CM3120: Module 4

Diffusion and Mass Transfer II

I. Classic diffusion and mass transfer: d) EMCD
II. Classic diffusion and mass transfer: e) Penetration model
III. Unsteady macroscopic species A mass balances (Intro)
IV. Interphase species A mass transfers—To an interface—k_x, k_c, k_p
V. Unsteady macroscopic species A mass balances (Redux)
VI. Interphase species A mass transfers—Across multiple resistances—K_L, K_G
VII. Dimensional analysis
VIII. Data correlations

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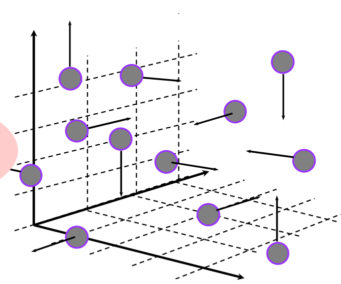
CM3120: Module 4

Module 4 Lecture VII

Dimensional Analysis

in Mass Transfer



Professor Faith A. Morrison
Department of Chemical Engineering
Michigan Technological University

www.chem.mtu.edu/~fmorriso/cm3120/cm3120.html

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Unsteady Macroscopic Species A Mass Balance

Michigan Tech

CM3110
Transport II
Part II: Diffusion and Mass Transfer

Unsteady State Microscopic Solutions to Diffusion Problems

Professor Faith A. Morrison

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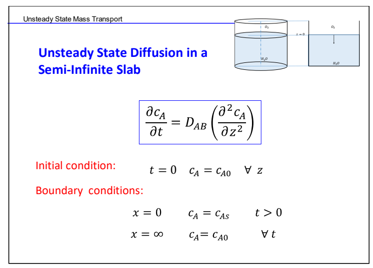
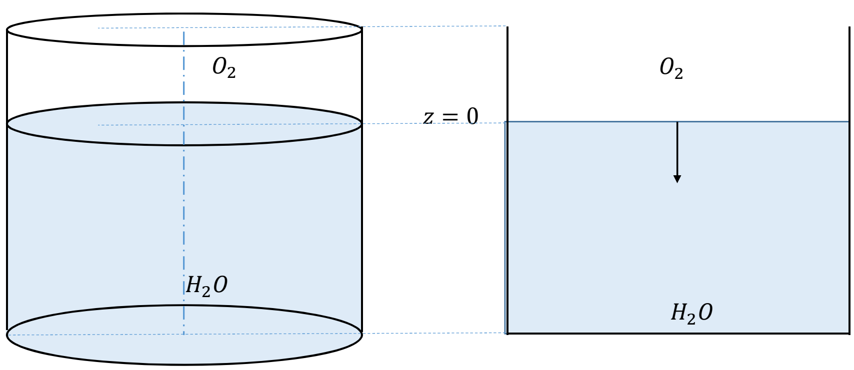
Michigan Technological University



Unsteady State Mass Transport

Example 14: A very long, very wide tank of water is suddenly exposed to oxygen atmosphere. Oxygen diffuses into the water. What is the concentration profile of the oxygen in the water as a function of time?



WRF, p534

Unsteady State Mass Transport

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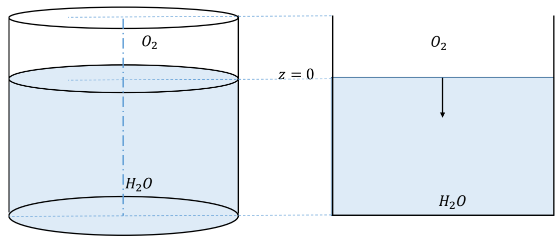

You try.

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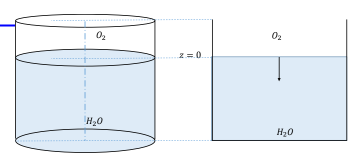
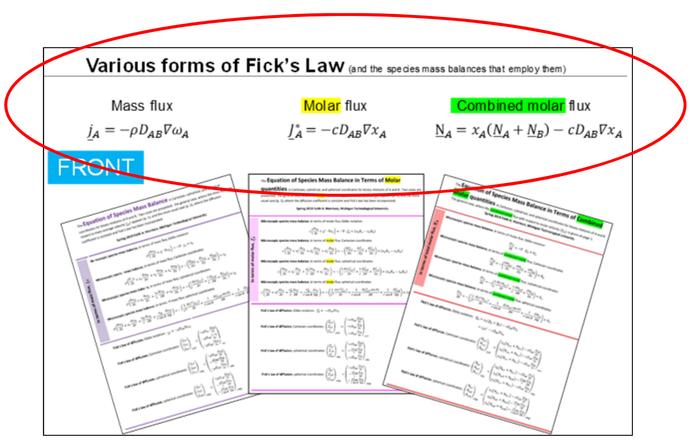
Unsteady State Mass Transport



Which microscopic species A mass balance is best for solving the model?



BSL2, p552

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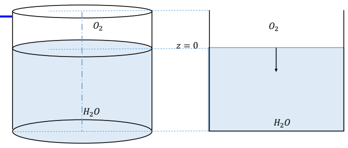
Unsteady State Mass Transport

Unsteady State Diffusion in a Semi-Infinite Slab



$$\boxed{\frac{\partial c_A}{\partial t} = \mathcal{D}_{AB}\left(\frac{\partial^2 c_A}{\partial z^2}\right)}$$

The "diffusion equation"

Initial condition: $t = 0 \quad c_A = c_{A0} \quad \forall\ z$

Boundary conditions:

$$x = 0 \qquad c_A = c_{As} \qquad t > 0$$

$$x = \infty \qquad c_A = c_{A0} \qquad \forall\ t$$

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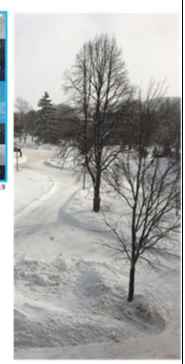
Unsteady State Heat Transfer: Lecture 6

Earlier ....

