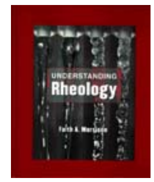
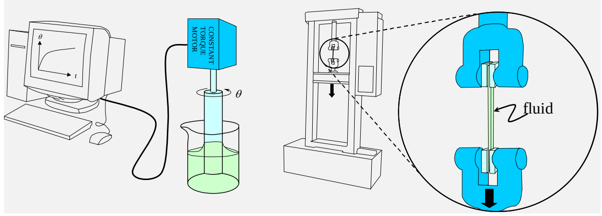
Chapter 4: Standard Flows

CM4650
Polymer Rheology
Michigan Tech



Newtonian fluids: $\underline{\underline{\tau}} = -\mu \dot{\underline{\underline{\gamma}}}$ *vs.* non-Newtonian fluids: $\underline{\underline{\tau}} \neq -\mu \dot{\underline{\underline{\gamma}}}$

How can we investigate non-Newtonian behavior?

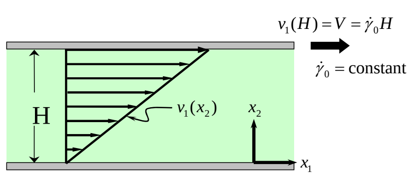


1

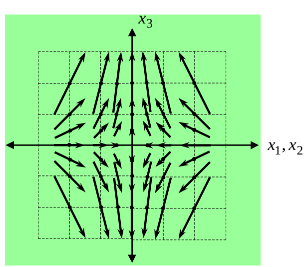
© Faith A. Morrison, Michigan Tech U.

Chapter 4: Standard Flows for Rheology

CM4650
Polymer Rheology
Michigan Tech



shear



elongation

2

© Faith A. Morrison, Michigan Tech U.

On to . . . Polymer Rheology . . .

We now know how to model Newtonian fluid motion, $\underline{v}(\underline{x},t),\ p(\underline{x},t):$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \underline{v}\right) = 0$$

Continuity equation

$$\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v}\right) = -\nabla p - \nabla \cdot \underline{\underline{\tau}} + \rho \underline{g}$$

Cauchy momentum equation

$$\underline{\underline{\tau}} = -\mu\left(\nabla \underline{v} + \left(\nabla \underline{v}\right)^T\right)$$

Newtonian constitutive equation

3

© Faith A. Morrison, Michigan Tech U.

Rheological Behavior of Fluids – Non-Newtonian

How do we model the motion of Non-Newtonian fluid fluids?

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \underline{v}\right) = 0$$

Continuity equation

$$\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v}\right) = -\nabla p - \nabla \cdot \underline{\underline{\tau}} + \rho \underline{g}$$

Cauchy Momentum Equation

$$\underline{\underline{\tau}} = f(\underline{x},t)$$

Non-Newtonian constitutive equation

4

© Faith A. Morrison, Michigan Tech U.

Rheological Behavior of Fluids – Non-Newtonian

How do we model the motion of Non-Newtonian fluid fluids?

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{v}) = 0$$ Continuity equation

$$\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v}\right) = -\nabla p - \nabla \cdot \underline{\underline{\tau}} + \rho \underline{g}$$ Cauchy Momentum Equation

$$\underline{\underline{\tau}} = f(\underline{x}, t)$$ **Non-Newtonian constitutive equation**

This is the missing piece

5

© Faith A. Morrison, Michigan Tech U.

Chapter 4: Standard Flows for Rheology

Chapter 4: Standard flows
Chapter 5: Material Functions
Chapter 6: Experimental Data

To get to constitutive equations, we must first **quantify** how non-Newtonian fluids behave

Chapter 7: GNF
Chapter 8: GLVE
Chapter 9: Advanced

New Constitutive Equations

$$\underline{\underline{\tau}} = f(\underline{x}, t)$$

6

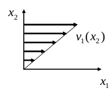
© Faith A. Morrison, Michigan Tech U.

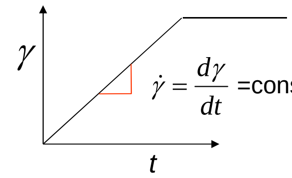
What do we observe?

Rheological Behavior of Fluids – **Newtonian**

1. Strain response to imposed shear stress

•shear rate is constant

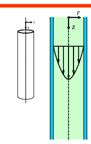


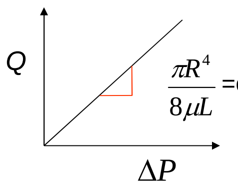


$$\dot{\gamma} = \frac{d\gamma}{dt} \text{ =constant}$$

2. Pressure-driven flow in a tube (Poiseuille flow)

•viscosity is constant





$$Q = \frac{\pi \Delta P R^4}{8 \mu L}$$

$$\frac{\pi R^4}{8 \mu L} \text{ =constant}$$

3. Stress tensor in shear flow

•only two components are nonzero

$$\underline{\underline{\tau}} = \begin{pmatrix} 0 & \tau_{12} & 0 \\ \tau_{21} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{123}$$

7

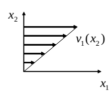
© Faith A. Morrison, Michigan Tech U.

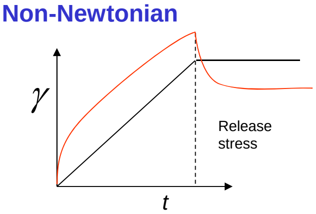
What do we observe?

Rheological Behavior of Fluids – **Non-Newtonian**

1. Strain response to imposed shear stress

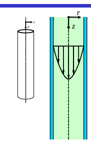
•shear rate is variable

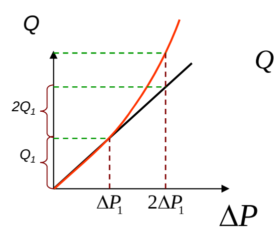




2. Pressure-driven flow in a tube (Poiseuille flow)

•viscosity is variable



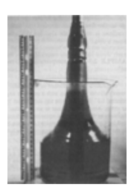


$$Q = f(\Delta P)$$

3. Stress tensor in shear flow

•all 9 components are nonzero

Normal stresses



$$\underline{\underline{\tau}} = \begin{pmatrix} \tau_{11} & \tau_{12} & \tau_{13} \\ \tau_{21} & \tau_{22} & \tau_{23} \\ \tau_{31} & \tau_{32} & \tau_{33} \end{pmatrix}_{123}$$

8

© Faith A. Morrison, Michigan Tech U.

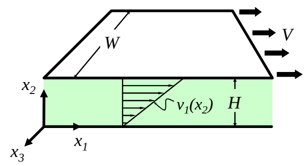
Non-Newtonian Constitutive Equations

- We have observations that some materials are not like Newtonian fluids.
- How can we be systematic about developing new, unknown models for these materials?

Need measurements

For Newtonian fluids, measurements were **easy**:

- independent of flow (use shear flow)
- one stress, τ_{21}
- one material constant, μ (viscosity)



$$\underline{\underline{\tau}} = -\mu\left(\nabla \underline{v} + (\nabla \underline{v})^T\right)$$

9

© Faith A. Morrison, Michigan Tech U.

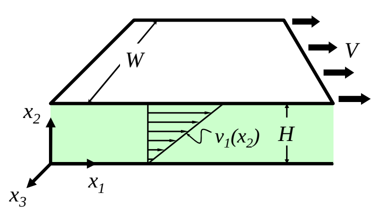
Non-Newtonian Constitutive Equations

Need measurements

For **non-Newtonian** fluids, measurements are **not easy**:

- Depends on the flow (shear flow is not the only choice)
- Four non-zero stresses even in shear, $\tau_{21}, \tau_{11}, \tau_{22}, \tau_{33}$
- Unknown number of material constants in $\underline{\underline{\tau}}(\underline{v})$
- Unknown number of material *functions* in $\underline{\underline{\tau}}(\underline{v})$

$$\underline{\underline{\tau}} = ???$$



10

© Faith A. Morrison, Michigan Tech U.

