ESTIMATION OF ELONGATIONAL VISCOITY FROM CONTRACTION FLOW

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ABSTRACT

Elongational viscosities of commercial linear low-density polyethylene, a metallocene polyethylene and an acrylic polycarbonate alloy were estimated using a finite-element analysis of flow into an abrupt contraction. In the calculations the strain-rate dependence of shear viscosity is modelled using the Carreau-Yassuda model, whereas a newly proposed equation has been used to model the elongational viscosity. The entrance pressure-loss predicted by the finite-element flow simulation was matched with the corresponding experimental data to predict the parameters in the elongational viscosity model. Qualitative differences in the measured elongational viscosities are compared with the shear behaviour of these polymers.

KEYWORDS: ELONGATIONAL VISCOSITY, ENTRANCE PRESSURE LOSS

INTRODUCTION

The elongational viscosity of a polymer can significantly affect the flow in manufacturing processes such as extrusion, blow molding, and fiber spinning, and therefore an accurate knowledge of the elongational viscosity of polymers is important for simulation of these processes. However, due to the inherently unsteady nature of an elongational flow, experimental determination of elongational viscosity is difficult and is limited to low elongation rates, $\dot{\gamma}$\textsuperscript{1}.

Due to the difficulties associated with direct measurement of elongational viscosity, the flow in a tube with an abrupt contraction (entrance flow) has often been used for an indirect measurement of elongational viscosity. When polymeric liquids flow into an abrupt contraction, the fluid usually channels towards the centerline and forms regions of recirculation in the corners of the contraction. On the boundaries of the central, funnel-shaped region (defined by the lines $R(z)$ shown in Figure 1), the velocity is not zero, but it is small compared to the velocity at the centerline [2]. The recirculating regions or vortices dissipate energy, and these vortices, as well as elongational effects and the rearrangement of the velocity profile, are reflected in the measurement of large entrance pressure-losses, $\Delta P_e$\textsuperscript{1}. Measurements of $\Delta P_e$ combined with a model for the entry flow can lead to an estimate of elongational viscosity.

The simplest analysis of entrance flow is that due to Cogswell [2][3], who assumed that $\Delta P_e$ can be written as the direct summation of two pressure drops, one due to shear viscosity and the other due to elongational viscosity. Cogswell then solved for each of these two pressure drops individually, applying a force balance on a differential section of the funnel-shaped entry-flow region and integrating over the entire entry section. The details of the analysis may be found in the literature [2]. In order to arrive at a tractable solution, Cogswell makes several assumptions in addition to those already mentioned. All of the assumptions invoked in Cogswell’s analysis are listed below:

Assumptions for the Cogswell Analysis
1. incompressible fluid
2. funnel-shaped flow; no-slip ($v_z = 0$) on funnel surface
3. unidirectional flow in the funnel region
4. well developed flow upstream and downstream
5. $\theta$-symmetry
6. pressure drops due to shear and elongation may be calculated separately and summed to give the total entrance pressure-loss
7. neglect Weissenberg-Rabinowitch correction
8. shear stress is related to shear-rate through a power-law, $\tau_R = m\dot{\gamma}^n$
9. elongational viscosity is constant
10. shape of the funnel is determined by the minimum generated pressure drop
11. no effect of elasticity (shear normal stresses neglected)
12. neglect inertia

Employing the assumptions above, the average elongational rate in flow into an abrupt contraction is calculated to be

$$\dot{\gamma}_e = \frac{\tau_R f_e}{2(\tau_{11} - \tau_{22})} \quad (1)$$

where $\tau_R = \frac{\Delta P_e}{2L}$ and $f_e = \frac{4Q}{\pi R^3}$, and both are associated with a particular entrance pressure-loss \cite{4}. The average stress difference $(\tau_{11} - \tau_{22})$ is obtained from the entry pressure-loss:

$$\tau_{11} - \tau_{22} = -\frac{3}{8}\Delta P_{en}(n + 1) \quad (2)$$

where $n$ is the power-law parameter for shear viscosity, $\eta$. The elongational viscosity, $\dot{\eta}$, is then calculated from the elongational stress and the elongational rate.

$$\dot{\eta} = \left(\frac{\tau_{11} - \tau_{22}}{\dot{\gamma}_e}\right) = \frac{9}{32}(n + 1)^2\Delta P_{en}^2}{\tau_R f_e} \quad (3)$$

Thus, if the shear viscosity function, $\eta = m\dot{\gamma}^{-1}$, is known, the elongational viscosity can be calculated from measurements of $\Delta P_{en}$ vs. flow rate, $Q$. When compared to other techniques for calculating $\dot{\eta}$ for polymers the
Cogswell analysis is fairly accurate at high rates, but it fails at lower rates [5].