We've seen this mathematics problem before.

Example:
When will my pipes freeze?

The temperature has been 35ºF for a while now, sufficient to chill the ground to this temperature for many tens of feet below the surface. Suddenly the temperature drops to -20ºF. How long will it take for freezing temperatures (32ºF) to reach my pipes, which are 8 *ft* under ground?



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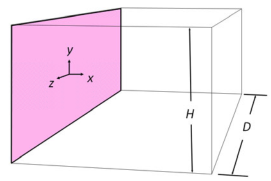
Unsteady State Heat Transfer: Lecture 6

Earlier ....

Develop a model:



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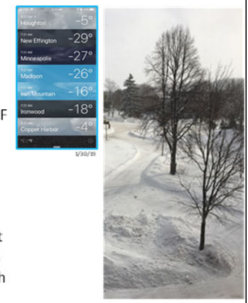

Example 1: **Unsteady Heat Conduction in a Semi-infinite solid**

A very long, very wide, very tall slab is initially at a temperature T_0. At time $t = 0$, the left face of the slab is exposed to a vigorously mixed gas at temperature T_1. What is the time-dependent temperature profile in the slab?

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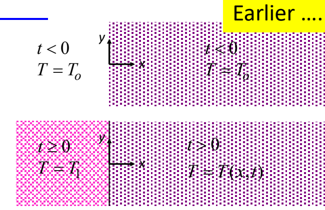
Unsteady State Heat Transfer: Lecture 6

Earlier ....

Unsteady State Heat Conduction in a Semi-Infinite Slab



$$\frac{\partial T}{\partial t} = \frac{k}{\rho \hat{C}_p}\left(\frac{\partial^2 T}{\partial x^2}\right) = \alpha\left(\frac{\partial^2 T}{\partial x^2}\right)$$

thermal diffusivity

$$\alpha \equiv \frac{k}{\rho \hat{C}_p}$$

Initial condition: $t = 0 \quad T = T_0 \quad \forall\ x$

Boundary conditions:

$$x = 0 \qquad \frac{q_x}{A} = -k\frac{dT}{dx} = h(T_1 - T) \qquad t > 0$$

$$x = \infty \qquad T = T_0 \qquad \forall\, t$$

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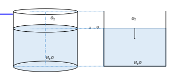
Unsteady State Mass Transport

The mathematics of unsteady state mass transfer is, in many cases, directly analogous to problems in unsteady state heat transfer.

We do not need to solve the differential equations again; just re-use the solutions, including Heissler charts. *Provided BC and IC are the same.*

Unsteady State Mass Transport



Unsteady State Diffusion in a Semi-Infinite Slab

$$\boxed{\frac{\partial c_A}{\partial t} = D_{AB}\left(\frac{\partial^2 c_A}{\partial z^2}\right)}$$

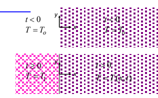
Initial condition: $t = 0 \quad c_A = c_{A0} \quad \forall\ z$

Boundary conditions:

$x = 0 \quad c_A = c_{As} \quad t > 0$

$x = \infty \quad c_A = c_{A0} \quad \forall\ t$

Unsteady State Heat Transfer: Lecture 6



Unsteady State Heat Conduction in a Semi-Infinite Slab

$$\boxed{\frac{\partial T}{\partial t} = \frac{k}{\rho \hat{C}_p}\left(\frac{\partial^2 T}{\partial x^2}\right) = \alpha\left(\frac{\partial^2 T}{\partial x^2}\right)}$$

thermal diffusivity $\alpha \equiv \frac{k}{\rho \hat{C}_p}$

Initial condition: $t = 0 \quad T = T_0 \quad \forall\ x$

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$x = 0 \quad \frac{q_x}{A} = -k\frac{dT}{dx} = h(T_1 - T) \quad t > 0$

$x = \infty \quad T = T_0 \quad \forall\ t$

(For the pipe-freezing problem we did the Newton's law of cooling BC case, but the soln with temperature BCs is in the literature too.)

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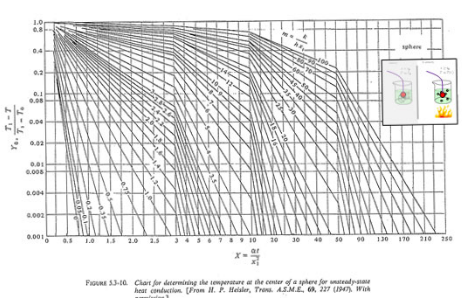
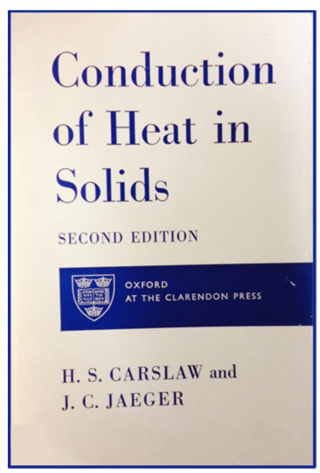
Unsteady State Mass Transport

The mathematics of unsteady state **mass** transfer is, in many cases, **directly analogous** to problems in unsteady state **heat** transfer.

