1. Continuous Culture with Recycle for Industrial Wastewater Treatment.

   Problem 9.11 of the text.

2. Secondary Metabolite (for example penicillin) Production in a 2-Stage Chemostat

   Problem 9.2 of the text.

3. Fed-Batch Bioreactor for Production of Penicillin

   Problem 9.4 of the text. Note that “quasi-steady state” means dX/dt = 0 in the cell mass balance, which is the assumption we used to derive the equations in lecture.

Due Fri. 19 Oct., '07
9.11 Industrial Wastewater Treatment
Continuous Culture w/ Recycle.

\[ F = 100 \text{ L/hr}, \quad x_0 = 0, \quad S_0 = 5,000 \frac{\text{mg}}{\text{L}} \]

\[ \mu_m = 0.25 \text{ h}^{-1} \]
\[ K_S = 200 \frac{\text{mg}}{\text{L}} \]
\[ Y_{X/S} = 0.4 \]

\[ C X_1 = X_R \]
\[ C = 2 \]

\[ F (1 + \alpha) \]
\[ S_1 = 100 \frac{\text{mg}}{\text{L}} \]
\[ X_2 \]
\[ S_2 = 100 \frac{\text{mg}}{\text{L}} \]

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a) Volume of Reactor, \( V_1 \)

\[ S_1 = \frac{K_S D_1 (1 + \alpha - \alpha C)}{\mu_m - D_1 (1 + \alpha - \alpha C)} \]

\[ 100 = \frac{(200) D_1 (1.6 - 0.6(2))}{0.25 - D_1 (1.6 - 0.6(2))} \Rightarrow 0.5 = \frac{0.4 D_1}{0.25 - 0.4 D_1} \]

\[ D_1 = 0.208 \text{ h}^{-1} \Rightarrow V_1 = \frac{F}{D_1} = \frac{100 \frac{\text{L}}{\text{h}}}{0.208 \text{ h}^{-1}} = \boxed{481 \text{ L}} \]

b) \( X_1 \) and \( X_R \)

\[ X_1 = \frac{Y_{X/S} (S_0 - S_1)}{1 + \alpha - \alpha C} = \frac{(0.4)(5000 - 100)}{(1.6 - 0.6(2))} = \boxed{4900 \frac{\text{mg}}{\text{L}}} \]

\[ X_R = C X_1 = 2(4900) = \boxed{9800 \frac{\text{mg}}{\text{L}}} \]
c) Sedimentation Tank.

\[ F(1+\alpha) \]

\[ V_{ST} \rightarrow F \]

\[ \alpha F \]

Residence Time, \( \theta_{ST} = \frac{V_{ST}}{(1+\alpha)F} = 2 \text{ hr} \)

\[ V_{ST} = (2 \text{ hr})(1 + 0.6) 100 \frac{L}{h} = 320 \frac{L}{h} \]

\[ X_2 = (1 + \alpha)X_1 - \alpha C X_1 \quad ; \text{from Example 9.1} \]

\[ = (1.6)X_1 - (1.2)X_1 \]

\[ = 0.4X_1 = 0.4(4,900 \text{ mg/L}) \]

\[ = 1,960 \text{ mg/L} \]
Problem 9.2 \( \mu_m = 0.3 \text{ h}^{-1} \) \( K_S = 0.1 \text{ g/L} \) \( Y_{x/S} = 0.4 \text{ g/g} \)

\[
F = 100 \text{ L/h} \\
S_0 = 5 \text{ g/L} \\
x_0 = 0
\]

\[ V_i = 500 \text{ L} \]

\[ \frac{S_i}{V_i} + \frac{S_1}{V_1} = 0 \]

\[ V_a = 300 \text{ L} \]

\[ S_a, X_a \]

\[ P_a \]

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a) \( D_i = \frac{F}{V_i} = \frac{100 \text{ L/h}}{500 \text{ L}} = 0.2 \text{ h}^{-1} = \mu_i \) at steady state

\( \therefore 0.2 \text{ h}^{-1} = \mu_m \frac{S_i}{K_S + S_i} = 0.3 \text{ h}^{-1} \left( \frac{S_i}{0.1 \text{ g/L} + S_i} \right) \)

Solving for \( S_i \):

\( S_i = 0.2 \text{ g/L} \)

\( x_1 = Y_{x/S} (S_o - S_i) = 0.4 \text{ g/g} (5 - 0.2 \text{ g/L}) = 1.92 \text{ g/L} \)

b) \( \mu_a = 0 \) \( \therefore x_a = x_1 \)

\( q_p = 0.02 \text{ g/(g cell \cdot h)} \)

\( Y_{p/S} = 0.6 \text{ g/g} \)

\( D_a = \frac{F}{V_a} = \frac{100 \text{ L/h}}{300 \text{ L}} = 0.333 \text{ h}^{-1} \)