A more complicated but more accurate analysis of contraction flow is that due to Binding [6]. Binding also makes many of the same assumptions as Cogswell, but he allows the elongational viscosity to vary with deformation rate, and he does not neglect the Weissenberg-Rabinowitsch correction. To make the problem solvable, however, Binding resorts to some assumptions about the shape of the funnel, and, like Cogswell, he neglects any influence of elasticity on the normal stresses calculated.

**Assumptions for the Binding Analysis**

1. incompressible fluid
2. funnel-shaped flow; no-slip \( v_z = 0 \) on funnel surface
3. unidirectional flow in the funnel region (see assumption 10)
4. well developed flow upstream and downstream
5. \( \theta \)-symmetry
6. shear viscosity is related to shear-rate through a power-law, \( \eta = m \dot{\gamma}^{\alpha-1} \)
7. elongational viscosity is given by the power law \( \eta = l \dot{\gamma}^{\alpha-1} \)
8. shape of the funnel is determined by the minimum work to drive flow
9. no effect of elasticity (shear normal stresses neglected)
10. the quantities \( (dR/dz)^2 \) and \( a^2 R \dot{z}^2 \), related to the shape of the funnel, are neglected; implies that the radial velocity is neglected when calculating the rate of deformation
11. neglect energy required to maintain the corner circulation
12. \( \tau_{px} - \tau_{px} = 0 \)
13. neglect inertia

In the Binding analysis, \( \Delta \rho_{out} \) is related to the shear- and elongational-viscosity functions through the following equation:

\[
\Delta \rho_{out} = \frac{2m(1+t)^2}{3\lambda(1+n)} \left[ \lambda(3n + 1)w \int_{n}^{t} \int_{m}^{1} \left( \frac{3n + 1}{2} \right)^{n+1} \eta \dot{\gamma}^{\alpha-1} \right] \left( \int_{m}^{1} \bar{\eta} \dot{\gamma}^{\alpha-1} \right)
\]

where \( m \) and \( n \) are the parameters associated with the shear viscosity power law, \( l \) and \( t \) are the parameters associated with the elongational viscosity power law, \( \alpha = R_o/R_t \), \( R_i \) is the downstream radius, \( R_o \) is the upstream radius, \( Q \) is the flow rate, and \( I_{nt} \) and \( \bar{\eta}_t \) are defined below:

\[
I_{nt} = \int_{0}^{1} \left( \int_{0}^{1} \bar{\eta} \dot{\gamma}^{\alpha-1} \right) d\phi
\]

\[
\bar{\eta}_t = \frac{(3n + 1)Q}{n \pi R_o^2}
\]

To calculate \( \Delta \rho_{out} \) we first need to know \( t \), the power-law index for elongational viscosity. From equations 4 and 6 we see that a plot of \( (\log \Delta \rho_{out}) \) versus \( Q \) will give a straight line of slope \( n(\alpha + 1)/(1+n) \), from which we can calculate \( t \) if the shear-viscosity power-law is known. Once \( t \) is known, the integral \( I_{nt} \) may be calculated, and finally the elongational viscosity coefficient \( l \) may be calculated from equation 4. The final outcome of the analysis is the function \( \bar{\eta} = l \dot{\gamma}^{\alpha-1} \).

The elongational viscosities calculated using the Binding analysis are quite reasonable for shear-thinning fluids, but for elastic fluids with constant viscosity (Boger fluids) the analysis is not accurate. The Binding analysis is more accurate than the Cogswell analysis for commodity polymers such as linear low-density polyethylene (LLDPE) [7]. Neither analysis is accurate at low shear rates.

