PART I and
PART II

CALCULATING
FORCE ON A SURFACE
IN FLOW
FROM VELOCITY FIELD

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\[ \text{Force} = A \cdot \text{Stress} = \left( \frac{\text{force}}{\text{area}} \right) (\text{Area}) \]

\[ \text{Stress is uniform} \]

\[ \text{total force} \]

\[ F = \text{stress} \cdot A_1 + \text{stress} \cdot A_2 \ldots \]

\[ F = \sum_{i=1}^{n} (\text{stress}) \cdot A_i \]
Total force \( F \) is given by:

\[
F = \int_A \text{stress} \, dA
\]

at the surface as a function of position.
STEADY FLOW DOWN INCLINE

\[ u_z(x) = \frac{pg \cos \beta}{2\mu} (H^2 - x^2) \]
\[ F = \int \int \int \text{stress} \, \text{plat surface} \, dy \, dz \]

Newton's Law

\[ T_{xz} = -\nu \frac{dV_{x}}{dx} \]

flux of \( z \)-momentum in the \( x \)-dir

stress on \( x \)-surface in \( z \)-direction

\( x \)-surface = surface whose unit normal is \( \hat{e}_x \)

\[ F = \int \int_0^L \int_0^W -T_{xz} \, dy \, dz \text{ surface} \]
\[ F = \int_0^2 \int_0^x 2 - x \ dx \ d\gamma \text{ surface} \]

\[ = \int_0^2 \int_0^x \left. 2 - x \right|_{x=H} \ dx \ d\gamma \]

\[ T_{x^2} = -\gamma \frac{dV_z}{dx} \]

\[ V_z = \frac{pg \cos \beta}{2\mu} (H^2 - x^2) \]

\[ \frac{dV_z}{dx} = \frac{pg \cos \beta}{\mu} (-2x) \]

\[ T_{x^2} = -\gamma \left( \frac{pg \cos \beta}{\mu} x \right) \]

\[ T_{x^2} = pg \cos \beta x \]

\[ \text{Newton's Law of Viscosity} \]
\[ F = \iiint_{D} \sigma_{zz} \, dy \, dz \quad \text{where} \quad \sigma_{zz} = \rho g \cos \beta x \]

\[ F = \int_{0}^{L} \int_{0}^{w} \rho g \cos \beta y \, dy \, dz \]

\[ F = -\rho g \cos \beta H W L \]

Compressive force \( F = \text{tensile force} \)