Problem 9A.9

Problem 10A.1

Problem 10A.6

Problem 10A.8 Derive equation 10.2-23 as part of the solution to this problem.

Problem 10B.1 Do not do part d.

Due Fri. 03 Mar., 2006.
10A.1 Heat Loss: Insulated Pipe

\[
\begin{align*}
  k_{o1} &= 26.1 \text{ [Btu/(hr.ft.°F)]} \\
  k_{12} &= 0.04 \\
  k_{23} &= 0.03 \\
  T_3 &= 90°F
\end{align*}
\]

\[
\begin{align*}
  r_0 &= 2.067/2 = 1.0335 \text{ in} \\
  r_1 &= r_0 + 0.154 = 1.19 \text{ in} \\
  r_2 &= r_1 + 2 = 3.19 \text{ in} \\
  r_3 &= r_2 + 2 = 5.19 \text{ in}
\end{align*}
\]

Eqn. 10.6-29

\[
\frac{Q_o}{L} = \frac{2 \pi (T_0 - T_3)}{\left[ \frac{\ln \left( \frac{r_1}{r_0} \right)}{k_{o1}} + \frac{\ln \left( \frac{r_2}{r_1} \right)}{k_{12}} + \frac{\ln \left( \frac{r_3}{r_2} \right)}{k_{23}} \right]}
\]
\[
\frac{Q_0}{L} = \frac{2\pi (250-90)^\circ F}{\left[ \frac{\ln\left(\frac{1.19}{1.0335}\right)}{26.1} + \frac{\ln\left(\frac{3.19}{1.19}\right)}{.04} + \frac{\ln\left(\frac{5.19}{3.19}\right)}{.03} \right] Btu \left(\frac{\text{hr. ft.}}{\text{deg F}}\right)}
\]

\[= 24 [\text{Btu} / (\text{hr. ft. pipe})]\]
10A.6 Heat Loss from a Wall:

\[ T_3 = 0^\circ F \]

\[ T_2 = 61^\circ F \quad \ominus \quad X_2 \]

\[ T_1 = 69^\circ F \quad \ominus \quad X_1 \]

Wall L plastic panel, \( k_{12} = 0.075 \frac{\text{Btu}}{\text{hr} \cdot \text{ft} \cdot ^\circ \text{F}} \)

\( a) \) Shell Balance for Panel: \( u_x = u_y = u_z = 0 \)

\[ P, k \text{ constant.} \]

\[ \frac{dg_x}{dx} = 0 \quad \text{eqn 10.6-2} \]

Integrate: \( g_x = \text{constant} = g_0 \)

\[-k_{12} \frac{dT}{dx} = g_0 \quad \Rightarrow \quad \int dT = -\frac{g_0}{k_{12}} \int^x_{x_1} dT \]

\[ T_2 - T_1 = -\frac{g_0}{k_{12}} (X_2 - X_1) \quad \text{or} \]

\[ g_0 = k_{12} \frac{(T_1 - T_2)}{(X_2 - X_1)} = (0.075)(\frac{(69 - 61)^{\circ F}}{(0.502/_{12})^{\text{ft}}}) \]
\[ q_0 = 14.3 \frac{\text{Btu}}{\text{hr} \cdot \text{ft}^2} \]

b) Wall Thermal Resistance, \( R_{23} \)

\[
R_{23} = \frac{(X_3 - X_2)}{k_{23}} \quad \text{but} \quad k_{23} = \frac{q_0 (X_3 - X_2)}{(T_2 - T_3)}
\]

\[
= \frac{(T_2 - T_3)}{q_0} = \frac{(61 - 0) \, ^\circ F}{14.3 \frac{\text{Btu}}{\text{hr} \cdot \text{ft}^2}}
\]

\[
= 4.2 \left[ \frac{^\circ F}{\text{Btu} / (\text{hr} \cdot \text{ft}^2)} \right]
\]
10A.8 Steady-State Temp. Rise in an Electric Wire

\[ T_a = 25^\circ C \]