We do not need to solve the differential equations again; we can just re-use the solutions (including Heisler charts).



From Geankoplis, 4th edition, page 374



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Unsteady State Mass Transport

Example 14

O_2 Diffusion Solution:

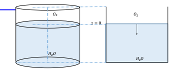
The oxygen concentration as a function of time and depth into the water is given by (adapted from the heat-transfer solution, by analogy):

$$\frac{c_{As}-c_A}{c_{As}-c_{A0}} = \operatorname{erfc}\left(\frac{z}{2\sqrt{D_{AB}t}}\right) = \operatorname{erfc}\zeta$$

$$\zeta \equiv \frac{z}{2\sqrt{D_{AB}t}}$$

Unsteady State Mass Transport

Unsteady State Diffusion in a Semi-Infinite Slab



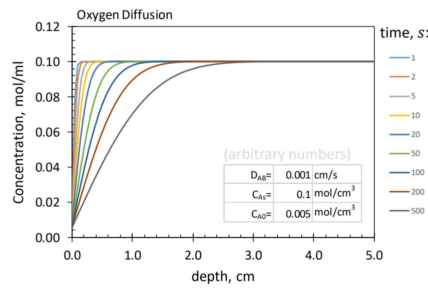
$$\frac{\partial c_A}{\partial t} = D_{AB}\left(\frac{\partial^2 c_A}{\partial z^2}\right)$$

Initial condition: $t=0 \quad c_A = c_{A0} \quad \forall\ z$

Boundary conditions:

$x=0 \quad c_A = c_{As} \quad t>0$

$x=\infty \quad c_A = c_{A0} \quad \forall\ t$



WRF, p534

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Unsteady State Mass Transport

Summary of Unsteady Diffusion:

- The microscopic balances of energy and mass of species A are quite similar mathematically:

$$\left(\frac{\partial T}{\partial t} + \underline{v}\cdot\nabla T\right) = \alpha\nabla^2 T + S_e$$

$$\left(\frac{\partial \omega_A}{\partial t} + \underline{v}\cdot\nabla \omega_A\right) = \mathcal{D}_{AB}\nabla^2 \omega_A + r_A$$

- Some of the boundary conditions are also similar, e.g.:

$t=0$ $\quad T$ or ω_A = known value

$z=0,\infty$ $\quad T$ or ω_A = known value

$z=0,\infty$ $\quad \frac{\partial T}{\partial z}$ or $\frac{\partial \omega_A}{\partial z}$ = known value

$z=z_1$ $\quad \frac{\partial T}{\partial z}$ or $\frac{\partial \omega_A}{\partial z}$ = linear driving force expression (h or k_c)

- Literature results for heat transfer can be repurposed for species A mass transfer

- Intuition for heat transfer is plausible to use for species A mass transfer

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Dimensional analysis and data correlations

Now that we have solved an idealized problem of a system of interest (mass transfer of species A in a semi-infinite slab) we can pursue the dimensionless groups to use in creating data correlations

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mass transfer?

~~heat transfer?~~

What do we do to understand complex ~~flows?~~

Same strategy as:

flows

- *Turbulent tube flow*
- *Noncircular conduits*
- *Drag on obstacles*
- *Boundary Layers*

heat transfer

- *Forced-convection heat transfer coefficients*
- *Natural-convection heat transfer coefficients*
- *Problems with multiple kinds of physics*

1. Find a simple problem that allows us to identify the physics
2. Nondimensionalize
3. Explore that problem
4. Take data and correlate
5. Solve real problems

Solve Real Problems.

Powerful.

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Solve Real Problems. Powerful.

What do we do to understand complex ~~flows?~~ ~~heat transfer?~~ mass transfer?

Same strategy as:

flows
- *Turbulent tube flow*
- *Noncircular conduits*
- *Drag on obstacles*
- *Boundary Layers*

heat transfer
- *Forced-convection heat transfer coefficients*
- *Natural-convection heat transfer coefficients*
- *Problems with multiple kinds of physics*

Mass transfer
- *From fluid to plate*
- *To a falling film*
- *In pipes and ducts*
- *Past submerged objects*
- *To/from bubbles, drops*
- *In agitated systems*
- *In fixed and fluidized beds*
- *In packed 2-phase contactors (absorption, distillation, cooling towers)*

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Dimensional Analysis in Mass Transfer

Let's review *our review* of dimensional analysis…

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Dimensional Analysis in Mass Transfer

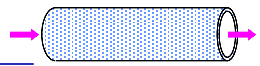
Review of dimensional analysis

CM3120 module 2

Choose "typical" or "characteristic" values; can only know if they are the right choices if the D.A. works.

steady heat transfer

Forced Convection Heat Transfer



CM3110 REVIEW

Pipe flow

z-component of the Navier-Stokes Equation:

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)$$

$$=-\frac{\partial P}{\partial z}+\mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right)+\rho g_z$$

Choose:

D = characteristic length
V = characteristic velocity
D/V = characteristic time
ρV^2 = characteristic pressure

- Choose "typical" values (*scale factors*)
- Use them to scale the equations
- Deduce which terms dominate

Choose "characteristic" values

Dimensional Analysis in Mass Transfer

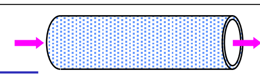
Review of dimensional analysis

CM3120 module 2

Choose "typical" or "characteristic" values; can only know if they are the right choices if the D.A. works.