Non-Newtonian Constitutive Equations

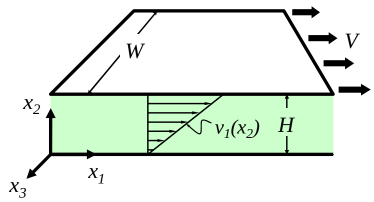
➡ Need measurements

For **non-Newtonian** fluids, measurements are **not easy**:

We know we need to make measurements to know more,

- Depends on the flow (shear flow is not the only choice)
- Four non-zero stresses even in shear, $\tau_{21}, \tau_{11}, \tau_{22}, \tau_{33}$
- Unknown number of material constants in $\underline{\underline{\tau}}(\underline{v})$
- Unknown number of material *functions* in $\underline{\underline{\tau}}(\underline{v})$

$\underline{\underline{\tau}} = ???$



11

© Faith A. Morrison, Michigan Tech U.

Non-Newtonian Constitutive Equations

➡ Need measurements

For **non-Newtonian** fluids, measurements are **not easy**:

We know we need to make measurements to know more,

- Depends on the flow (shear flow is not the only choice)
- Four non-zero stresses even in shear, $\tau_{21}, \tau_{11}, \tau_{22}, \tau_{33}$
- Unknown number of material constants in $\underline{\underline{\tau}}(\underline{v})$
- Unknown number of material *functions* in $\underline{\underline{\tau}}(\underline{v})$

$\underline{\underline{\tau}} = ???$

But, because we do not know the functional form of $\underline{\underline{\tau}}(\underline{v})$, we don't know what we need to measure to know more!

12

© Faith A. Morrison, Michigan Tech U.

Non-Newtonian Constitutive Equations

What should we do?

13

© Faith A. Morrison, Michigan Tech U.

Non-Newtonian Constitutive Equations

What should we do?

1. Pick a small number of simple flows — Chapter 4: Standard flows
   - Standardize the flows
   - Make them easy to calculate with
   - Make them easy to produce in the lab

14

© Faith A. Morrison, Michigan Tech U.

Non-Newtonian Constitutive Equations

What should we do?

1. Pick a small number of simple flows — Chapter 4: Standard flows
   - Standardize the flows
   - Make them easy to calculate with
   - Make them easy to produce in the lab
2. Make calculations
3. Make measurements

(2–3: Chapter 5: Material Functions; Chapter 6: Experimental Data)

15

© Faith A. Morrison, Michigan Tech U.

Non-Newtonian Constitutive Equations

What should we do?

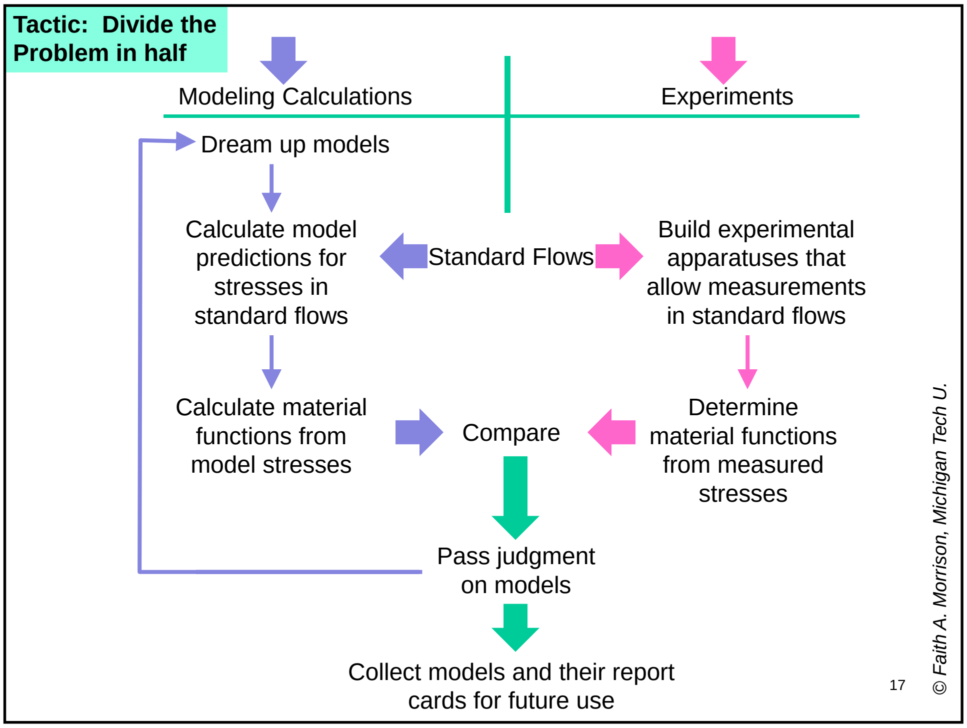
1. Pick a small number of simple flows — Chapter 4: Standard flows
   - Standardize the flows
   - Make them easy to calculate with
   - Make them easy to produce in the lab
2. Make calculations
3. Make measurements
4. Try to deduce $\underline{\underline{\tau}}(\underline{v})$

(2–3: Chapter 5: Material Functions; Chapter 6: Experimental Data)

(4: Chapter 7: GNF; Chapter 8: GLVE; Chapter 9: Advanced)

16

© Faith A. Morrison, Michigan Tech U.



Standard flows – choose a velocity field (not an apparatus or a procedure)

- For model predictions, calculations are straightforward
- For experiments, design can be optimized for accuracy and fluid variety

Material functions – choose a common vocabulary of stress and kinematics to report results

- Make it easier to compare model/experiment
- Record an "inventory" of fluid behavior (expertise)

18

© Faith A. Morrison, Michigan Tech U.

Newtonian fluids:

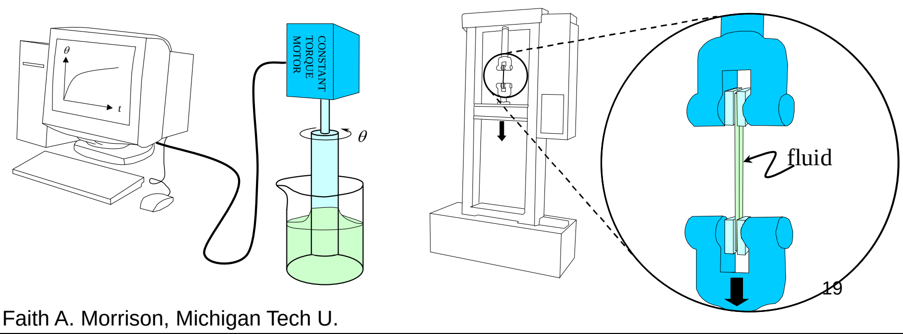
$$\underline{\underline{\tau}} = -\mu \dot{\underline{\underline{\gamma}}}$$

vs.

non-Newtonian fluids:

$$\underline{\underline{\tau}} \neq -\mu \dot{\underline{\underline{\gamma}}}$$

How can we investigate non-Newtonian behavior?

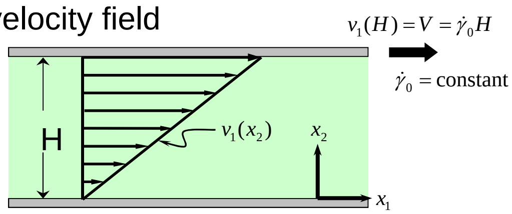


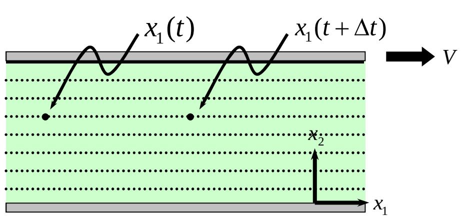
19

© Faith A. Morrison, Michigan Tech U.