Assumptions: \[ \nu_r = \nu_\theta = \nu_z = 0 \]
\[ e_r = q_r, \quad e_\theta = e_z = 0 \]
(conduction only in \( r \)-direction)
\[ T = T(r) \]

A shell balance results in,

\[ \frac{d}{dr} (r q r) = S_e r \quad \text{eqn 10.2-5} \]

where \[ S_e = \frac{I^2}{k_e} \]
\[ I = \text{current density (}\frac{\text{ohm}}{\text{cm}^2}) \]
\[ k_e = \text{electrical conductivity} \]
\[ \frac{(V k_e)^2}{k_e L^2} = \frac{1}{1.72 \times 10^6} \text{ ohm}^{-1} \text{ cm}^{-1} \text{ (copper)} \]
(see prob. 9A.6)
\[ 0.581 \]

\[ V = \frac{I}{L k_e} \]

note: \[ \frac{V}{L} = \frac{I}{k_e} \]
Integrating eqn. 10.2-5 and using BC1

\[ q_r = \frac{Se}{2} r \]  \hspace{1cm} \text{eqn 10.2-9}

\[-k \frac{dT}{dr} = -\frac{Se}{2} r \]  \Rightarrow  \hspace{.5cm} T = -\frac{Se}{4k} r^2 + C_2

BC2 \hspace{.5cm} r = R, \hspace{.5cm} -k \frac{dT}{dr} = h(T - T_a)

\[ \Rightarrow \text{for } r = R \]

\[ \frac{SeR}{2} = h\left(-\frac{Se}{4k} R^2 + C_2 - T_a\right) \]

\[ \frac{SeR}{2} = -\frac{hSeR^2}{4k} + hC_2 - hT_a \]

\[ C_2 = T_a + \frac{SeR}{2h} \left(1 + \frac{hR}{2k}\right) \]

\[ T = -\frac{Se}{4k} r^2 + T_a + \frac{SeR}{2h} \left(1 + \frac{hR}{2k}\right) \]

\[ \boxed{S = T_a + \frac{SeR^2}{4k} \left(1 - \left(\frac{r}{R}\right)^2\right) + \frac{SeR}{2h}} \]  \hspace{1cm} \text{eqn. 10.2-23}
Maximum temperature at $r = 0$

\[
T_{\text{max}} = T_a + \left(\frac{S_e R^2}{4k} + \frac{S_e R}{2h}\right) \left(\frac{12 \text{ cm/m}}{\text{amp.ohm}}\right)^2
\]

\[
S_e = \frac{V^2 k_e}{L^2} = \left(\frac{0.6 \text{ Volt}}{15 \text{ ft}}\right)^2 \left(\frac{5.83 \times 10^5 \text{ ohm}^{-1} \text{ cm}^{-1}}{0.3048 \frac{\text{m}}{\text{ft}}}\right)^2
\]

\[
= 1.0 \times 10^6 \text{ amp}^2 \cdot \text{ohm/m}^3
\]

\[
= 1.0 \times 10^6 \text{ W/m}^3 \quad \text{(note 1 W = 1 amp}^2 \cdot \text{ohm)}
\]

\[
k = 384.1 \text{ W/(m} \cdot \text{K)}
\]

\[
h = 5.7 \text{ Btu/(hr. ft}^2 \cdot \text{°F)}
\]

\[
\gamma = 32.37 \text{ W/(m}^2 \cdot \text{°K)} \quad \text{App. F conversion factors}
\]

\[
T_{\text{max}} = 25°C + \left(\frac{10^6 \text{ W/m}^3 (2.5 \times 10^{-3} \text{ m})^2}{4 (384.1 \text{ W/(m} \cdot \text{K))}} + \frac{10^6 \text{ W/m}^3 (2.5 \times 10^{-3} \text{ m})}{2 (32.37 \text{ W/(m}^2 \cdot \text{°K)})}\right)
\]