steady heat transfer

Forced Convection Heat Transfer



CM3110 REVIEW

Pipe flow

non-dimensional variables:

time:	position:	velocity:	driving force:
$t^* \equiv \frac{tV}{D}$	$r^* \equiv \frac{r}{D}$	$v_z^* \equiv \frac{v_z}{V}$	$P^* \equiv \frac{P}{\rho V^2}$
	$z^* \equiv \frac{z}{D}$	$v_r^* \equiv \frac{v_r}{V}$	$g_z^* \equiv \frac{g_z}{g}$
		$v_\theta^* \equiv \frac{v_\theta}{V}$	

- Choose "typical" values (*scale factors*)
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Choose "characteristic" values

Dimensional Analysis in Mass Transfer

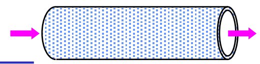
Review of dimensional analysis

CM3120 module 2

Oops, re-used the "*" notation; *here* it is dimensionless variable, not molar average velocity

steady heat transfer

Forced Convection Heat Transfer



CM3110 REVIEW

Pipe flow

non-dimensional variables:

time:

$$t^* \equiv \frac{tV}{D}$$

position:

$$r^* \equiv \frac{r}{D} \qquad z^* \equiv \frac{z}{D}$$

velocity:

$$v_z^* \equiv \frac{v_z}{V} \qquad v_r^* \equiv \frac{v_r}{V} \qquad v_\theta^* \equiv \frac{v_\theta}{V}$$

driving force:

$$P^* \equiv \frac{P}{\rho V^2} \qquad g_z^* \equiv \frac{g_z}{g}$$

- Choose "typical" values (*scale factors*)
- Use them to scale the equations
- Deduce which terms dominate

Choose "characteristic" values

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Dimensional Analysis in Mass Transfer

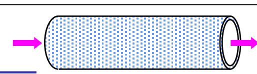
Review of dimensional analysis

CM3120 module 2

Choose "typical" or "characteristic" values; can only know if they are the right choices if the D.A. works.

steady heat transfer

Forced Convection Heat Transfer



CM3110 REVIEW

Energy

Microscopic energy balance:

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right) + S$$

non-dimensional variables:

position:

$$r^* \equiv \frac{r}{D} \qquad z^* \equiv \frac{z}{D}$$

temperature:

$$T^* \equiv \frac{T - T_o}{(T_1 - T_o)}$$

source:

$$S^* \equiv \frac{S}{S_0}$$

Choose:
T – use a characteristic interval (since distance from T = $0K$ is not part of this physics)
S – use a reference source, S_0

$$S_0 \equiv \frac{(T_1 - T_0) V \rho \hat{C}_p}{D} [=] \frac{W}{m^2}$$

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Dimensional Analysis in Mass Transfer

Review of dimensional analysis

CM3120 module 2

steady heat transfer

Micro E-Balance produces $Pe = PrRe$

Complex Heat Transfer – Dimensional Analysis

CM3110 REVIEW

Non-dimensional Energy Equation

$$\left(\frac{\partial T^*}{\partial t^*} + v_r^*\frac{\partial T^*}{\partial r^*} + \frac{v_\theta^*}{r^*}\frac{\partial T^*}{\partial \theta} + v_z^*\frac{\partial T^*}{\partial z^*}\right) = \frac{1}{Pe}\left(\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r\frac{\partial T^*}{\partial r^*}\right) + \frac{1}{r^{*2}}\frac{\partial^2 T^*}{\partial \theta^2} + \frac{\partial^2 T^*}{\partial z^{*2}}\right) + S^*$$

$$Pe = Pr\,Re = \frac{\hat{C}_p \mu}{k}\frac{\rho V D}{\mu}$$

$$Pr \equiv \frac{\hat{C}_p \mu}{k}$$

Non-dimensional Navier-Stokes Equation

$$\frac{Dv_z^*}{Dt^*} = -\frac{\partial P^*}{\partial z^*} + \frac{1}{Re}\left(\nabla^2 v_z\right)^* + \frac{1}{Fr} g^*$$

Non-dimensional Continuity Equation

$$\frac{\partial v_x^*}{\partial x^*} + \frac{\partial v_y^*}{\partial y^*} + \frac{\partial v_z^*}{\partial z^*} + = 0$$

$$\frac{Dv_z}{Dt} \equiv \frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}$$

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Dimensional Analysis in Mass Transfer

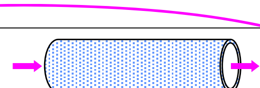
Review of dimensional analysis

CM3120 module 2

The D.A. goes with a *particular* problem (a *particular* physics)

steady heat transfer

Forced Convection Heat Transfer



Here, the "engineering property of interest" is the heat transferred across the boundary, $\dot{Q}$.

Linear driving force model $\left|\frac{q_x}{A}\right| = h|T_1 - T_0|$

Apply at the interface:

$$(2\pi RL)(h)(T_1 - T_0) = Q = \iint_S \left[\hat{e}_r \cdot \tilde{q}\right]_{surface} dS$$

$$(2\pi RL)(h)(T_1 - T_0) = Q = \int_0^{2\pi}\int_0^{L} -k\left.\frac{\partial T}{\partial r}\right|_{r=R} R\,dz\,d\theta$$

Now, non-dimensionalize this expression as well.