Simple Shear Flow

velocity field





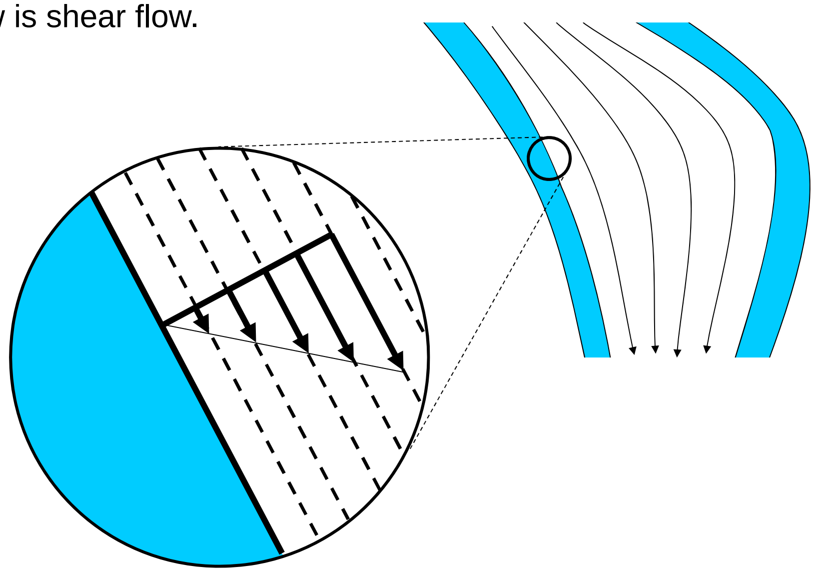
path lines

$$\underline{v} \equiv \begin{pmatrix} \dot{\varsigma}(t) x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$$

20

© Faith A. Morrison, Michigan Tech U.

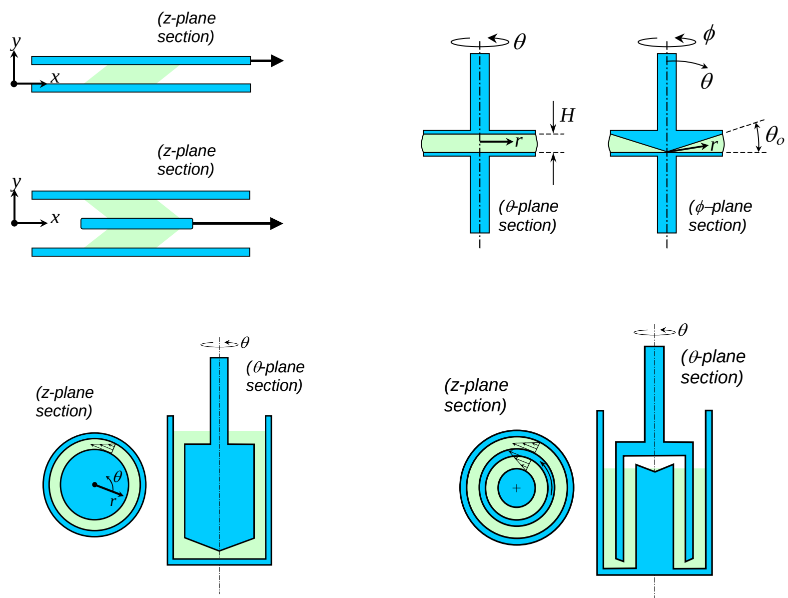
Near solid surfaces, the flow is shear flow.



21

© Faith A. Morrison, Michigan Tech U.

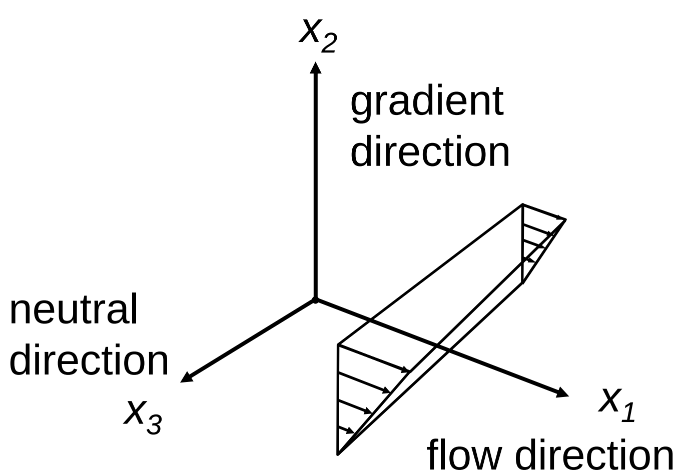
Experimental Shear Geometries



22

© Faith A. Morrison, Michigan Tech U.

Standard Nomenclature for Shear Flow

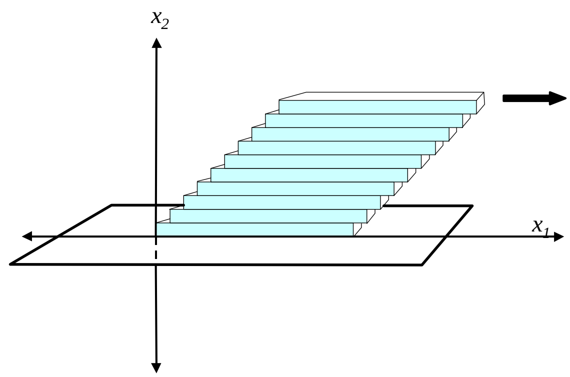


23

© Faith A. Morrison, Michigan Tech U.

Why is shear a standard flow?

- simple velocity field
- represents all sliding flows
- simple stress tensor

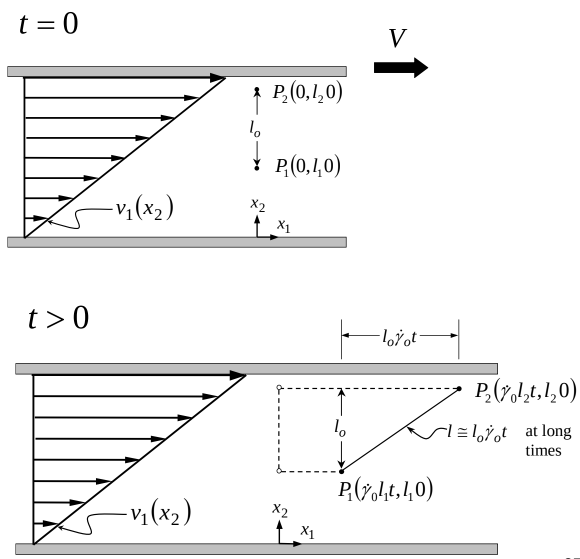


24

© Faith A. Morrison, Michigan Tech U.

How do particles move apart in shear flow?

Consider two particles in the same x_1-x_2 plane, initially along the x_2 axis.



25

© Faith A. Morrison, Michigan Tech U.

How do particles move apart in shear flow?

$$\underline{v} = \begin{pmatrix} \dot{\gamma}_0 x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$$

Each particle has a different velocity depending on its x_2 position:

$$v_1 = \dot{\gamma}_0 x_2$$

$$P_1: \quad v_1 = \dot{\gamma}_0 l_1$$

$$P_2: \quad v_1 = \dot{\gamma}_0 l_2$$

Consider two particles in the same x_1-x_2 plane, initially along the x_2 axis ($x_1=0$).

The initial x_1 position of each particle is $x_1=0$. After t seconds, the two particles are at the following positions:

$$P_1(t): \quad x_1 = \dot{\gamma}_0 l_1 t$$

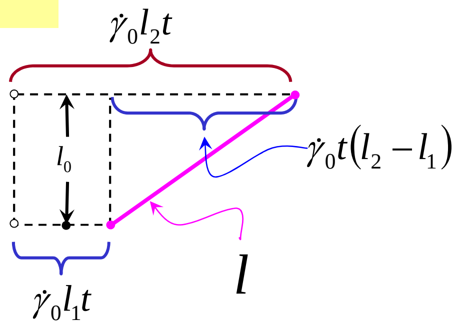
$$P_2(t): \quad x_1 = \underbrace{\dot{\gamma}_0 l_2}\underbrace{t}$$

$$location = initial + \left(\frac{length}{time}\right)(time)$$

26

© Faith A. Morrison, Michigan Tech U.

