1. Temperature Profile at a Thermocouple Junction.
A thermocouple junction consists of two wires connected and embedded at the center of a small spherical ball of solder as shown in the accompanying figure below. The ball has a diameter \( D = 0.71 \) mm. The thermocouple is used to measure temperature in a gas stream. Under the conditions used, the convective heat transfer coefficient is \( h \). The initial temperature of the thermocouple junction is 25°C and the temperature of the gas stream is 200°C. It is desired to know the temperature at the center of the ball of solder where the two thermocouple wires join. In your analysis, neglect the small effect that the thin thermocouple wires have on heat transfer in the solder ball. In this problem, derive the governing equations and list all boundary/initial conditions for this problem, but do not solve.

2. Problem 11B.11 of BSL: Temperature Distribution in a Spherical Catalyst
Parts b - f.
1. Temperature Profile in a Thermocouple

\[ T(0, r) = 25^\circ C, \quad 0 \leq r \leq R \]

Energy Eqn: Spherical coordinates, \( U_r = U_\theta = U_\phi = 0 \)

\[ \rho C_p \left( \frac{\partial T}{\partial t} + \rho r \frac{\partial T}{\partial r} + \frac{\partial}{\partial \theta} \left( \rho r^2 \frac{\partial T}{\partial \theta} \right) + \frac{\partial}{\partial \phi} \left( \rho r^2 \sin \theta \frac{\partial T}{\partial \phi} \right) \right) = \]

\[ k \left[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \phi^2} \right] + f(\theta, \phi) \]

\[ \rho C_p \frac{\partial T}{\partial t} = k \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) \]

IC: \( t=0, \quad 0 \leq r \leq R \)

BC1 \( t>0, \quad r=0 \) \quad T = T_0 \quad (T \text{ is finite or } \frac{\partial T}{\partial r} = 0)

BC2 \( t>0, \quad r=R \) \quad -k \frac{\partial T}{\partial r} |_{r=R} = h (T_{|R} - 200) \]
2. Temperature Rise in a Catalyst Pellet (P. 113, 11 of BSL)

Assumptions:
- \( u_r = u_\theta = u_\phi = 0 \)
- \( \rho, k, C_p \) constant
- Steady-state
- Conduction only
- \( T = T(r) \) only.

\[ S_c = \frac{\dot{q}}{d} \]

Spherical catalyst pellet

- A chemical reaction within the pellet generates heat at a constant rate per unit volume, \( S_c \).
- Gas, at constant \( T_g \).
- Heat loss from pellet: \( \dot{q}_R = h(T_R - T_g) \)

Table B.9: All terms are zero except...

b.) \( \dot{Q} = k \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) + S_c \); \( BC1 \) \( r = 0 \), \( T \) is finite
\( BC2 \) \( r = R \), \( -k \frac{\partial T}{\partial r} |_{R} = h(T_R - T_g) \)

c.) Integrate
\[ \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) = -\frac{S_c}{k} r^2 \quad \Rightarrow \quad \int \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) = -\frac{S_c}{k} \int r^2 dr \]
\[ r^2 \frac{\partial T}{\partial r} = -\frac{S_c}{3k} r^3 + C_1 \quad \Rightarrow \quad \int r^2 \frac{\partial T}{\partial r} = -\frac{S_c}{3k} \int r^2 dr + C_1 \int \frac{dr}{r^2} \]
\[ T = -\frac{S_c}{6k} r^2 - \frac{C_1}{r} + C_2 \]

BC1: \( T \) is finite \( = -\frac{S_c}{6k} (0)^2 - \frac{C_1}{0} + C_2 \quad \therefore C_1 = 0 \)

BC2: \( -k \frac{\partial T}{\partial r} |_{R} = -k \left( -\frac{S_c}{3k} R + \frac{C_1}{R^2} \right) = h(T_R - T_g) \)
but \( T_{1R} = -\frac{S_c}{6k} R^2 + C_2 \); BC2 now is

\[
\frac{S_c R}{3} = h\left(-\frac{S_c}{6k} R^2 + C_2 - T_g\right) \Rightarrow C_2 = \frac{S_c R}{3h} + \frac{S_c R^2}{6k} + T_g
\]

\[
T = -\frac{S_c}{6k} R^2 + C_2 = -\frac{S_c}{6k} R^2 + \frac{S_c R}{3h} + \frac{S_c R^2}{6k} + T_g
\]

\[
i = T_g + \frac{S_c R^2}{6k} \left(1 - \left(\frac{r}{R}\right)^2\right) + \frac{S_c R}{3h}
\]

\[
\frac{dT}{dr} = 0
\]

\[
n r \quad T
\]

\[
T_g + \frac{S_c R}{3h}
\]

\[
O
\]

\[
R
\]

d) for \( h \to \infty \), the last term drops out.

\[
T = T_g + \frac{S_c R^2}{6k} \left(1 - \left(\frac{r}{R}\right)^2\right)
\]

e) Maximum Temperature in the pellet, \( T_{max} \)

\[
T_{max} = T_g + \frac{S_c R^2}{6k} + \frac{S_c R}{3h}
\]

\[
f) \text{ If } k = k(r) \text{ and } S_c = S_c(r) ; \text{ from Table B.8,}
\]

\[
0 = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 k(r) \frac{\partial T}{\partial r} \right) + S_c(r)
\]