the heat loss

The heat loss depends on the heat-transfer coefficient from the part to the environment

$$\dot{Q}_{in} = Ah(T_\infty - T)$$

You solve.

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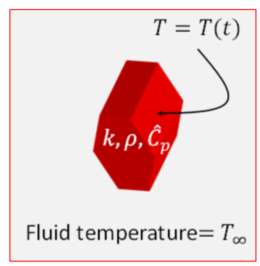
Unsteady State Heat Transfer: Low Biot Number



Unsteady Macroscopic Energy Balance Applied to cooling steel part:

$$\frac{(T_\infty - T)}{(T_\infty - T_0)} = e^{-\left(\frac{hA}{\rho \hat{C}_p V}\right) t}$$

$V_{sys} = V$

$$\ln\left(\frac{(T_\infty - T)}{(T_\infty - T_0)}\right) = -\left(\frac{hA}{\rho \hat{C}_p V}\right) t$$

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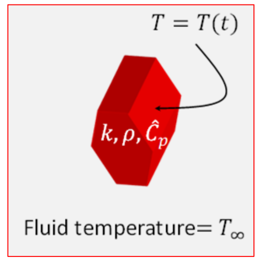
Unsteady State Heat Transfer: Low Biot Number

Unsteady Macroscopic Energy Balance Applied to cooling steel part:



$$\frac{(T_\infty - T)}{(T_\infty - T_0)} = e^{-\left(\frac{hA}{\rho \hat{C}_p V}\right)t}$$

$V_{sys} = V$

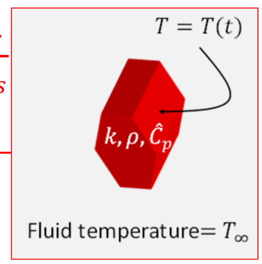
$$\ln\left(\frac{(T_\infty - T)}{(T_\infty - T_0)}\right) = -\left(\frac{hA}{\rho \hat{C}_p V}\right) t$$

In dimensionless form? →

Unsteady State Heat Transfer: Low Biot Number

$$\mathrm{Bi}_{LP} = \frac{hV_{sys}}{kA_{sys}}$$

Unsteady Macroscopic Energy Balance Applied to cooling steel part:



$$\frac{(T_\infty - T)}{(T_\infty - T_0)} = e^{-\left(\frac{hA}{\rho \hat{C}_p V}\right)t}$$

$$\frac{hAt}{\rho \hat{C}_p V} = \left(\frac{h}{k}\right)\left(\frac{k}{\rho \hat{C}_p}\right)\left(\frac{A}{V}\right)t = \left(\frac{\mathrm{Bi}}{D_{char}}\right)\alpha\left(\frac{t}{D_{char}}\right) = \mathrm{Bi}\,\mathrm{Fo}$$

LP good for: $\mathrm{Bi}_{LP} < 0.1$

$D_{char} \equiv \frac{\text{volume}}{\text{area}} = \frac{V}{A}$

thermal diffusivity $\alpha \equiv \frac{k}{\rho \hat{C}_p}$

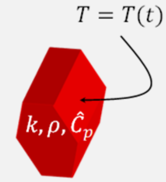
$\mathrm{Bi} - \text{Biot Number} = \frac{hD_{\text{char}}}{k}$

$\mathrm{Fo} - \text{Fourier Number} = \frac{\alpha t}{D_{\text{char}}^2}$

Unsteady State Heat Transfer: Low Biot Number

$$\mathrm{Bi}_{LP} = \frac{hV_{sys}}{kA_{sys}}$$

Unsteady Macroscopic Energy Balance
Applied to cooling steel part:



Fluid temperature= T_∞

$$Y = \frac{(T_\infty - T)}{(T_\infty - T_0)} = e^{-\mathrm{Bi}\,\mathrm{Fo}}$$

Geankoplis p359
WRF p279

Lumped parameter analysis

https://en.wikipedia.org/wiki/Lumped_element_model

Lumped parameter analysis:

LP good for:
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$$\mathrm{Bi} - \text{Biot Number} = \frac{hD_{\text{char}}}{k} = \frac{hV}{kA}$$

$$\mathrm{Fo} - \text{Fourier Number} = \frac{\alpha t}{D_{\text{char}}^2}$$

$$D_{char} \equiv \frac{\text{volume}}{\text{surf. area}} = \frac{V}{A}$$

thermal diffusivity

$$\alpha \equiv \frac{k}{\rho \hat{C}_p}$$

Summary

Unsteady State Heat Transfer: Ranges of Biot Number

Lumped parameter analysis
(negligible internal resistance)

Low Bi:
high k,
low h

At low Bi, the temperature is uniform in a finite body; heat transfer is limited by rate of heat transfer to the surface (h).

$$D_{char} \equiv \frac{\text{volume}}{\text{area}} = \frac{V_{sys}}{A_{sys}}$$

This is always the D_{char} we use for the Biot number in lumped parameter analysis. We use different D_{char} in other cases, however.

$$\mathrm{Bi}_{LP} = \frac{hV_{sys}}{kA_{sys}}$$

The test for applicability of the lumped parameter analysis is:

$$\mathrm{Bi}_{LP} = \frac{hV_{sys}}{kA_{sys}} < 0.1$$

Two things to remember about lumped parameter analysis:

1. $\mathrm{D}_{\mathrm{char,LP}} \equiv \frac{\mathrm{V}}{\mathrm{A}}$
2. Only valid for $\mathrm{Bi}_{\mathrm{LP}} < 0.1$

!

Third thing: D_{char} is *different* for other ranges of Biot number Bi.

Next: *Moderate* and *High* Biot number behavior

We indicated that there are three ranges of Biot number to consider:

$\text{Bi} - \text{Biot Number} = \frac{hD}{k}$

$\text{Bi} = \frac{D_{char}/k}{1/h}$

At high Bi, the surface temperature equals the bulk temperature; heat transfer is limited by conduction in the body.

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