What is the separation of the particles after time t?



$$l^2 = l_0^2 + [\dot{\gamma}_0 t(l_2 - l_1)]^2$$
$$= l_0^2 + \dot{\gamma}_0^2 t^2 l_0^2$$
$$= l_0^2 (1 + \dot{\gamma}_0^2 t^2)$$
$$l = l_0 \sqrt{1 + \dot{\gamma}_0^2 t^2} \approx l_0 \dot{\gamma}_0 t$$

negligible as $t \to \infty$

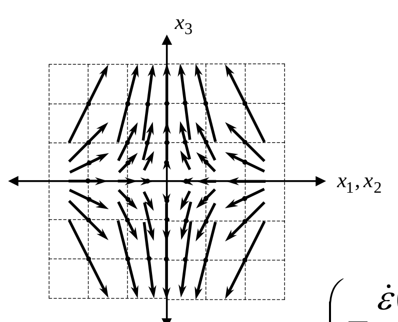
$$l \approx l_0 \dot{\gamma}_0 t$$

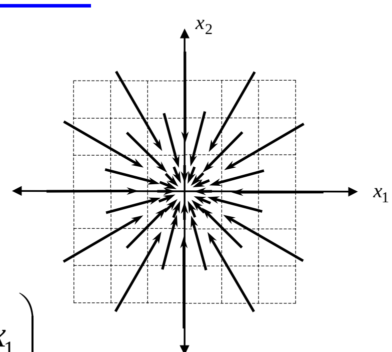
In shear the distance between points is directly proportional to time

27

© Faith A. Morrison, Michigan Tech U.

Uniaxial Elongational Flow





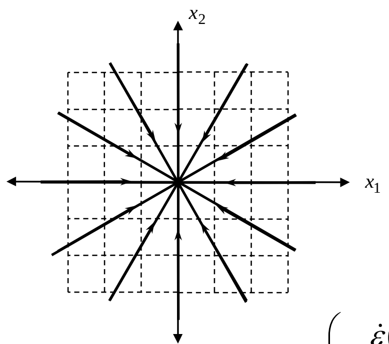
$$\underline{v} \equiv \begin{pmatrix} -\frac{\dot{\varepsilon}(t)}{2} x_1 \\ -\frac{\dot{\varepsilon}(t)}{2} x_2 \\ \dot{\varepsilon}(t) x_3 \end{pmatrix}_{123} \qquad \dot{\varepsilon}(t) > 0$$

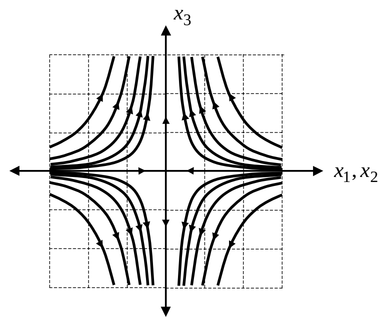
velocity field

28

© Faith A. Morrison, Michigan Tech U.

Uniaxial Elongational Flow





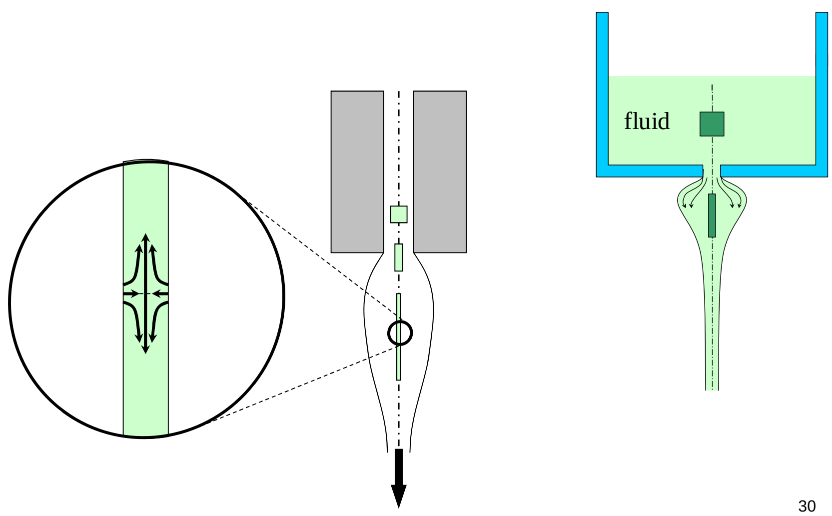
$$\underline{v} \equiv \begin{pmatrix} -\frac{\dot{\varepsilon}(t)}{2} x_1 \\ -\frac{\dot{\varepsilon}(t)}{2} x_2 \\ \dot{\varepsilon}(t) x_3 \end{pmatrix}_{123} \qquad \dot{\varepsilon}(t) > 0$$

path lines

29

© Faith A. Morrison, Michigan Tech U.

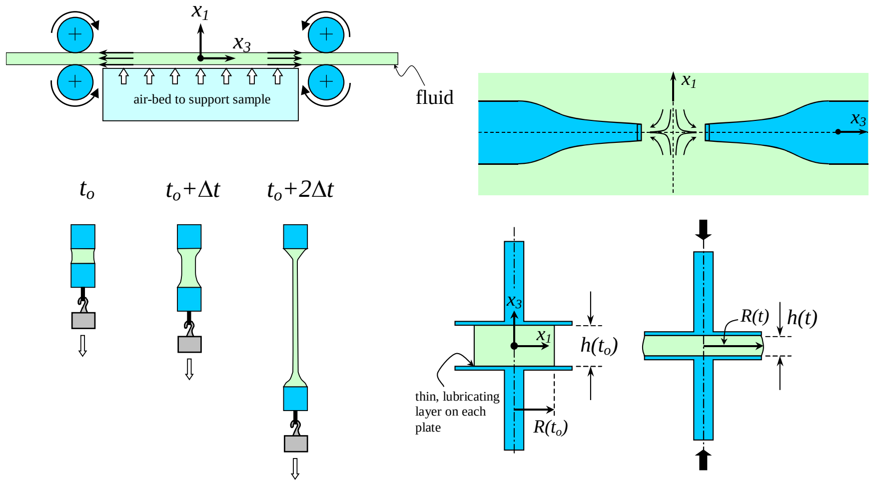
Elongational flow occurs when there is stretching - die exit, flow through contractions



30

© Faith A. Morrison, Michigan Tech U.

Experimental Elongational Geometries

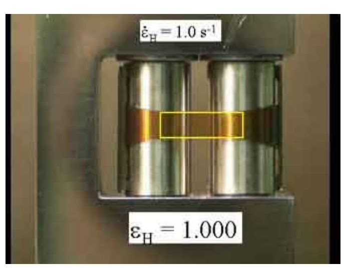


31

© Faith A. Morrison, Michigan Tech U.

Sentmanat Extension Rheometer (2005)

- •Originally developed for rubbers, good for melts
- •Measures elongational viscosity, startup, other material functions
- •Two counter-rotating drums
- •Easy to load; reproducible



www.xpansioninstruments.com

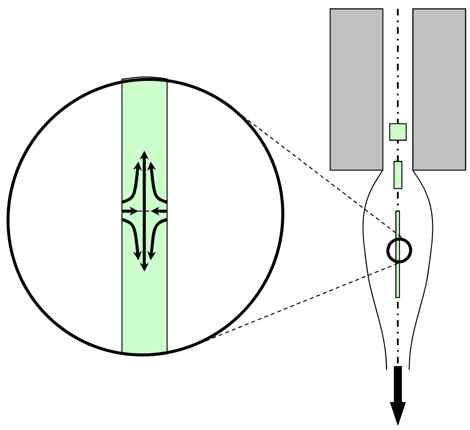
http://www.xpansioninstruments.com/rheo-optics.htm

32

© Faith A. Morrison, Michigan Tech U.

