1. Heat Loss from the Trans-Alaska Pipeline
(Open Book, But Open Notes, HW, and HW Soln.)

The main Trans-Alaska Pipeline runs north to south 800 miles (1,300 km), from the Arctic Ocean at Prudhoe Bay, Alaska to the Gulf of Alaska at Valdez, Alaska. There are 12 pump stations along the pipeline, each containing four electric pumps driven by diesel or natural gas generators. The pumps keep the oil moving at a rate of approximately 300,000 m³/day (conversion factor of 258.1 gal/m³). Viscous heating within the turbulent flowing oil keeps the oil at a constant temperature of 140°F (60°C) within the cross section of the pipe along the entire length of its journey. The pipeline is 4 feet (1.219 m) in diameter with a 2 foot (.6096 m) thick insulation layer surrounding the pipe having a thermal conductivity of 0.03 W/(m•°K) or 0.0173 Btu/(ft•hr•°F). The annual average air temperature for Alaska can be approximated to be 25°F (-4°C) and the heat transfer coefficient at the interface of the insulated pipeline and the air is 50 W/(m²•°C) or 12 Btu/(ft²•hr•°F).

a) List all simplifying assumptions and write down the applicable Equations of Change for the insulation layer.
b) List boundary conditions.
c) Solve for the temperature profile in the thermal insulation surrounding the oil pipe.
   Calculate the temperature at the outer surface of the insulation layer.
d) Calculate the heat loss rate per unit length of the pipeline.
e) Calculate the total heat loss rate over the entire length of the pipeline.
f) Given that the lower heating value of crude oil is approximately 140,000 Btu/gal crude oil (42x10⁶ Joules/kg crude oil, density is 800 kg/m³), estimate the percent of the energy contained in the crude oil that is lost as heat to the atmosphere during the trans-Alaska transport in the pipeline.
1. Alaska Pipeline Heat Losses

\[ q = h (T - T_a) \]

\[ T_a \text{ constant with } z, \theta, \tau \]

Crude Oil,

\[ T_0 = 140^\circ F \]

Thermal Insulation

**a) Assumptions:**  
- \( T = T(r) \) only due to symmetry
- \( u_r = u_\theta = u_z = 0 \) in thermal insulation
- \( \rho, k, C_p \) are constant, also \( h \)
- neglect radiation because at \( r = R_2, T \) is low.

**Eqns. of Change:**

Continuity Eqn. and Eqn. of Motion not needed.

Energy: Cylindrical Coordinates / Insulation Layer

\[ \rho C_p \left( \frac{\partial T}{\partial t} + u_r \frac{\partial T}{\partial r} + u_\theta \frac{\partial T}{\partial \theta} + u_z \frac{\partial T}{\partial z} \right) = \]

Steady-state

\[ k \left[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{1}{r^2} \frac{\partial^2 T}{\partial z^2} \right] + \rho C_p \frac{\partial T}{\partial t} = 0 \]

\[ k \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) = 0 \]

\[ \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) = 0 \]

**BC1**  
\( r = R_1, \ T = T_0 \)

**BC2**  
\( r = R_2, \ q_r = h (T - T_a) \)

where \( q_r = -k \frac{\partial T}{\partial r} \) at \( r = R_2 \)
c) Integrating once.

\[ \int_0^r \frac{dT}{dr} = \int_0^r \frac{C_1}{r} \, dr + C_1 \]

\[ r \frac{dT}{dr} = C_1, \quad \Rightarrow \quad \frac{dT}{dr} = \frac{C_1}{r} \]

Integrating again.

\[ \int C_1 \, dr + C_2 = T \]

\[ T = C_1 \ln r + C_2 \]

BC1: \( T_0 = C_1 \ln R_1 + C_2 \)

BC2: \[ -k \left. \frac{dT}{dr} \right|_{R_2} = h (T_{1R_2} - T_a) \]

\[ \frac{C_1}{R_2} \]

\[ C_1 \ln R_2 + C_2 \]

\[ T_0 - C_1 \ln R_1, \text{ from BC1!} \]

\[ -k \frac{C_1}{R_2} = h (C_1 \ln R_2 + T_0 - C_1 \ln R_1 - T_a) \]

\[ = h C_1 (\ln R_2 - \ln R_1) + h (T_0 - T_a) \]

\[ C_1 \left( \frac{-k}{R_2} - h (\ln R_2 - \ln R_1) \right) = h (T_0 - T_a) \]

\[ C_1 = -\frac{h (T_0 - T_a)}{\left( \frac{k}{R_2} + h(\ln R_2 - \ln R_1) \right)} \]

\[ (\ln R_2 - \ln R_1 = \ln \frac{R_2}{R_1}) \]

