Problem 9A.1

Problem 9A.5

Problem 10B.2

**Term Project**
You have three weeks to complete this part of HW#4 (due Fri. Mar. 14). Give some thought and do some reading in the literature if necessary on your term transport project. One possibility is to consult with your research advisor to decide on a project that would aid in your research. Alternatively, you could choose a project that you are just interested in (pure selfish gratification!). You should address these aspects in the work you submit in three weeks.

a) Short description of the problem with a diagram
b) State assumptions (velocity components and/or temperature and the coordinate directions they depend on, boundary conditions, assumptions on fluid properties, etc.
c) Governing equations for the problem
d) Strategy for solution of the governing equations.

Due Fri. 29 Feb., 2008.
9A.1 Prediction of thermal conductivities of gases at low density.

a. Since Argon is monatomic, we use Eq. 9.3-13 to predict its $k$ in the low-density gas region:

$$k = 1.9891 \times 10^{-4} \frac{\sqrt{T/M}}{\sigma^2 \Omega_M}$$

Here $T = 100 + 273.15 = 373.15$K, and Table E.1 gives $M = 39.948$, $\sigma = 3.432$Å, $\epsilon/\kappa = 122.4$K for Argon. Then $\kappa T/\epsilon = 373.15/122.4 = 3.049$, and interpolation in Table E.2 gives $\Omega_M = 1.0344$. Equation 9.3-13 then gives

$$k = 1.9891 \times 10^{-4} \frac{\sqrt{373.15/39.948}}{3.432^2 \times 1.0344} = 499 \times 10^{-7} \text{ cal/s\cdot cm\cdot K}$$

which is within 1.5 percent of the observed value.

b. Equation 9.3-15, Eucken’s formula, gives

$$k = \left(\tilde{C}_p + 1.25 R/M\right)\mu = (\tilde{C}_p + 1.25 R)\mu/M$$

with $R = 1.987$ cal/g-mole-K. Insertion of the data for $\tilde{C}_p$ and $\mu$, and for $M$ from Table E.1, gives

for NO, \hspace{1cm} k = (7.15 + 1.25 \times 1.987) \frac{1929 \times 10^{-7}}{30.01} = 620 \times 10^{-7} \text{ cal/s\cdot cm\cdot K} \\
\hspace{1cm} = 0.02595 \text{ W/m\cdot K} \\
\hspace{1cm} \text{vs. 0.02590 W/m\cdot K from Table 9.1-2.}

for CH$_4$, \hspace{1cm} k = (8.55 + 1.25 \times 1.987) \frac{1116 \times 10^{-7}}{16.04} = 768 \times 10^{-7} \text{ cal/s\cdot cm\cdot K} \\
\hspace{1cm} = 0.03212 \text{ W/m\cdot K} \\
\hspace{1cm} \text{vs. 0.03427 W/m\cdot K from Table 9.1-2.}
9A.5 Estimation of the thermal conductivity of a pure liquid.

We first calculate the derivative \((\partial p/\partial \rho)_T\) required for Eq. 9.3-4:

\[
(\partial p/\partial \rho)_T = \rho^{-1} / [\rho^{-1}(\partial \rho/\partial p)_T]^{-1} \\
= (1/0.9938)[38 \times 10^{-6}]^{-1} = 2.648 \times 10^4 \text{ megabar cm}^3/\text{g} \\
= 2.648 \times 10^{10} \text{ cm}^2/\text{s}^2
\]

Inserting this result into Eq. 9.3-4 and setting \(C_p \approx C_v\), we obtain

\[
v_s = \sqrt{2.648 \times 10^{10}} = 1.627 \times 10^5 \text{ cm/s}
\]

Equation 9.4-3 then gives the following estimate of the thermal conductivity:

\[
k = 2.80 (\tilde{N}\rho/M)^{2/3} k_v \\
= 2.80 \left[\frac{6.02214 \times 10^{23} \times 0.9938}{18.02}\right]^{2/3} \times 1.38066 \times 10^{-16} \times 1.627 \times 10^5 \\
= 6.50 \times 10^4 \text{ g cm/s}^3 \cdot \text{K} \\
= 0.650 \text{ W/m} \cdot \text{K} \\
= 0.375 \text{ Btu/hr} \cdot \text{ft} \cdot \text{F}
\]
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\[ v(z) = v_b \left( \frac{x}{b} \right) \]

Eqn. of Change for Energy

\[ q_x - \mu x \left( \frac{v_b}{b} \right)^2 = C_1 \quad \text{Eqn. 10.4-5} \]

\[ \text{BC1} \quad x = b \quad q_x = 0 \]

\[ C_1 = -\mu b \left( \frac{v_b}{b} \right)^2 \]

\[ -k \frac{dT}{dx} - \mu x \left( \frac{v_b}{b} \right)^2 + \mu b \left( \frac{v_b}{b} \right)^2 = 0 \]

\[ \frac{dT}{dx} = \frac{\mu b}{k} \left( \frac{v_b}{b} \right)^2 (1 - \frac{x}{b}) \]

\[ T = \frac{\mu b}{k} \left( \frac{v_b}{b} \right)^2 \left( x - \frac{x^2}{2b} \right) + C_2. \]

\[ \text{BC2} \quad x = 0 \quad T = T_0 \]

\[ T_0 = 0 + C_2 \quad \Rightarrow \quad C_2 = T_0. \]

\[ T - T_0 = \frac{\mu b^2 \left( \frac{v_b}{b} \right)^2}{k} \frac{x}{b} \left( 1 - \frac{1}{2} \left( \frac{x}{b} \right)^2 \right) = \frac{\mu v_b^2}{k} \left( \frac{x}{b} - \frac{1}{2} \left( \frac{x}{b} \right)^2 \right) \]