Why is elongation a standard flow?

- simple velocity field
- represents all stretching flows
- simple stress tensor

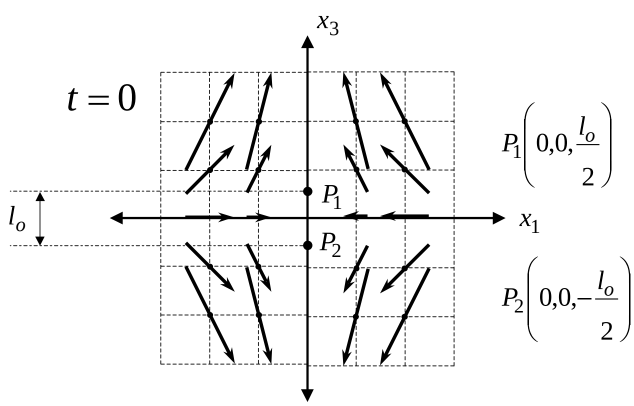


33

© Faith A. Morrison, Michigan Tech U.

How do particles move apart in elongational flow?

Consider two particles in the same x_1-x_3 plane, initially along the x_3 axis.



34

© Faith A. Morrison, Michigan Tech U.

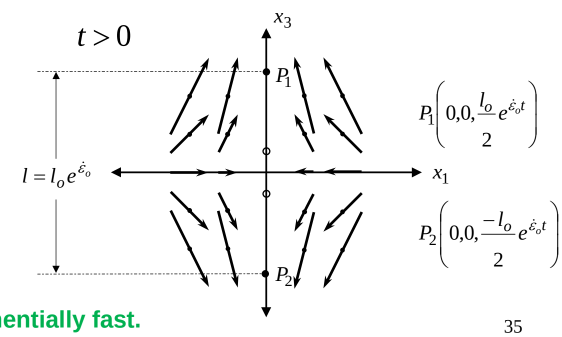
How do particles move apart in elongational flow?

Consider two particles in the same x_1-x_3 plane, initially along the x_3 axis.

$$x_1 = 0$$
$$x_2 = 0$$
$$x_3 \text{ varies}$$

$$\underline{v} = \begin{pmatrix} -\frac{\dot\varepsilon_0}{2}x_1 \\ -\frac{\dot\varepsilon_0}{2}x_2 \\ \dot\varepsilon_0 x_3 \end{pmatrix}_{123} = \begin{pmatrix} 0 \\ 0 \\ \dot\varepsilon_0 x_3 \end{pmatrix}_{123}$$

$$v_3 = \frac{dx_3}{dt} = \dot\varepsilon_0 x_3$$
$$\frac{dx_3}{x_3} = \dot\varepsilon_0 dt$$
$$\ln x_3 = \dot\varepsilon_0 t + C_1$$
$$x_3 = x_3(0)e^{\dot\varepsilon_0 t}$$



$$l = l_0 e^{\dot\varepsilon_0 t}$$

Particles move apart exponentially fast.

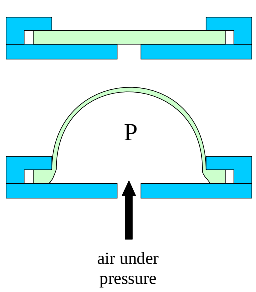
35

© Faith A. Morrison, Michigan Tech U.

A second type of shear-free flow: **Biaxial Stretching**

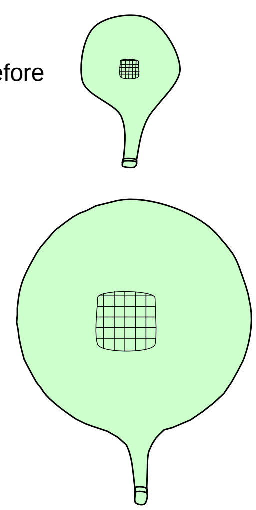
before

after



before

after



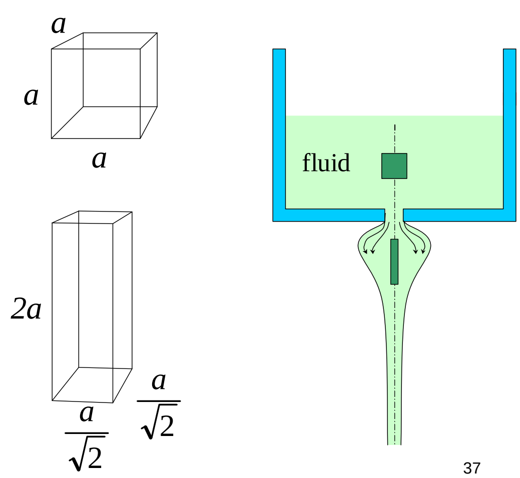
$$\underline{v} \equiv \begin{pmatrix} -\frac{\dot\varepsilon(t)}{2}x_1 \\ -\frac{\dot\varepsilon(t)}{2}x_2 \\ \dot\varepsilon(t)x_3 \end{pmatrix}_{123} \qquad \dot\varepsilon(t) < 0$$

36

© Faith A. Morrison, Michigan Tech U.

How do uniaxial and biaxial deformations differ?

Consider a uniaxial flow in which a particle is doubled in length in the flow direction.

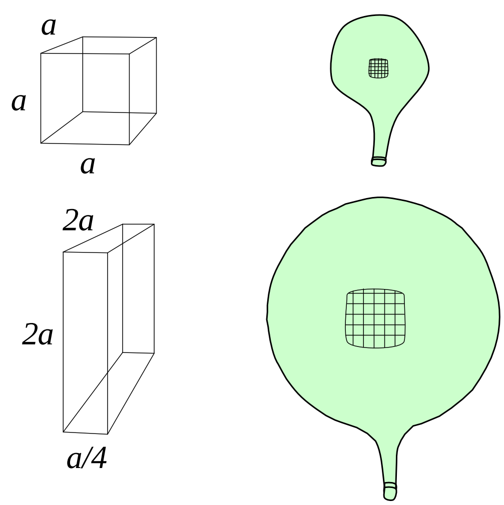


37

© Faith A. Morrison, Michigan Tech U.

How do uniaxial and biaxial deformations differ?

Consider a biaxial flow in which a particle is doubled in length in the flow direction.

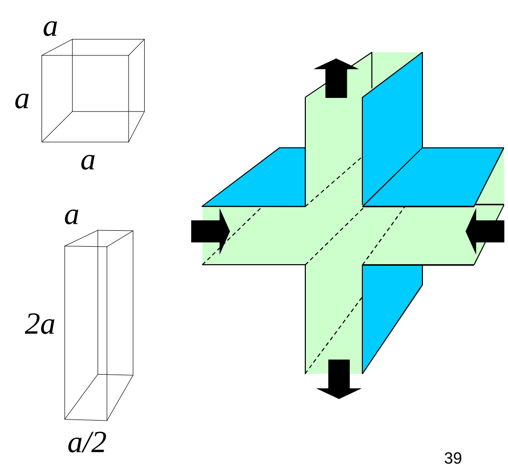


38

© Faith A. Morrison, Michigan Tech U.

A third type of shear-free flow:
Planar Elongational Flow

$$\underline{v} \equiv \begin{pmatrix} -\dot{\varepsilon}(t)x_1 \\ 0 \\ \dot{\varepsilon}(t)x_3 \end{pmatrix}_{123} \qquad \dot{\varepsilon}(t) > 0$$



39

© Faith A. Morrison, Michigan Tech U.