With modern computational tools it is now possible to perform more in-depth simulations of the entry flow problem and to obtain an estimate of elongational viscosity without resorting to as many assumptions as are required by the Cogswell and Binding analyses [8][9]. In our finite-element simulations of contraction flow we use the Carreau-Yassuda model to capture the details of the shear viscosity function.

\[
\eta - \eta_w = \left[ 1 + \left( \frac{\dot{\gamma}}{\dot{\gamma}_c} \right)^m \right]^{\frac{n-1}{m}}
\]

where \( \eta \) is the shear viscosity, \( \eta_w \) is the zero-shear viscosity, \( \eta_w \) is the viscosity limit at high shear rates, \( \lambda \) is a relaxation time, \( \dot{\gamma}_c \) is the magnitude of the shear rate tensor, \( \dot{\gamma}_c \) is a power-law index related to the slope of the high-shear region, and \( m \) is a parameter that affects the shape of the transition from the zero-shear region to the power-law region. For the elongational viscosity, we use an equation recently proposed by Sarkar and Gupta [11]:

\[
\bar{\eta} = \left[ 1 + \left( \frac{\dot{\gamma}_c}{\dot{\gamma}_c} \right)^m \right]^{\frac{n-1}{m}} + \frac{1}{\sqrt[1+n]{\dot{\gamma}_c}}
\]

which is an empirical equation that combines a Carreau-Yassuda part (with elongational parameters \( \dot{\gamma}_c \) and \( m \)), and \( \alpha \) replaced by 2) and an additional term that allows a maximum to appear in elongational viscosity; the parameters \( \delta \) and \( \dot{\gamma}_c \) control the shape of the elongational viscosity maximum. We use a finite-element calculation to obtain the flow field in the contraction, making no assumptions about the shape of the funnel-flow region. Currently the program does not take into account any shear-normal-stress contributions to the flow.

**EXPERIMENTAL:**

Viscosity and entrance pressure-loss were measured on a Goettfert Rheotester 1000. Capillaries with diameter 1mm and lengths of 20, 30, and 40mm were used to extrapolate \( \Delta \rho_{out} \) [4]. All capillary data have been
corrected for non-parabolic velocity profile. Low-shear-rate viscosities in steady shear were measured on a Rheometrics RDAII or a Bohlin VOR using 25mm parallel plates.

Elongational viscosity functions were calculated for three commercial polymers, a low-density polyethylene (Dow 132I, 190°C), an acrylic polycarbonate alloy (Cyrex 200-8000, 235°C), and a metallocene polyethylene (Dow Affinity PL1880, 190°C). High-shear-rate viscosities and entrance pressure-losses for Cyrex 200-8000 were provided by Datapoints Lab, Inc., Ithaca, NY.

COMPUTATIONAL METHOD:

The procedure for estimation of elongational viscosity starts with an initial guess for the values of the parameters in equation 8. Using the elongational viscosity thus predicted and the Carreau-Yasuda model for shear viscosity, the finite-element software reported in references [12,13] was employed to predict $\Delta p_{\text{ent}}$. The simplex optimization scheme [14] was then used to iteratively improve the values of the parameters such that the least-square error between the experimental data for entrance loss and the corresponding estimation from the finite-element simulation is minimized. The simplex optimization scheme was used for a penalty-based constrained optimization in a semi-infinite search domain.

RESULTS AND DISCUSSION

The entry-pressure-losses for the three polymers are shown in Figure 2. Shear viscosity and calculated elongational viscosity functions are shown in Figure 3. The fit obtained to the experimental $\Delta p_{\text{ent}}$ values was acceptable in all cases. The main limitations of the results are in the availability of $\Delta p_{\text{ent}}$ over a wide range of flow rates and the accuracy of the values of $\Delta p_{\text{ent}}$ obtained experimentally.

The technique is shown to be successful within the limitations of available data. Improvements on the estimates of $\bar{\gamma}$ will come with improvements in the range and quality of experimental data (on going), in including normal-stress effects in the finite-element model (on going).

REFERENCES:

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Figure 1: Schematic of the flow geometry addressed by Cogswell [2]and Binding[6].

Figure 2: Entry pressure losses for the three polymers plus the fits obtained through the optimization.
Figure 3: Shear and elongational viscosities for the three polymers.