\[
= 25°C + (0.0041 + 38.7)°C
\]

\[
= 63.7041°C \quad \text{at } r = 0
\]
b) \( r = R, \ T = T_R \)

\[
T_R = T_a + \frac{S_e R^2}{4k} \left( 1 - \left( \frac{R_0}{R} \right)^2 \right) + \frac{S_e R}{2h}
\]

\[
= 25^\circ C + 38.7^\circ C = 63.7^\circ C
\]

\[
\Delta T_{\text{wax}} = T_{\text{max}} - T_R = 63.7041 - 63.7 = 0.0041^\circ C
\]

\[
\Delta T_{\text{air film}} = 63.7^\circ C - 25^\circ C = 38.7^\circ C
\]

The largest \( \Delta T \) is across the air film!
10B.1 Heat Conduction From a Sphere to a Stagnant Fluid.

BC1: \( r = R, \quad T = T_R \)

BC2: \( r = \infty, \quad T = T_\infty \)

where \( T_\infty < T_R \)

Assumptions:

\( v_r = v_\theta = v_\phi = 0 \)

\( e_\theta = e_\phi = 0 \) (uniform \( T_R \) over sphere)

a) Shell Energy Balance

\[ e_r l r + dr = q_r l r + dr \]

\[ e_r l r = q_r l r \]

rate of heat in by conduction

rate of heat out by conduction

\[ 4\pi \left( (r^2 q_r) l r - (r^2 q_r) l r + dr \right) = 0 \]
\[ \frac{d}{dr} (r^2 q r) = 0 \]

\[ \frac{d}{dr} (r^2 (-k \frac{dT}{dr})) = 0 \]

\[ \frac{d}{dr} (r^2 \frac{dT}{dr}) = 0 \]

\[ \mathrm{eqn\ 10.3-7} \]

\[ \div \text{by } -k \]

\[ \mathrm{b) \ Integrating,} \]

\[ \int d (r^2 \frac{dT}{dr}) = \int 0 \Rightarrow r^2 \frac{dT}{dr} = C_1 \]

\[ \frac{dT}{dr} = \frac{C_1}{r^2} \Rightarrow \int dT = \int \frac{C_1}{r^2} dr + C_2 \]

\[ T = -\frac{C_1}{r} + C_2 \]

\[ \text{BC1} \quad T_R = -\frac{C_1}{R} + C_2 \]

\[ \text{BC2} \quad T_\infty = -\frac{C_1}{\infty} + C_2 \Rightarrow C_2 = T_\infty \]

\[ \therefore C_1 = R (T_\infty - T_R) \]
\[ T = -\frac{C_1}{r} + C_2 = -(T_\infty - T_R)(\frac{R}{r}) + T_\infty \]

\[
\frac{T - T_\infty}{T_R - T_\infty} = \frac{R}{r}
\]

\[ \begin{aligned}
\left. q_{\text{r}} \right|_R &= -k \frac{dT}{dr} \bigg|_R = -k \left. \frac{d}{dr} \left[ T_\infty + (T_R - T_\infty)\left(\frac{R}{r}\right) \right] \right|_R \\
&= -k (T_R - T_\infty) \left. \left(\frac{R}{r^2}\right) \right|_R \\
&= k \left. \frac{(T_R - T_\infty)}{R} \right|_R
\end{aligned} \]

Newton's Law of Cooling,

\[ q_{\text{r}} \big|_R = h (T_R - T_\infty) \]

\[ k (T_R - T_\infty) \left. \frac{1}{R} \right|_R = h (T_R - T_\infty) \]

\[ \frac{k}{R} = h \]

\[ \frac{k}{D/2} = h \Rightarrow \frac{hD}{k} = 2 \]

Nusselt No. (Nu)