All three shear-free flows can be written together as:

$$\underline{v} = \begin{pmatrix} -\frac{1}{2}\dot{\varepsilon}(t)(1+b)x_1 \\ -\frac{1}{2}\dot{\varepsilon}(t)(1-b)x_2 \\ \dot{\varepsilon}(t)x_3 \end{pmatrix}_{123}$$

Elongational flow: b=0, $\dot{\varepsilon}(t) > 0$

Biaxial stretching: b=0, $\dot{\varepsilon}(t) < 0$

Planar elongation: b=1, $\dot{\varepsilon}(t) > 0$

40

© Faith A. Morrison, Michigan Tech U.

Why have we chosen these flows?

ANSWER: Because these simple flows have **symmetry**.

And symmetry allows us to draw conclusions about the stress tensor that is associated with these flows ***for any fluid*** subjected to that flow.

41

© Faith A. Morrison, Michigan Tech U.

In general:

$$\underline{\underline{\tau}} = \begin{pmatrix} \tau_{11} & \tau_{12} & \tau_{13} \\ \tau_{21} & \tau_{22} & \tau_{23} \\ \tau_{31} & \tau_{32} & \tau_{33} \end{pmatrix}_{123}$$

But the stress tensor is symmetric – leaving 6 independent stress components.

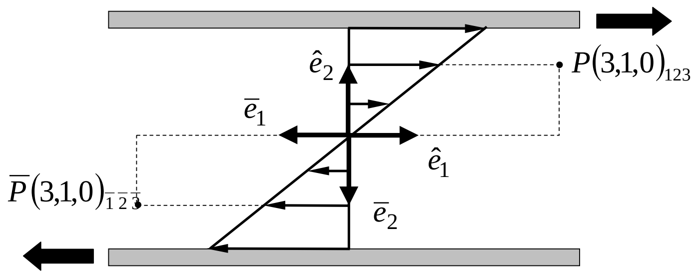
Can we choose a flow to use in which there are fewer than 6 independent stress components?

Yes we can – **symmetric flows**

42

© Faith A. Morrison, Michigan Tech U.

How does the stress tensor simplify for shear (*and later, elongational*) flow?

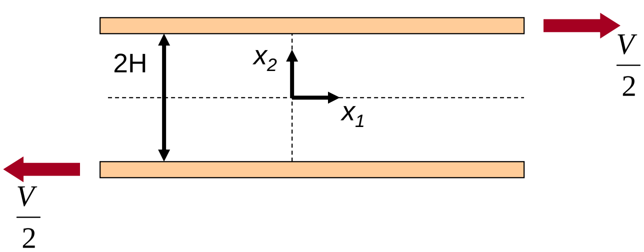


43

© Faith A. Morrison, Michigan Tech U.

What would the velocity function be for a Newtonian fluid in this coordinate system?

$$\underline{v} = \begin{pmatrix} v_1 \\ 0 \\ 0 \end{pmatrix}_{123}$$

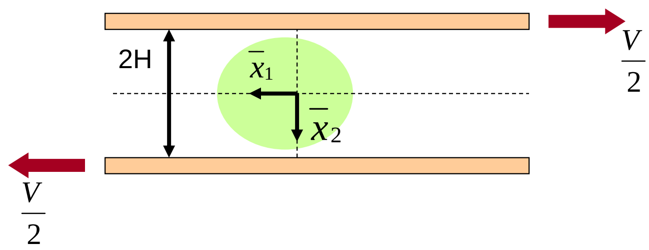


44

© Faith A. Morrison, Michigan Tech U.

What would the velocity function be for a Newtonian fluid in **this** coordinate system?

$$\underline{v} = \begin{pmatrix} v_1 \\ 0 \\ 0 \end{pmatrix}_{123}$$



45

© Faith A. Morrison, Michigan Tech U.

Vectors are independent of coordinate system, but in general the coefficients will be different when the same vector is written in two different coordinate systems:

$$\underline{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}_{123} = \begin{pmatrix} \bar{v}_1 \\ \bar{v}_2 \\ \bar{v}_3 \end{pmatrix}_{\bar{1}\bar{2}\bar{3}}$$

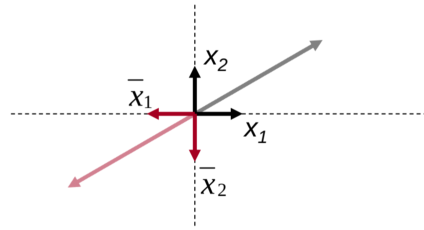
For shear flow and the two particular coordinate systems we have just examined, however:

$$\underline{v} = \begin{pmatrix} \frac{V}{2H} x_2 \\ 0 \\ 0 \end{pmatrix}_{123} = \begin{pmatrix} \frac{V}{2H} \bar{x}_2 \\ 0 \\ 0 \end{pmatrix}_{\bar{1}\bar{2}\bar{3}}$$

46

© Faith A. Morrison, Michigan Tech U.

$$\underline{v} = \begin{pmatrix} \frac{V}{2H} x_2 \\ 0 \\ 0 \end{pmatrix}_{123} = \begin{pmatrix} \frac{V}{2H} \bar{x}_2 \\ 0 \\ 0 \end{pmatrix}_{\bar{1}\bar{2}\bar{3}}$$



If we plug in the **same number** in for x_2 and $\bar{x}_2$, we will NOT be asking about the same point in space, but we WILL get the same exact velocity vector.

Since stress is calculated from the velocity field, we will get the **same exact stress components** when we calculate them from either vector representation.

$$v_n = \bar{v}_n$$
$$\tau_{pk} = \bar{\tau}_{pk}$$

This is an unusual circumstance only true for the particular coordinate systems chosen.

47

© Faith A. Morrison, Michigan Tech U.

What do we learn if we formally transform $\underline{V}$ from one coordinate system to the other?

48

© Faith A. Morrison, Michigan Tech U.

What do we learn if we formally transform $\underline{\underline{\tau}}$ from one coordinate system to the other?

$$\hat{e}_1 = -\bar{e}_1$$
$$\hat{e}_2 = -\bar{e}_2$$
$$\hat{e}_3 = \bar{e}_3$$

49

© Faith A. Morrison, Michigan Tech U.

What do we learn if we formally transform $\underline{V}$ from one coordinate system to the other?

$$\underline{\underline{\tau}} = \tau_{ms}\hat{e}_m\hat{e}_s = \bar{\tau}_{ms}\bar{e}_m\bar{e}_s$$

(now, substitute from previous slide and simplify)

You try.

50

© Faith A. Morrison, Michigan Tech U.

Conclusion:

Because of symmetry, there are only 5 nonzero components of the extra stress tensor in shear flow.

SHEAR:

$$\underline{\underline{\tau}} = \begin{pmatrix} \tau_{11} & \tau_{12} & 0 \\ \tau_{21} & \tau_{22} & 0 \\ 0 & 0 & \tau_{33} \end{pmatrix}_{123}$$

This greatly simplifies the experimentalists tasks as only four stress components must be measured instead of 6 (recall $\tau_{21} = \tau_{12}$).

51

© Faith A. Morrison, Michigan Tech U.

Summary:

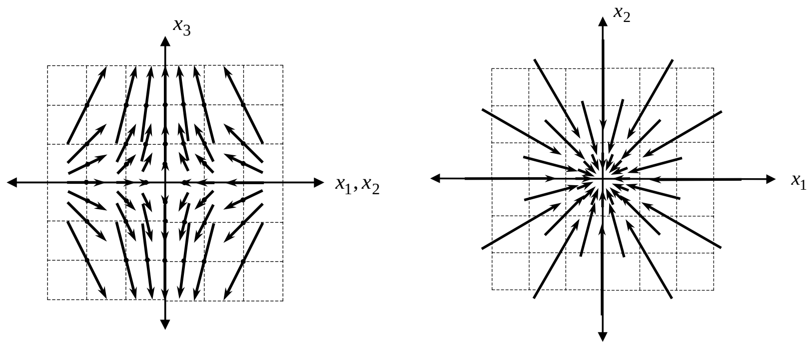
We have found a coordinate system (the shear coordinate system) in which there are only 5 non-zero coefficients of the stress tensor. In addition, $\tau_{21} = \tau_{12}$.