Yields correlations for nondimensional heat transfer coefficient, Nu

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Dimensional Analysis in Mass Transfer

Review of dimensional analysis

CM3120 module 2

steady heat transfer

Complex Heat Transfer – Dimensional Analysis

CM3110 REVIEW

$$h(\cancel{\pi}DL)\cancel{(T_1-T_o)} = \int_0^{2\pi}\int_0^{L/D} -k\left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1/2} \frac{\cancel{(T_1-T_o)}}{\cancel{D}}\frac{\cancel{D^2}}{2}\,dz^*\,d\theta$$

This is a function of Re and Pr through fluid $\underline{v}$ distribution and energy balance

$$2\pi\underbrace{\left(\frac{hD}{k}\right)}\left(\frac{L}{D}\right) = \int_0^{2\pi}\int_0^{L/D} -\left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1/2} dz^*\,d\theta$$

Nusselt number, Nu
(dimensionless heat-transfer coefficient)

$$\boxed{Nu = Nu\left(T^*, \frac{L}{D}\right)}$$

one additional dimensionless group

The engineering quantity of interest produces Nu

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Dimensional Analysis in Mass Transfer

Review of dimensional analysis

CM3120 module 2

The D.A. produces:

$$\mathrm{Nu} = \mathrm{Nu}\left(\mathrm{Re}, \mathrm{Pr}, \frac{L}{D}\right)$$

steady heat transfer

Complex Heat Transfer – Dimensional Analysis

CM3110 REVIEW

According to our **dimensional analysis** calculations, the dimensionless heat transfer coefficient should be found to be a function of ~~four~~ three dimensionless groups:

no free surfaces

$$\boxed{Nu = Nu\left(\mathrm{Re}, \mathrm{Pr}, \cancel{\mathrm{Fr}}, \frac{L}{D}\right)}$$

Peclet number

$$\mathrm{Pe} \equiv \frac{\rho \hat{C}_p V D}{k} = \frac{\hat{C}_p \mu}{k}\frac{\rho V D}{\mu}$$

Prandtl number

$$\mathrm{Pr} \equiv \frac{\hat{C}_p \mu}{k}$$

Now, do the experiments.

Can only know if the D.A. is right, if the D.A. works.

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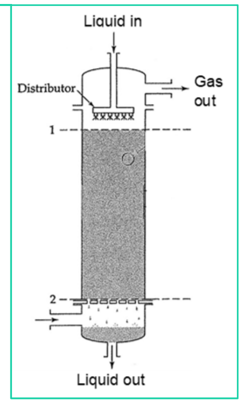
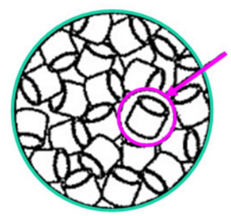
Dimensional Analysis in Mass Transfer

Returning to our question:

What do we do to understand complex <u>mass transfer</u>?

1. Find a simple problem that allows us to identify the physics
2. Non-dimensionalize:
   a. Choose characteristic values
   b. Produce a non-dimensional governing equation
   c. Produce a non-dimensional engineering quantity of interest
3. Explore that problem
4. Take data and correlate (confirm D.A. for chosen problem)
5. Solve real problems with the correlation





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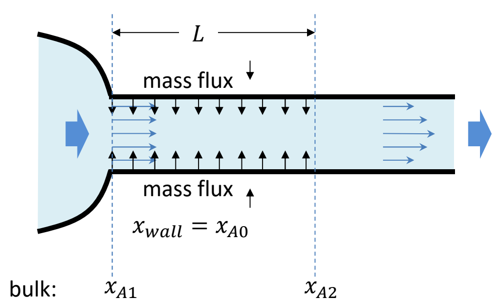
Dimensional Analysis in Mass Transfer

<u>Example 15</u>: What is the mass transfer through the walls of a permeable tube (laminar or turbulent flow)?



Assumptions:
1. Isothermal
2. Steady flow
3. Uniform inlet composition x_{A1}
4. Constant interfacial liquid composition of x_{A0}
5. ρ, μ, c, D_{AB} all constant
6. Radial mass flux (negative)

$$\text{Total mass in} = \int_0^L \int_0^{2\pi} +c\mathcal{D}_{AB} \left.\frac{\partial x_A}{\partial r}\right|_{r=R} R\,d\theta\,dz$$

$$= k_x(2\pi RL)(x_{A0} - x_{A1})$$

BSL2 p679

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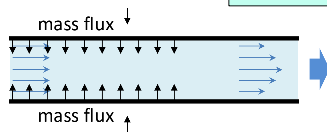
Dimensional Analysis in Mass Transfer

Example 15

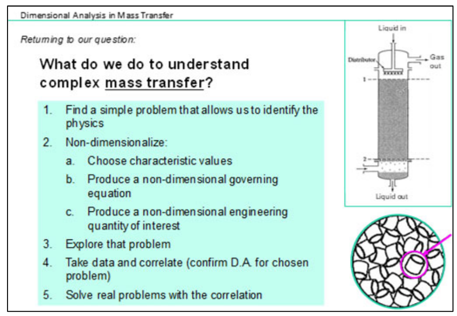
Forced Convection Mass Transfer

Pipe flow



$$k_x(2\pi RL)(x_{A0}-x_{A1}) = \int_0^L\int_0^{2\pi} +c\mathcal{D}_{AB}\left.\frac{\partial x_A}{\partial r}\right|_{r=R} Rd\theta dz$$

Next?


Dimensional Analysis in Mass Transfer

Returning to our question:

What do we do to understand complex mass transfer?

