FORCE ON THE WALLS OF A TUBE — PRESSURE-DRIVEN TURBULENT FLOW

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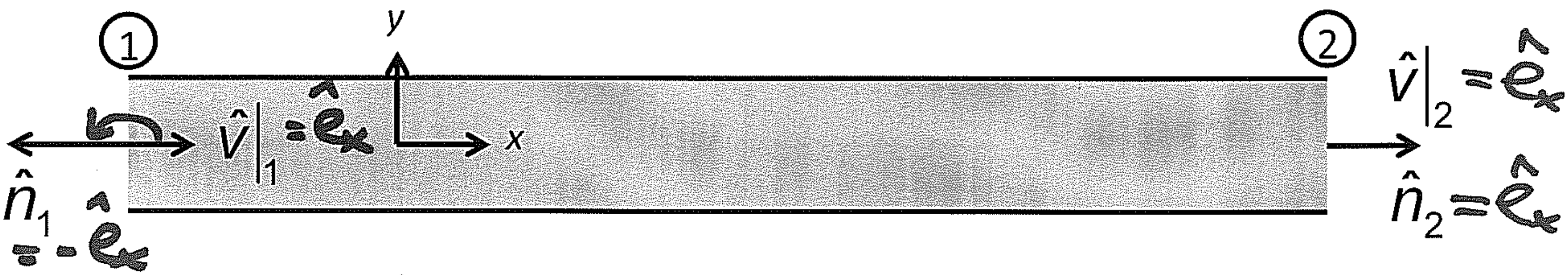
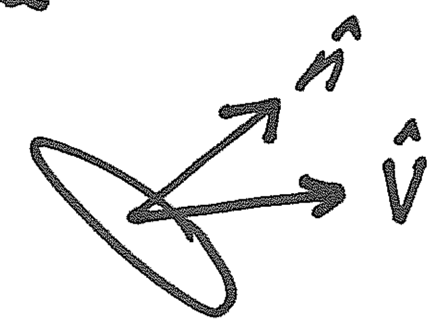
Example: Use a macroscopic momentum balance to relate pressure drop to vector fluid force on the walls in a horizontal straight pipe.





$$\underline{g} = \begin{pmatrix} 0 \\ -mg \\ 0 \end{pmatrix}_{xyz}$$

$\theta_2 = 0$

$\cos\theta_2 = 1$

$\theta_1 = 180°$

$\cos\theta_1 = -1$

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Steady-State Macroscopic Momentum Balance

pressure wall force gravity

$$0 = \sum_{i=1}^{N} \left[\frac{-\rho A \langle v \rangle^2 \cos\theta}{\beta} \hat{v} \right]_i + F_p + F_v + F_g$$

$$0 = \sum_{i=1}^{N} \left[\frac{-\dot{m} \langle v \rangle \cos\theta}{\beta} \hat{v} \right]_i + F_p + F_v + F_g$$

$$\dot{m} = \rho A \langle v \rangle$$

= 1 turbulent

= 3/4 laminar

$$0 = \cancel{-\frac{\rho A \langle v \rangle^2 (-1)}{1} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_{x=2}} + \cancel{\frac{-\rho A \langle v \rangle^2 (1)}{1} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}} + \underline{F}_p + \underline{F}_v + \underline{F}_g$$

$$\boxed{0 = \underline{F}_p + \underline{F}_v + \underline{F}_g}$$

$$\underline{F}_{p_{inlet}} = -P \hat{n} A = -P_{in} A \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$$

$$\underline{F}_{p\,exit} = -P_{out} A \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

⑤

$$0 = \underline{F}_p + \underline{F}_v + \underline{F}_g$$

$$0 = -P_{in}A\begin{pmatrix}-1\\0\\0\end{pmatrix}_{xyz} + -P_{out}A\begin{pmatrix}1\\0\\0\end{pmatrix}_{xyz} + \begin{pmatrix}0\\-mg\\0\end{pmatrix}_{xyz} + \begin{pmatrix}R_x\\R_y\\R_z\end{pmatrix}_{xyz}$$

z-component: $\boxed{R_z = 0}$

y-component: $mg + R_y = 0$

$$\boxed{R_y = +mg}$$

x-component: $P_{in}A - P_{out}A + R_x = 0$

$$\boxed{R_x = (P_{out} - P_{in})A}$$

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corrected 25 Oct 2011
FAM