This leaves only four stress components to be measured for this flow, expressed in this coordinate system.

52

© Faith A. Morrison, Michigan Tech U.

How does the stress tensor simplify for elongational flow?



There is 180º of symmetry around all three coordinate axes.

53

© Faith A. Morrison, Michigan Tech U.

Because of symmetry, there are only 3 nonzero components of the extra stress tensor in elongational flows.

ELONGATION:

$$\underline{\underline{\tau}} = \begin{pmatrix} \tau_{11} & 0 & 0 \\ 0 & \tau_{22} & 0 \\ 0 & 0 & \tau_{33} \end{pmatrix}_{123}$$

This greatly simplifies the experimentalists tasks as only three stress components must be measured instead of 6.

54

© Faith A. Morrison, Michigan Tech U.

Standard Flows Summary

Choose velocity field:

$$\underline{v} \equiv \begin{pmatrix} \dot{\varsigma}(t)x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$$

$$\underline{v} = \begin{pmatrix} -\frac{1}{2}\dot{\varepsilon}(t)(1+b)x_1 \\ -\frac{1}{2}\dot{\varepsilon}(t)(1-b)x_2 \\ \dot{\varepsilon}(t)x_3 \end{pmatrix}_{123}$$

Symmetry alone implies:
(no constitutive equation needed yet)

$$\underline{\underline{\tau}} = \begin{pmatrix} \tau_{11} & \tau_{12} & 0 \\ \tau_{21} & \tau_{22} & 0 \\ 0 & 0 & \tau_{33} \end{pmatrix}_{123}$$

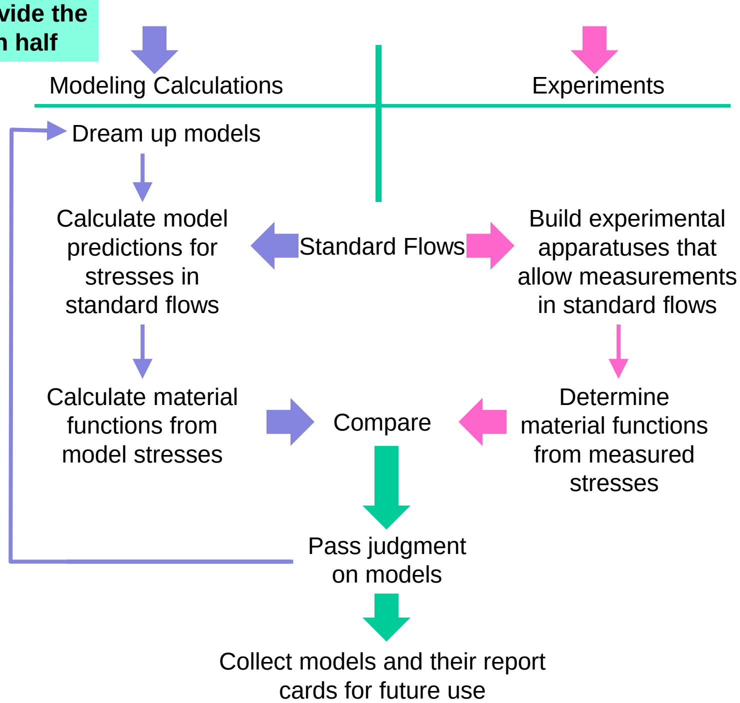
$$\underline{\underline{\tau}} = \begin{pmatrix} \tau_{11} & 0 & 0 \\ 0 & \tau_{22} & 0 \\ 0 & 0 & \tau_{33} \end{pmatrix}_{123}$$

By choosing these symmetric flows, we have reduced the number of stress components that we need to measure.

55

© Faith A. Morrison, Michigan Tech U.

Tactic: Divide the Problem in half



56

© Faith A. Morrison, Michigan Tech U.

Next, build and assume this

Choose velocity field:

$$\underline{v} \equiv \begin{pmatrix} \dot{\varsigma}(t)x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$$

$$\underline{v} = \begin{pmatrix} -\frac{1}{2}\dot{\varepsilon}(t)(1+b)x_1 \\ -\frac{1}{2}\dot{\varepsilon}(t)(1-b)x_2 \\ \dot{\varepsilon}(t)x_3 \end{pmatrix}_{123}$$

Symmetry alone implies:
(no constitutive equation needed yet)

$$\underline{\underline{\tau}} = \begin{pmatrix} \tau_{11} & \tau_{12} & 0 \\ \tau_{21} & \tau_{22} & 0 \\ 0 & 0 & \tau_{33} \end{pmatrix}_{123}$$

$$\underline{\underline{\tau}} = \begin{pmatrix} \tau_{11} & 0 & 0 \\ 0 & \tau_{22} & 0 \\ 0 & 0 & \tau_{33} \end{pmatrix}_{123}$$

Measure and predict this

57

© Faith A. Morrison, Michigan Tech U.

One final comment on measuring stresses. . .

What is measured is the total stress, $\underline{\underline{\Pi}}$:

$$\underline{\underline{\Pi}} = \begin{pmatrix} p+\tau_{11} & \tau_{12} & \tau_{13} \\ \tau_{21} & p+\tau_{22} & \tau_{23} \\ \tau_{31} & \tau_{32} & p+\tau_{33} \end{pmatrix}_{123}$$

For the normal stresses we are faced with the difficulty of separating *p* from t_{ii}.

Compressible fluids:

$$p = \frac{nRT}{V}$$

Get *p* from measurements of *T* and *V*.

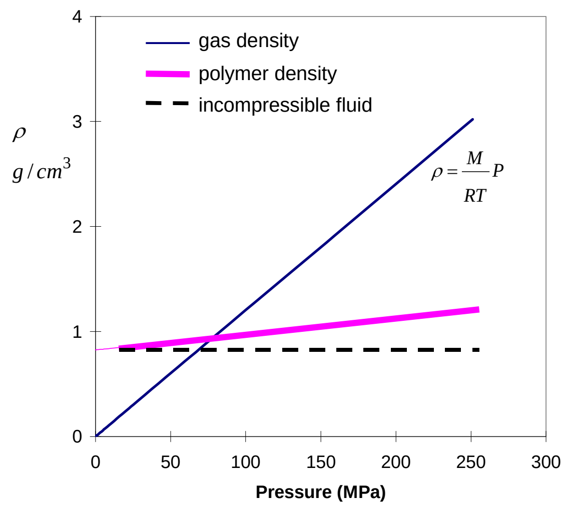
Incompressible fluids:

?

58

© Faith A. Morrison, Michigan Tech U.

Density does not vary (much) with pressure for polymeric fluids.



59

© Faith A. Morrison, Michigan Tech U.

For incompressible fluids it is not possible to separate *p* from t_{ii}.

Luckily, this is not a problem since we only need $\nabla \cdot \underline{\underline{\Pi}} = \nabla p + \nabla \cdot \underline{\underline{\tau}}$

Equation of motion

$$\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v} = -\nabla \underline{\underline{\Pi}} + \rho \underline{g}$$
$$= -\nabla P - \nabla \cdot \underline{\underline{\tau}} + \rho \underline{g}$$

We do not need t_{ii} directly to solve for velocities

Solution? *Normal stress differences*

60

© Faith A. Morrison, Michigan Tech U.

Normal Stress Differences

First normal stress difference $N_1 \equiv \Pi_{11} - \Pi_{22} = \tau_{11} - \tau_{22}$

Second normal stress difference $N_2 \equiv \Pi_{22} - \Pi_{33} = \tau_{22} - \tau_{33}$

In shear flow, three stress quantities are measured τ_{21}, N_1, N_2

In elongational flow, two stress quantities are measured $\tau_{33} - \tau_{11}, \tau_{22} - \tau_{11}$

61

© Faith A. Morrison, Michigan Tech U.