1. Find a simple problem that allows us to identify the physics
2. Non-dimensionalize:
   a. Choose characteristic values
   b. Produce a non-dimensional governing equation
   c. Produce a non-dimensional engineering quantity of interest
3. Explore that problem
4. Take data and correlate (confirm D.A. for chosen problem)
5. Solve real problems with the correlation

BSL2 p679

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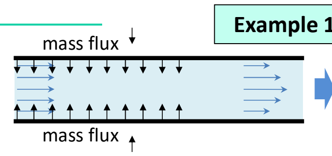
Dimensional Analysis in Mass Transfer

Example 15

Forced Convection Mass Transfer

Pipe flow



non-dimensional variables:

time:

$$t^* \equiv \frac{tV}{D}$$

position:

$$r^* \equiv \frac{r}{D}$$

$$z^* \equiv \frac{z}{D}$$

velocity:

$$v_z^* \equiv \frac{v_z}{V}$$

$$v_r^* \equiv \frac{v_r}{V}$$

$$v_\theta^* \equiv \frac{v_\theta}{V}$$

driving force:

$$P^* \equiv \frac{P}{\rho V^2}$$

$$g_z^* \equiv \frac{g_z}{g}$$

- Choose "typical" values (*scale factors*)
- Use them to scale the equations
- Deduce which terms dominate

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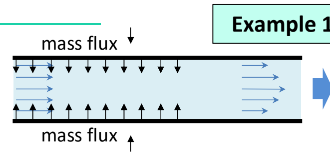
Dimensional Analysis in Mass Transfer

Example 15

Forced Convection Mass Transfer



Species A Mass

Microscopic species A mass balance (no reaction):

$$c\left(\frac{\partial x_A}{\partial t}+v_r\frac{\partial x_A}{\partial r}+\frac{v_\theta}{r}\frac{\partial x_A}{\partial \theta}+v_z\frac{\partial x_A}{\partial z}\right)=c\mathcal{D}_{AB}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial x_A}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 x_A}{\partial \theta^2}+\frac{\partial^2 x_A}{\partial z^2}\right)$$

non-dimensional variables:

position:

$$r^*\equiv\frac{r}{D}$$

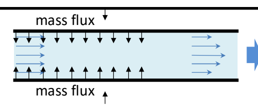
$$z^*\equiv\frac{z}{D}$$

composition

$$x_A^*=\frac{(x_A-x_{A0})}{(x_{A1}-x_{A0})}$$

Choose:
x_A —use a characteristic interval

Dimensional Analysis in Mass Transfer—Forced Convection



Non-dimensional Species A Mass Equation

$$\left(\frac{\partial x_A^*}{\partial t^*}+v_r^*\frac{\partial x_A^*}{\partial r^*}+\frac{v_\theta^*}{r^*}\frac{\partial x_A^*}{\partial \theta}+v_z^*\frac{\partial x_A^*}{\partial z^*}\right)=\frac{1}{\mathrm{Pe}_m}\left(\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r\frac{\partial x_A^*}{\partial r^*}\right)+\frac{1}{r^{*2}}\frac{\partial^2 x_A^*}{\partial \theta^2}+\frac{\partial^2 x_A^*}{\partial z^2}\right)$$

$$\mathrm{Pe}_m=\mathrm{ReSc}=\frac{VD}{\mathcal{D}_{AB}}$$

Non-dimensional Navier-Stokes Equation

$$\frac{Dv_z^*}{Dt^*}=-\frac{\partial P^*}{\partial z^*}+\frac{1}{\mathrm{Re}}\left(\nabla^2 v_z\right)^*+\frac{1}{Fr}g^*$$

$$\mathrm{Sc}=\frac{\mu}{\rho\mathcal{D}_{AB}}$$

Schmidt number
(a material property)

Non-dimensional Continuity Equation

$$\frac{\partial v_x^*}{\partial x^*}+\frac{\partial v_y^*}{\partial y^*}+\frac{\partial v_z^*}{\partial z^*}+=0$$

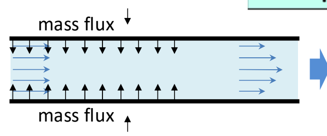
Oops! This is dimensionless v, NOT molar average velocity; sorry!

$$\frac{Dv_z}{Dt}\equiv\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}$$

Dimensional Analysis in Mass Transfer

Example 15

Forced Convection Mass Transfer



Pipe flow

Now, non-dimensionalize this expression as well.

$$k_x(2\pi RL)(x_{A0}-x_{A1})=\int_0^L\int_0^{2\pi}+c\mathcal{D}_{AB}\left.\frac{\partial x_A}{\partial r}\right|_{r=R}R\,d\theta dz$$

$$\text{Sh}=\text{Nu}_{AB}=\frac{k_x D}{c\mathcal{D}_{AB}}=\frac{1}{2\pi\left(\frac{L}{D}\right)}\int_0^L\int_0^{2\pi}\left.-\frac{\partial x_A^*}{\partial r}\right|_{r^*=\frac{1}{2}}d\theta dz^*$$

This is a function of Re and Sc through fluid $\underline{v}$ distribution and species A mass balance

Sherwood number, Sh (dimensionless mass-transfer coefficient)

$$\boxed{\text{Sh}=\text{Sh}\left(x_A^*,\frac{L}{D}\right)}$$

one additional dimensionless group

BSL2 p680

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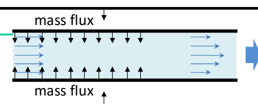
Dimensional Analysis in Mass Transfer

Example 15



According to our **dimensional analysis** calculations, the dimensionless mass-transfer coefficient Sh should be found to be a function of <u>three</u> dimensionless groups:

Peclet number

$$\text{Pe}_m=\text{ReSc}=\frac{VD}{\mathcal{D}_{AB}}$$

Schmidt number

$$\text{Sc}=\frac{\mu}{\rho\mathcal{D}_{AB}}$$

$$\boxed{\text{Sh}=\text{Sh}\left(\text{Re},\text{Sc},\frac{L}{D}\right)}$$

Now, do the experiments.

