Problem 1. Radiant Energy Flux from an H Bomb
The fireball of a hydrogen bomb radiates as a black body at a temperature $T = 7,200$ K. Take the diameter of the fireball as 1 mile. Assume that air is transparent to this radiation. Calculate the radiant energy flux incident on an area of a vertical wall of a house 25 miles from the center of a blast occurring at an altitude of 1 mile. Use an emissivity of $e = 0.7$ for the house wall (Problem 15.24 of the Middleman text “An Introduction to Heat and Mass Transfer, Wiley, 1998). Use simplifying assumptions to estimate the solution to this problem, similar to the sun-earth example from lecture.

Problem 2. Surface Temperature of a Car Roof
Estimate the temperature at a point 10 cm back from the front edge of the roof on an automobile traveling at 55 mph in air at 30°C. The solar radiation flux at the time is 800 W/m². The roof may be regarded as flat, 1 m wide and 1.5 m long. Take the absorptivity for solar radiation as $a = 0.8$, and the emissivity (at the surface temperature) as $\varepsilon = 0.4$. What is the rooftop temperature if the automobile is not moving? (Problem 15.32 of the Middleman text “An Introduction to Heat and Mass Transfer, Wiley, 1998)

Problem 3. 16B.4 of the text.

Due Fri. 28 Mar., 2008
Radiant Energy Flux From a H. Bomb: P15.24 of Middleman

\[ D = 1 \text{ mi.} \]

\[ \Theta_1 = \Theta_2 \approx 0 \]

\[ E = \text{emissivity of wall} = 0.1 \]

\[ r_{12} \text{ is assumed constant at a value equal to the distance from the center of the H Bomb to the base of the House wall,} \]

\[ r_{12} = \sqrt{(25^2 + 1^2)} \text{ mi}^2 \approx \sqrt{726} \text{ mi}^2 \approx 25 \text{ mi}\]

The radiant heat flux from the H Bomb to the house wall

\[ q_{12} = \int \frac{dQ_{12}}{A_1 \cos \Theta_2} \, dA_2 = \int \frac{\sigma T_1^4}{\pi r_{12}^2} \frac{r_{12}^2}{\cos \Theta} \, dA, \]

\[ S = \frac{\sigma T_1^4}{\pi r_{12}^2} \frac{\pi D_1^2}{4} \text{ (emissivity or absorptivity of wall is not important to calculate "incident" radiation).} \]
\[ g_{12} = \frac{\sigma T_1^4}{\pi r_{12}^2} \cdot \left( \frac{\pi D^2}{4} \right) = \frac{\sigma T_1^4 D^2}{4 r_{12}^2} \]

\[ = \frac{1.355 \times 10^{-12} \text{cal} \cdot (17,200 \text{K})^4 \cdot (1 \text{mi} \cdot 1600 \frac{\text{m}}{\text{mi}} \cdot 100 \frac{\text{cm}}{\text{m}})^2}{4 \cdot (25 \text{mi} \cdot 1600 \frac{\text{m}}{\text{mi}} \cdot 100 \frac{\text{cm}}{\text{m}})^2} \]

\[ = 1.46 \ \text{cal} \cdot \frac{\text{cm}^2}{\text{s}} \]

\[ = 610.9 \ \text{W} \cdot \frac{\text{m}^2}{\text{m}^2} \]

\[ 4.184 \ \text{J/\text{cal}} \]

\[ 100 \ \text{cm}^2/\text{m}^2 \]

about the same radiant flux as the sun’s radiation incident on earth.
Surface Temperature of a Car Roof. (Problem 15. of Middleman)

\[ V_0 = 55 \text{ mph} \quad q_{\text{solar}} = 800 \text{ W/m}^2 \]

\[ T_0 = 30^\circ C \]

\[ x \quad a_{\text{roof}} = 0.8 \quad \varepsilon_{\text{roof}} = 0.4 \]

10 cm \[ T_{\text{roof}} \]

Assumptions:
Sky = black body
Car Roof = grey body

Energy Balance:
"rate of Solar radiation absorbed" = "rate of radiation emitted by roof + rate of cooling by Newton's Law of cooling"

\[ a_{\text{roof}} \cdot 800 \text{ W/m}^2 = \varepsilon_{\text{roof}} \sigma \left( T_{\text{roof}}^4 - T_0^4 \right) + h \left( T_{\text{roof}} - T_0 \right) \]