Normal Stress Differences

First normal stress difference $N_1 \equiv \Pi_{11} - \Pi_{22} = \tau_{11} - \tau_{22}$

Second normal stress difference $N_2 \equiv \Pi_{22} - \Pi_{33} = \tau_{22} - \tau_{33}$

In shear flow, three stress quantities are measured τ_{21}, N_1, N_2

Are shear normal stress differences real?

In elongational flow, two stress quantities are measured $\tau_{33} - \tau_{11}, \tau_{22} - \tau_{11}$

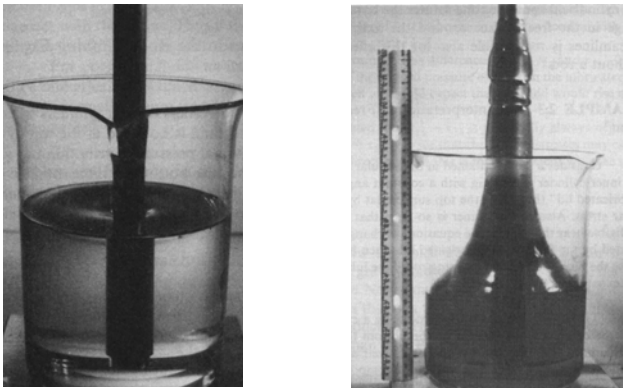
62

© Faith A. Morrison, Michigan Tech U.

First normal stress effects: rod climbing

$$\tau_{11} - \tau_{22} < 0$$

Extra tension in the 1-direction pulls azimuthally and upward (see DPL p65).



Newtonian - glycerin

Viscoelastic - solution of polyacrylamide in glycerin

Bird, et al., *Dynamics of Polymeric Fluids*, vol. 1, Wiley, 1987, Figure 2.3-1 page 63. (DPL)

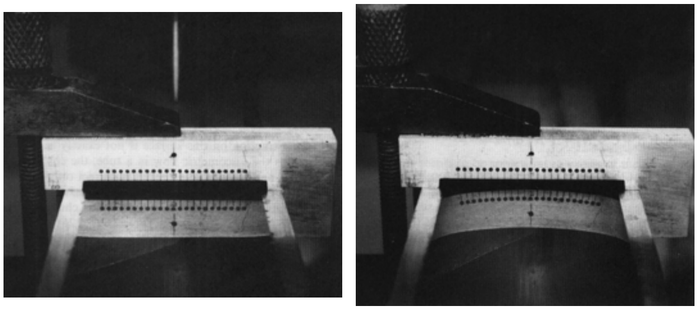
63

© Faith A. Morrison, Michigan Tech U.

Second normal stress effects: inclined open-channel flow

$$\tau_{22} - \tau_{33} > 0$$

Extra tension in the 2-direction pulls down the free surface where dv_1/dx_2 is greatest (see DPL p65).



Newtonian - glycerin

Viscoelastic - 1% soln of polyethylene oxide in water

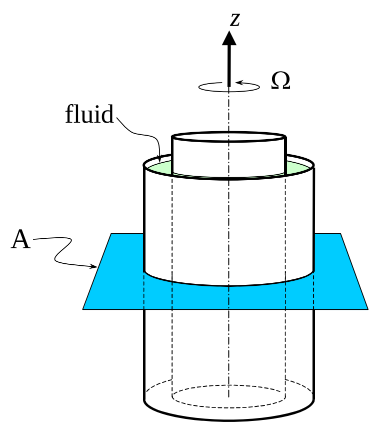
$N_2 \simeq -N_1/10$

R. I. Tanner, *Engineering Rheology*, Oxford 1985, Figure 3.6 page 104

64

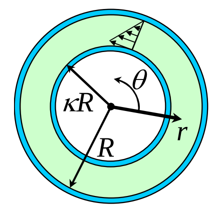
© Faith A. Morrison, Michigan Tech U.

Example: Can the equation of motion predict rod climbing for typical values of N_1, N_2?



$$\underline{v} = \begin{pmatrix} 0 \\ v_\theta \\ 0 \end{pmatrix}_{r\theta z}$$

cross-section A:



What is $\dfrac{d\Pi_{zz}}{dr}$?

www.chem.mtu.edu/~fmorriso/cm4650/rod_climb.pdf

Bird et al. p64

65

© Faith A. Morrison, Michigan Tech U.

What's next?

Shear	Shear-free (elongational, extensional)
$$\underline{v} \equiv \begin{pmatrix} \dot{\varsigma}(t) x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$$	$$\underline{v} = \begin{pmatrix} -\frac{1}{2}\dot{\varepsilon}(t)(1+b)x_1 \\ -\frac{1}{2}\dot{\varepsilon}(t)(1-b)x_2 \\ \dot{\varepsilon}(t)x_3 \end{pmatrix}_{123}$$

Even with just these 2 (or 4) standard flows, we can still generate an *infinite* number of flows by varying $\dot{\varsigma}(t)$ and $\dot{\varepsilon}(t)$.

Elongational flow: b=0, $\dot{\varepsilon}(t) > 0$
Biaxial stretching: b=0, $\dot{\varepsilon}(t) < 0$
Planar elongation: b=1, $\dot{\varepsilon}(t) > 0$

66

© Faith A. Morrison, Michigan Tech U.

We seek to quantify the behavior of non-Newtonian fluids

Procedure:

1. Choose a flow type (shear or a type of elongation).
2. Specify $\dot{\varsigma}(t)$ or $\dot{\varepsilon}(t)$ as appropriate.
3. Impose the flow on a fluid of interest.
4. Measure stresses.

shear	τ_{21}, N_1, N_2
elongation	$\tau_{33}-\tau_{11}, \tau_{22}-\tau_{11}$

5. Report stresses in terms of material functions.

6a. Compare measured material functions with predictions of these material functions (from proposed constitutive equations).

7a. Choose the most appropriate constitutive equation for use in numerical modeling.

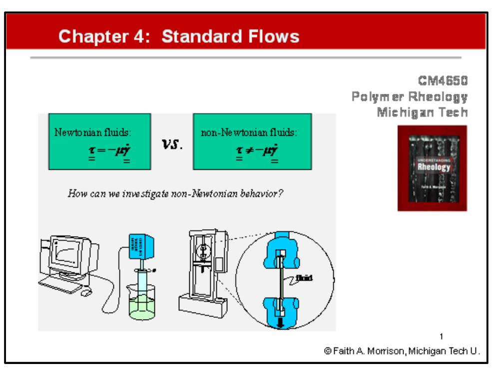
6b. Compare measured material functions with those measured on other materials.

7a. Draw conclusions on the likely properties of the unknown material based on the comparison.

© Faith A. Morrison, Michigan Tech U.

67

Done with
Standard Flows.



Let's move on to Material Functions

68

© Faith A. Morrison, Michigan Tech U.

Chapter 5: Material Functions

CM4650
Polymer Rheology
Michigan Tech

Steady Shear Flow Material Functions

Kinematics:

$$\underline{v} \equiv \begin{pmatrix} \dot{\varsigma}(t) x_2 \\ 0 \\ 0 \end{pmatrix}_{123} \qquad \dot{\varsigma}(t) = \dot{\gamma}_0 = \text{constant}$$

Material Functions:

$$\eta \equiv \frac{-\tau_{21}}{\dot{\gamma}_0}$$

Viscosity

First normal-stress coefficient

$$\Psi_1 \equiv \frac{-(\tau_{11} - \tau_{22})}{\dot{\gamma}_0^2}$$

Second normal-stress coefficient

$$\Psi_2 \equiv \frac{-(\tau_{22} - \tau_{33})}{\dot{\gamma}_0^2}$$

© Faith A. Morrison, Michigan Tech U.

69

© Faith A. Morrison, Michigan Tech U.