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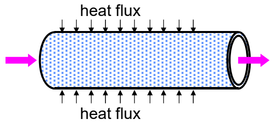
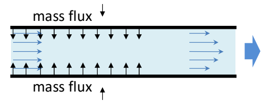
Dimensional Analysis in Mass Transfer

Example 15

<u>Note</u> this development has been exactly the same as a related heat transfer development:





Complex Heat Transfer – Dimensional Analysis

CM3110 REVIEW

According to our **dimensional analysis** calculations, the dimensionless heat transfer coefficient should be found to be a function of ~~four~~ three dimensionless groups:

no free surfaces

$$Nu = Nu\left(\mathrm{Re}, \mathrm{Pr}, \cancel{\mathrm{Fr}}, \frac{L}{D}\right)$$

Peclet number

$$\mathrm{Pe} \equiv \frac{\rho \hat{C}_p V D}{k} = \frac{\hat{C}_p \mu}{k}\frac{\rho V D}{\mu}$$

Prandtl number

$$\mathrm{Pr} \equiv \frac{\hat{C}_p \mu}{k}$$

Now, do the experiments.

Dimensional Analysis in Mass Transfer

Example 15

According to our **dimensional analysis** calculations, the dimensionless mass-transfer coefficient Sh should be found to be a function of <u>three</u> dimensionless groups:

$$\mathrm{Sh} = \mathrm{Sh}\left(\mathrm{Re}, \mathrm{Sc}, \frac{L}{D}\right)$$

Peclet number

$$\mathrm{Pe}_m = \mathrm{ReSc} = \frac{VD}{\mathcal{D}_{AB}}$$

Schmidt number

$$\mathrm{Sc} = \frac{\mu}{\rho \mathcal{D}_{AB}}$$

Now, do the experiments.

In ~~many~~ some cases, heat transfer and mass transfer are **analogous**

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Dimensional Analysis

These numbers tell us about the relative importance of the terms they precede in the governing equations.

Dimensionless numbers from the
Equations of Change *(microscopic balances)*

momentum

Non-dimensional Navier-Stokes Equation

$$\left(\frac{\partial v_z^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* v_z^*\right) = -\frac{\partial P^*}{\partial z^*} + \frac{1}{\mathrm{Re}}(\nabla^{*2} v_z^*) + \frac{1}{\mathrm{Fr}} g^*$$

Re – Reynolds
Fr – Froude

energy

Non-dimensional Energy Equation

$$\left(\frac{\partial T^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* T^*\right) = \frac{1}{\mathrm{RePr}}(\nabla^{*2} T^*) + S^*$$

Pe – Péclet$_h$ = RePr
Pr – Prandtl

mass

Non-dimensional Continuity Equation (species A)

$$\left(\frac{\partial x_A^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* x_A^*\right) = \frac{1}{\mathrm{ReSc}}(\nabla^{*2} x_A^*)$$

Pe – Péclet$_m$ = ReSc
Sc – Schmidt

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ref: BSL1, p581, 644

Dimensional Analysis

These numbers tell us about the relative importance of the terms they precede in the governing equations.

Dimensionless numbers from the
Equations of Change *(microscopic balances)*

momentum

Non-dimensional Navier-Stokes Equation

$$\left(\frac{\partial v_z^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* v_z^*\right) = -\frac{\partial P^*}{\partial z^*} + \frac{1}{\mathrm{Re}}\left(\nabla^{*2} v_z^*\right) + \frac{1}{\mathrm{Fr}} g^*$$

Re – Reynolds
Fr – Froude

energy

Non-dimensional Energy Equation

$$\left(\frac{\partial T^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* T^*\right) = \frac{1}{\mathrm{RePr}}\left(\nabla^{*2} T^*\right) + S^*$$

Pe – Péclet$_h$ = RePr
Pr – Prandtl

mass

Non-dimensional Continuity Equation (species A)

$$\left(\frac{\partial x_A^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* x_A^*\right) = \frac{1}{\mathrm{ReSc}}\left(\nabla^{*2} x_A^*\right)$$

Pe – Péclet$_m$ = ReSc
Sc – Schmidt

Oops! This is dimensionless $\underline{v}$, NOT molar average velocity; sorry!

ref: BSL1, p581, 644

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Dimensionless Numbers

Dimensionless numbers from the
Equations of Change

Re – Reynolds $= \dfrac{\rho V D}{\mu} = \dfrac{VD}{\nu}$

Fr – Froude $= \dfrac{V^2}{gD}$

Pe – Péclet$_h$ $= \mathrm{RePr} = \dfrac{\hat{C}_p \rho V D}{k} = \dfrac{VD}{\alpha}$

Pe – Péclet$_m$ $= \mathrm{ReSc} = \dfrac{VD}{\mathcal{D}_{AB}}$

These numbers tell us about the **relative importance of the terms** they precede in the microscopic balances (*scenario properties*).

Pr – Prandtl $= \dfrac{\hat{C}_p \mu}{k} = \dfrac{\nu}{\alpha}$

Sc – Schmidt $= \mathrm{LePr} = \dfrac{\mu}{\rho \mathcal{D}_{AB}} = \dfrac{\nu}{\mathcal{D}_{AB}}$

Le – Lewis $= \dfrac{\alpha}{\mathcal{D}_{AB}}$

These numbers compare the **magnitudes of the diffusive transport coefficients** $\nu, \alpha, \mathcal{D}_{AB}$ (*material properties*).

