Thermal Conductivity of Solids, 9.5

- mechanisms of energy transfer in solids is complex
- no general predictive method is available.
- heat is conducted by molecular collisions and
  by free electrons in pure metals.

Pure Metals: heat conduction and electrical conduction
occur by free electrons,

\[ \frac{k}{k_e T} = L = \text{constant} \]

"Wiedemann-
Franz-Lorenz"

Eqn.

\[ L = \text{Lorenz} \; N_0 \Rightarrow 22 \Rightarrow 29 \times 10^{-9} \frac{\text{volt}^2}{\text{K}^2} \; @ \; 0^\circ C. \]

Effective Thermal Conductivity of Composite Solids, 9.6

Spheres:

\[ \frac{k_{\text{eff}}}{k_0} = 1 + \frac{3\phi}{\left( \frac{k_1 + 2k_0}{k_1 - k_0} \right) - \phi}, \quad \phi = \text{volume fraction solid in continuous phase}
\]

\( k_0 - \text{continuous media} \)

"by Maxwell"
Spheres: large $\phi$, up to $\pi/6$

\[
\frac{\text{K}_{\text{eff}}}{k_0} = 1 + \frac{3\phi}{(\frac{k_1+2k_0}{k_1-k_0})-\phi + 1.569(\frac{k_1-k_0}{3k_1-4k_0})\phi^{10/3} + \ldots}
\]

Cylinders / Non-spherical Inclusions.

\[
\frac{\text{K}_{\text{eff,zz}}}{k_0} = 1 + \left(\frac{k_1-k_0}{k_0}\right)\phi
\]

\[
\frac{\text{K}_{\text{eff,xx}}}{k_0} = 1 + \frac{2\phi}{(\frac{k_1+k_0}{k_1-k_0})-\phi + (\frac{k_1-k_0}{k_1+k_0})(0.306\phi^4 + 0.0134\phi^8 + \ldots)}
\]
Packed Beds of Porous Matl: (packed bed reactors).

Fluid flows through bed.

\[ K_{\text{eff,rr}} = \frac{1}{10} \rho \hat{C}_p \nu_0 D_p \]  \text{particle diameter}

\[ \text{fluid density} \quad \text{fluid heat capacity} \quad \text{"superficial" velocity} = \frac{\dot{V}}{A} \quad \text{volumetric flowrate} \]

\[ K_{\text{eff,zz}} = \frac{1}{2} \rho \hat{C}_p \nu_0 D_p \]

\[ K_{\text{eff,rr}} = \frac{1}{10} \rho \hat{C}_p \nu_0 D_p \]

For \( Re = D_p \nu_0 \rho / \mu > 200 \)
Combined Energy Flux Vector, \( \mathbf{e} \)

1. convective flux vector
2. work flux vector
3. molecular heat flux vector

   - one important property of a fluid is its energy content.
   - kinetic energy per unit volume:
     \[
     \frac{1}{2} \rho \mathbf{v}^2 = \frac{1}{2} \rho \left( u_x^2 + u_y^2 + u_z^2 \right)
     \]
   - internal energy per unit volume due to intra- and intermolecular potential energies:
     \[
     \rho \mathbf{\hat{U}}
     \]
   - \( \left( \frac{1}{2} \rho \mathbf{v}^2 + \rho \mathbf{\hat{U}} \right) \mathbf{v} \) → convective energy flux vector

\[
\left( \mathbf{n} \cdot \left( \frac{1}{2} \rho \mathbf{v}^2 + \rho \mathbf{\hat{U}} \right) \mathbf{v} \right) \rightarrow \text{convective energy flux through a surface normal to} \ \mathbf{n}
\]

"a rate of flow of energy in a direction normal to} \ A_n."
\[ x - \text{component}, \]
\[ \delta_x \cdot \left( \frac{1}{2} \rho u^2 + \rho \hat{u} \right) v = \left( \frac{1}{2} \rho u^2 + \rho \hat{u} \right) v_x \]

similarly for \( y \)- and \( z \)-components

Molecular Work Flux Vector

- rate of work \( \left( \frac{dW}{dt} \right) \) by a force \( (F) \), \( \rightarrow \) \( F \cdot v \)

- in a fluid, the force per unit area that a fluid exerts on surrounding fluid is,
  \[ \pi_x = \rho \delta_x + \tau_x \quad - \text{vector force acting on} \ A_x \]
  \[ \pi_y = \rho \delta_y + \tau_y \quad - \quad A_y \]
  \[ \pi_z = \rho \delta_z + \tau_z \quad - \quad A_z \]

- rate of work done on \( x, y, \) and \( z \) planes.
  \[ (\pi_x \cdot v) = \pi_{xx} v_x + \pi_{xy} v_y + \pi_{xz} v_z \quad \rightarrow \quad A_x \]
  \[ (\pi_y \cdot v) = \pi_{yx} v_x + \pi_{yy} v_y + \pi_{yz} v_z \quad \rightarrow \quad A_y \]
  \[ (\pi_z \cdot v) = \pi_{zx} v_x + \pi_{zy} v_y + \pi_{zz} v_z \quad \rightarrow \quad A_z \]

or combining,

\[ [\pi \cdot v] = \delta_x (\pi_x \cdot v) + \delta_y (\pi_y \cdot v) + \delta_z (\pi_z \cdot v) \]
Combined Energy Flux Vector, $e$

$$e = \left( \frac{1}{2} \rho v^2 + \rho \dot{u} \right) v + [\tau \cdot v] + g$$

but $[\tau \cdot v] = pv + [\tau \cdot v]$}

$$e = \left( \frac{1}{2} \rho v^2 + \rho \dot{\rho} \right) v + [\tau \cdot v] + g$$

Note: $\rho \dot{u} v + pv \Rightarrow \rho (\dot{u} + \frac{p}{\rho}) v \Rightarrow \\
\rho (\dot{u} + \rho \dot{v}) v \Rightarrow \rho \dot{\rho} v$

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