Substrate balance around second stage:

\[ F (S_i - S_2) - \frac{q_p X_a V_a}{Y_{p/S}} = V_a \frac{dS_a}{dt} = 0 \) at steady state

Solving for \( S_a \):

\( S_a = S_i - \frac{q_p X_a V_a}{F Y_{p/S}} = S_i - \frac{q_p X_a}{D_a Y_{p/S}} \)

\( \therefore S_a = 0.2 \text{ g/L} - \frac{0.02 \text{ g/g} \cdot 1 \text{ h}^{-1} \cdot (1.92 \text{ g/L})}{0.333 \text{ h}^{-1} \cdot 0.6 \text{ g/g}} = 0.008 \text{ g/L} \)

\( P_2 = Y_{p/S} (S_1 - S_2) = 0.6 \frac{g}{g} (1.2 - 0.008)\% = 1.15 \text{ g/L} \)
2. P 9.4 Penicillin production in Fed-Batch

\[ \mu_m = 0.2 \text{ h}^{-1} \]
\[ K_S = 0.5 \text{ g/l} \]
\[ Y_{X/S} = 0.3 \text{ g dw cells/g glucose} \]

\[ F = 50 \text{l/h} \quad S_0 = 300 \text{ g glucose/l} \]

\[ V_0 = 500 \text{ l} \]
\[ X_0 = 20 \text{ g} \]

a) \[ V = V_0 + F t = 500 \text{l} + 50 \frac{\text{l}}{\text{hr}} \times (10 \text{ hr}) \]
\[ = 1000 \text{l} \]

b) \[ S \approx t = 10 \text{ hr} \quad (\text{quasi-steady state}) \]

\[ \mu = \frac{D_0}{1 + D_0 t} = \frac{F/V_0}{1 + F/V_0 t} = \frac{50/500 \text{ hr}^{-1}}{1 + \frac{50}{500} (10 \text{ hr})} \]

\[ = 0.05 \text{ hr}^{-1} \]

\[ 0.05 \text{ hr}^{-1} = \frac{\mu_m S}{K_S + S} \]

\[ S = \frac{K_S (0.05 \text{ hr}^{-1})}{\mu_m - (0.05 \text{ hr}^{-1})} = \frac{0.5 \frac{q}{S} (0.05 \text{ hr}^{-1})}{(0.2 - 0.05) \text{ hr}^{-1}} \]

\[ \approx 0.167 \text{ g/l} \]
c) \( X_t \) and \( X \) at \( t = 10 \) hrs

\[
X_t = X_0^t + FY_{x/s} S_0 t
\]
\[
= X_0 V_0 + FY_{x/s} S_0 t
\]
\[
= \left( 120 \frac{g}{l} \right) (500l) + \left( 50 \frac{g}{hr} \right) \left( 3 \frac{g\text{cells}}{g\text{gluc}} \right) \left( 300 \frac{g\text{gluc}}{l} \right) (10hr)
\]
\[
= 10,000 g + 45,000 = \boxed{55,000 \frac{g\text{cells}}{l}}
\]

\[
X(10\text{hrs}) = \frac{X_t}{V_0} = \frac{55,000 \frac{g\text{cells}}{l}}{1000 l} = \boxed{55 \frac{g\text{cells}}{l}}
\]

d) \( P \) at \( t = 10 \) hrs.

\[
Q_p = 0.05 \frac{g\text{penicillin}}{g\text{cells} \cdot \text{hr}}
\]

\[
P_0 = 0.1 \frac{g\text{penicillin}}{l}
\]

\[
\frac{dP_t}{dt} = Q_p X_t = Q_p \left( X_0^t + FY_{x/s} S_0 t \right)
\]

\[
\int_{P_0}^{P_t} dP_t = \int_0^{10hr} Q_p \left( X_0^t + FY_{x/s} S_0 t \right) dt
\]

\[
P_t - P_0 = Q_p \left( X_0^t t + FY_{x/s} S_0 \frac{t^2}{2} \right)
\]
\[
\frac{p_t}{p_0} = \frac{P_j}{V_0} + \frac{q}{P_j (10^4 \text{g})(10 \text{hr}) + (1050 \text{g})(300 \text{g}) (10^4 \text{cm}^2)}
\]

\[
= \frac{(1.8 \text{g}) (700 \text{g} + 0.05 \text{g})}{10^5 \text{g} \cdot \text{hr}} = \frac{(325,000 \text{g} \cdot \text{hr})}{16,300 \text{g} \cdot \text{Penicillin}}
\]

\[
\frac{P_j}{V_0} = \frac{16,300 \text{g} \cdot \text{P}}{1000 \times 10^2 \text{g} \cdot \text{P}} = 16.3 \text{ g P}
\]

\[
\frac{q}{P_j} = \frac{\varphi_f = \frac{M}{V}}{P_j}
\]