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Dimensionless Numbers

Dimensionless numbers from the
Equations of Change

$\text{Re} - \text{Reynolds} = \frac{\rho V D}{\mu} = \frac{VD}{\nu}$

$\text{Fr} - \text{Froude} = \frac{V^2}{gD}$

$\text{Pe} - \text{Péclet}_h = \text{RePr} = \frac{\hat{C}_p \rho V D}{k} = \frac{VD}{\alpha}$

$\text{Pe} - \text{Péclet}_m = \text{ReSc} = \frac{VD}{\mathcal{D}_{AB}}$

These numbers tell us about the **relative importance of the terms** they precede in the microscopic balances (*scenario properties*).

$\text{Pr} - \text{Prandtl} = \frac{\hat{C}_p \mu}{k} = \frac{\nu}{\alpha}$

$\text{Sc} - \text{Schmidt} = \text{LePr} = \frac{\mu}{\rho \mathcal{D}_{AB}} = \frac{\nu}{\mathcal{D}_{AB}}$

$\text{Le} - \text{Lewis} = \frac{\alpha}{\mathcal{D}_{AB}}$

These numbers compare the **magnitudes of the diffusive transport coefficients** $\nu, \alpha, \mathcal{D}_{AB}$ (*material properties*).

Transport coefficients

$\nu \equiv \frac{\mu}{\rho} = \text{kinematic viscosity}$

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Dimensional Analysis

These numbers are defined to help us build **transport data correlations** based on the fewest number of grouped (dimensionless) variables (*scenario properties*).

Dimensionless numbers from the
Engineering Quantities of Interest

momentum

Dimensionless Force on the Wall (Drag)

$$f = \frac{1}{\pi}\frac{D}{L}\frac{1}{\text{Re}} \int_0^{\frac{L}{D}} \int_0^{2\pi} \left(\frac{\partial v_z^*}{\partial r^*} \right) \Bigg|_{r^* = \frac{1}{2}} d\theta dz^*$$

(Fanning)

$f -$ Friction Factor

$\frac{L}{D} -$ Aspect Ratio

$$f = \frac{\mathcal{F}_{drag}}{\left(\frac{1}{2}\rho V^2\right) A_c}$$

energy

Newton's Law of Cooling

$$Nu = \frac{1}{2\pi L/D} \int_0^{2\pi} \int_0^{L/D} -\frac{\partial T^*}{\partial r^*}\Bigg|_{r^*=1/2} dz^* d\theta$$

$\text{Nu} - \text{Nusselt}$

$\frac{L}{D} -$ Aspect Ratio

$\text{St}_h = \frac{h}{\rho V \hat{C}_p} = \frac{\text{Nu}}{\text{RePr}}$

$$\text{Nu} = \frac{hD}{k}$$

mass xfer

Dimensionless Mass Transfer Coefficient

$$Sh = \frac{1}{2\pi}\frac{D}{L} \int_0^{\frac{L}{D}} \int_0^{2\pi} \left(-\frac{\partial x_A^*}{\partial r^*} \right) \Bigg|_{r^* = \frac{1}{2}} d\theta dz^*$$

$\text{Sh} - \text{Sherwood}$

$\frac{L}{D} -$ Aspect Ratio

$\text{St}_m = \frac{k_c}{V} = \frac{\text{Sh}}{\text{ReSc}}$

$$\text{Sh} = \frac{k_c D}{\mathcal{D}_{AB}}$$

$\text{St} - \text{Stanton}$

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momentum
energy
mass

Dimensionless Numbers

Re – Reynolds $= \frac{\rho V D}{\mu} = \frac{VD}{\nu}$

Fr – Froude $= \frac{V^2}{gD}$

Pe – Péclet$_h$ = RePr $= \frac{\hat{C}_p \rho V D}{k} = \frac{VD}{\alpha}$

Pe – Péclet$_m$ = ReSc $= \frac{VD}{\mathcal{D}_{AB}}$

These numbers from the governing equations tell us about the relative importance of the terms they precede in the microscopic balances (***scenario** properties*).

Pr – Prandtl $= \frac{\hat{C}_p \mu}{k} = \frac{\nu}{\alpha}$

Sc – Schmidt = LePr $= \frac{\mu}{\rho \mathcal{D}_{AB}} = \frac{\nu}{\mathcal{D}_{AB}}$

Le – Lewis $= \frac{\alpha}{\mathcal{D}_{AB}}$

These numbers compare the magnitudes of the diffusive transport coefficients ν, α, D_{AB} (***material** properties*).

f – Friction Factor $= \frac{\mathcal{F}_{drag}}{\left(\frac{1}{2}\rho V^2\right) A_c}$

Nu – Nusselt $= \frac{hD}{k}$

Sh – Sherwood $= \frac{k_c D}{\mathcal{D}_{AB}}$

These numbers are defined to help us build transport data correlations based on the fewest number of grouped (dimensionless) variables (***scenario** properties*).

St$_h$ = Nu/Pe$_h$, St$_m$ = Sh/Pe$_m$ – Stanton

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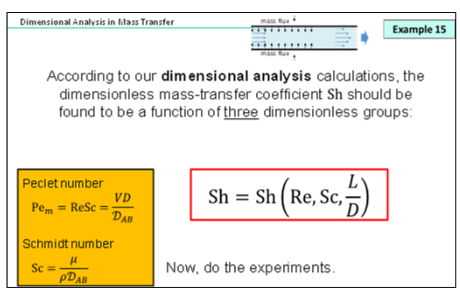
Dimensional Analysis in Mass Transfer

Next?



Tour the world of data correlations in mass transfer:

$$Sh(Re, Sc, \frac{L}{D})$$

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