From BC1, \[ C_2 = T_0 - C_1 \ln R_1 \]
\[ T(r) = C_1 \ln r + C_2 \]
\[ = \left( -\frac{h(T_0 - T_a)}{k + h \ln \frac{R_2}{R_1}} \right) \ln r + T_0 + \left( \frac{h(T_0 - T_a)}{k + h \ln \frac{R_2}{R_1}} \right) \ln R_1 \]

\[ T(r) = T_0 - \frac{h(T_0 - T_a)}{k + h \ln \frac{R_2}{R_1}} \ln \frac{r}{R_1} \]

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d) Heat loss per unit length, \( \frac{Q_r}{L} \)

\[ Q_r = 2\pi R_2 \cdot L \cdot q_r \Rightarrow \frac{Q_r}{L} = 2\pi R_2 \cdot q_r \bigg|_{R_2} \]

\[ q_r \bigg|_{R_2} = -k \frac{dT}{dr} \bigg|_{R_2} = -k \frac{C_1}{R_2} = -k \left( -\frac{h(T_0 - T_a)}{R_2^2} \right) \]

\[ = \frac{h(T_0 - T_a)}{1 + \frac{hR_2^2}{k} \ln \frac{R_2}{R_1}} \]

\[ \frac{Q_r}{L} = 2\pi R_2 \frac{h(T_0 - T_a)}{\left(1 + \frac{hR_2^2}{k} \ln \frac{R_2}{R_1}\right)} \]
\[
\frac{Q_r}{L} = 2\pi (1.219 \text{ m}) \left( \frac{50 \text{ W/m}^2}{	ext{K}} \right) \left( 60 - (-4) \right)^8 \left( \frac{50 \text{ W/m}^2}{\text{K}} \right) (1.219 \text{ m}) \ln \left( \frac{1.219}{0.03} \right) \ln \frac{1.219}{1.6096} \right]
\]

\[
\frac{\dot{Q}}{L} = 17.4 \text{ W/m} = 17.4 \frac{\text{J}}{\text{s.m}}
\]

e) \[
Q_r = (17.4 \frac{\text{J}}{\text{s.m}}) (1,300 \text{ km}) (10^3 \frac{\text{m}}{\text{km}}) = 22,6 \times 10^6 \frac{\text{J}}{\text{s}}
\]

\[
= 22.6 \frac{\text{MJ}}{\text{s}}
\]

f) \% of Energy in the Crude Oil Lost.

Rate of Energy Delivered by Pipeline: \( Q_{co} \)

\[
Q_{co} = \dot{V}_{co} \times H_{Vco} \times \rho_{co}
\]

\[
\downarrow \quad \text{volumetric flow rate of Crude Oil} \quad \downarrow \quad \text{Heating Value of Crude Oil} \quad \downarrow \quad \text{Density of Crude Oil}
\]
\[Q_{co} = (300,000 \text{ m}^2) \cdot \left( \frac{1 \text{ day}}{24 \text{ hr}} \right) \cdot \left( \frac{1 \text{ hr}}{60 \text{ min}} \right) \cdot \left( \frac{1 \text{ min}}{60 \text{ s}} \right) \cdot \left( 800 \frac{\text{kJ}}{\text{m}^2 \cdot \text{K}} \right) \cdot \left( 42 \frac{\text{MJ}}{\text{kg} \cdot \text{K}} \right)\]

\[= 1.167 \times 10^5 \frac{\text{MJ}}{\text{s}}\]

\[\% \text{ Heat Lost} = \frac{Q_r}{Q_{co}} \cdot 100\]

\[= \frac{22.6}{1.167 \times 10^5} \cdot 100 = 0.0194\%\]

C) continued.

\[T_1 R_2 = T_0 - \frac{h (T_0 - T_a)}{K + h \ln \frac{R_2}{R_1}} \ln \frac{R_2}{R_1}\]

\[= 60^\circ C - \frac{(50 \frac{\text{W}}{\text{m}^2 \cdot \text{K}}) (60 - (-4)^\circ C)}{[0.03 \frac{\text{W}}{\text{m} \cdot \text{K}}] + (50 \frac{\text{W}}{\text{m}^2 \cdot \text{K}}) \ln \frac{1.219}{1.6096}} \ln \frac{1.219}{1.6096}\]

\[\ln \frac{1.219}{1.6096}\]

\[= 60^\circ C - 63.95^\circ C = -3.95^\circ C\]

Only just above \(T_a\)!