\( h \) can be calculated from Boundary Layer Theory.

\[ h = \frac{2k \frac{1}{\text{Pr}^{\frac{1}{3}}}}{\sqrt{\frac{1260}{37} \left( \frac{\nu x}{T_0} \right)}} \]

For air at 30°C
\[ k = 0.028 \text{ W/(m}\cdot\text{K}) \]
\[ \text{Pr} = 0.74 \]
\[ \nu = 1.6 \times 10^{-5} \text{ m}^2/\text{s} \]
\[ h = \frac{2(0.028 \text{ W/(m.K)}) \cdot 1.74}{\sqrt{\frac{1260}{37} \left( \frac{1.6 \times 10^{-5} \text{ m/s}}{1 \text{ m}} \cdot \frac{1 \text{ m}}{55 \text{ mi/hr}} \cdot \frac{1 \text{ hr}}{1 \text{ mi}} \cdot \frac{1}{3600 \text{ s}} \right)}} = 27.8 \text{ W/m}^2\text{.K} \]

Substitute in for \( h \), and rearrange Energy Eqn.

\[ 0 = a_{\text{roof}} \cdot 800 \frac{\text{W}}{\text{m}^2} - \varepsilon_{\text{roof}} \sigma T_{\text{roof}}^4 - h (T_{\text{roof}} - T_0) \]

Solve for \( T_{\text{roof}} \) in Excel.

\[ T_{\text{roof}} = 317.86 \text{ °K} \]

If \( h = 0 \) (for \( u_0 = 0 \)): neglect natural convection effects.

Energy Balance is.

\[ 0 = a_{\text{roof}} \cdot 800 \frac{\text{W}}{\text{m}^2} - \varepsilon_{\text{roof}} \sigma T_{\text{roof}}^4 \]

Solve for \( T_{\text{roof}} \) in Excel.

\[ T_{\text{roof}} = 409.87 \text{ °K} \]
16B.4 Radiation and conduction through absorbing media

a. We begin by combining Eqs. 16.6-5 and 6 to get

\[ 0 = -\frac{d}{dz} q_z^{(r)} - m_a q_z^{(r)} \]

This may be integrated to give

\[ \ln q_z^{(r)} = -m_a z + C \]

The constant of integration may be obtained from the boundary condition at \( z = 0 \), so that

\[ q_z^{(r)} = q_0^{(r)} e^{-m_a z} \quad \text{and (from Eq. 16.6-6)} \quad Q = m_a q_0^{(r)} e^{-m_a z} \]

Next, use Eq. 16.6-4 to get

\[ 0 = k \frac{d^2 T}{dz^2} + m_a q_0^{(r)} e^{-m_a z} \]

Integration twice with respect to \( z \) gives

\[ T(z) = -\frac{q_0^{(r)}}{m_a k} e^{-m_a z} + C_1 z + C_2 \]

Next we apply the boundary conditions that \( T(0) = T_0 \) and \( T(\delta) = T_\delta \) to get

\[ T(z) - T_0 = \frac{q_0^{(r)}}{m_a k} (1 - e^{-m_a z}) + \left[ (T_\delta - T_0) - \frac{q_0^{(r)}}{m_a k} (1 - e^{-m_a \delta}) \right] \frac{z}{\delta} \]

b. The conductive heat flux is given by Fourier's law:

\[ q_z = -k \frac{dT}{dz} = -q_0^{(r)} e^{-m_a \delta} + k \frac{T_\delta - T_0}{\delta} - \frac{q_0^{(r)}}{m_a \delta} (1 - e^{-m_a \delta}) \]
For very large values of $m_a$, the first and third terms become negligibly small, and we are left with

Very large $m_a$:

$$q_z = -k \frac{T_0 - T_\delta}{\delta}$$

For very small values of $m_a$ we can expand the first and third terms in a Taylor series and get:

Very small $m_a$:

$$q_z = -q_0^{(r)} (1 - m_a z + \ldots) - k \frac{T_0 - T_\delta}{\delta}$$

$$- \frac{q_0^{(r)}}{m_a \delta} (1 - 1 + m_a \delta + \ldots)$$

$$\approx -k \frac{T_0 - T_\delta}{\delta}$$

Thus in both limits, the conductive heat flux is virtually unaffected by the radiation.

At intermediate values of $m_a$, $q_z$ increases with increasing $m_a$. 