CM3110
Transport I
Part II: Heat Transfer

Complex Heat Transfer – Dimensional Analysis

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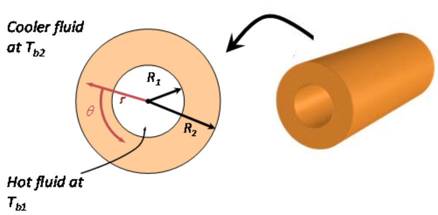
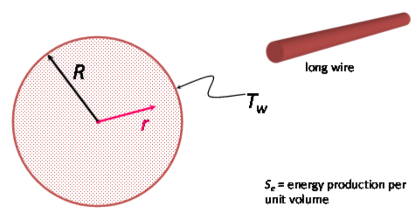
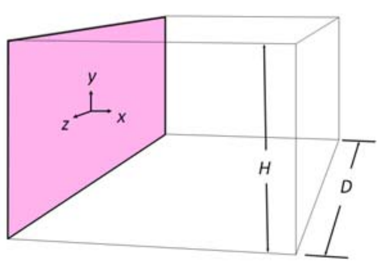
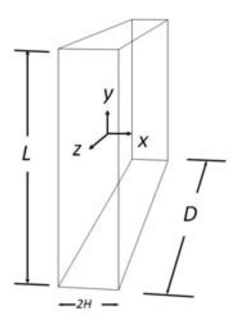
(what have we been up to?)

Examples of (simple) Heat Conduction









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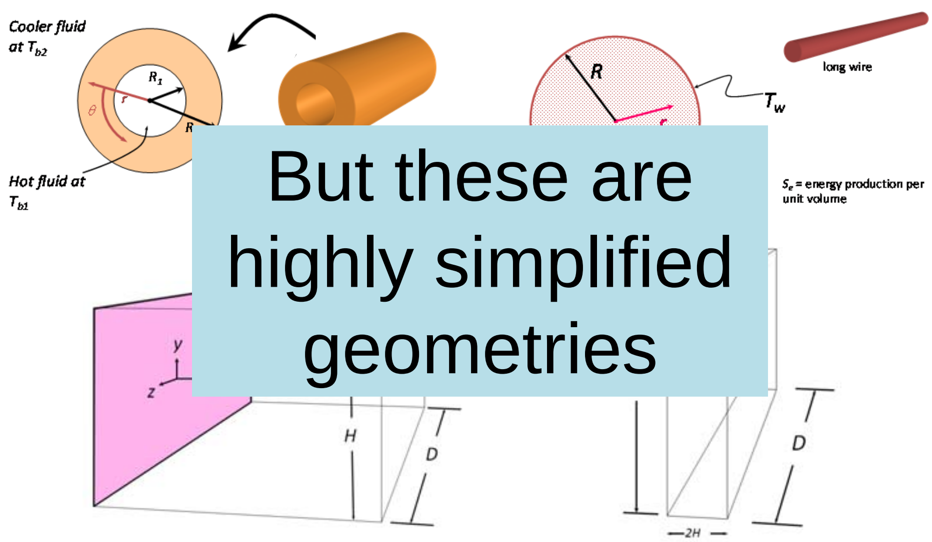
Examples of (simple) Heat Conduction



But these are highly simplified geometries

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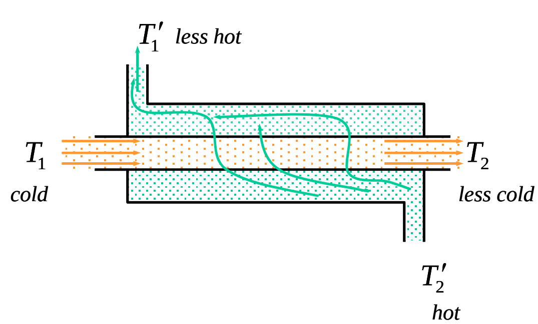
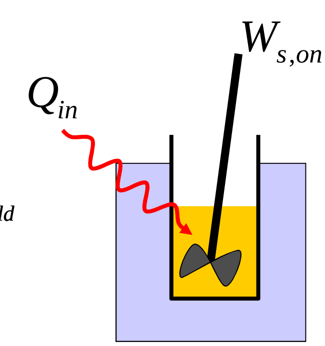
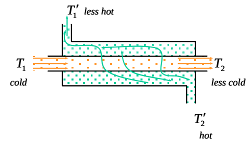
How do we handle complex geometries, complex flows, complex machinery?

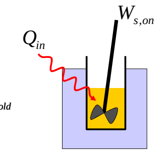




Process scale

(***Answer***: *Use the same techniques we have been using in fluid mechanics)*





Engineering Modeling

- •Choose an idealized problem and solve it
- •From insight obtained from **ideal** problem, identify governing equations of **real** problem
- •Nondimensionalize the governing equations; deduce dimensionless scale factors (e.g. Re, Fr for fluids)
- •Design experiments to test modeling thus far
- •Revise modeling (structure of dimensional analysis, identity of scale factors, e.g. add roughness lengthscale)
- •Design additional experiments
- •Iterate until useful correlations result

Experience with Dimensional Analysis thus far:

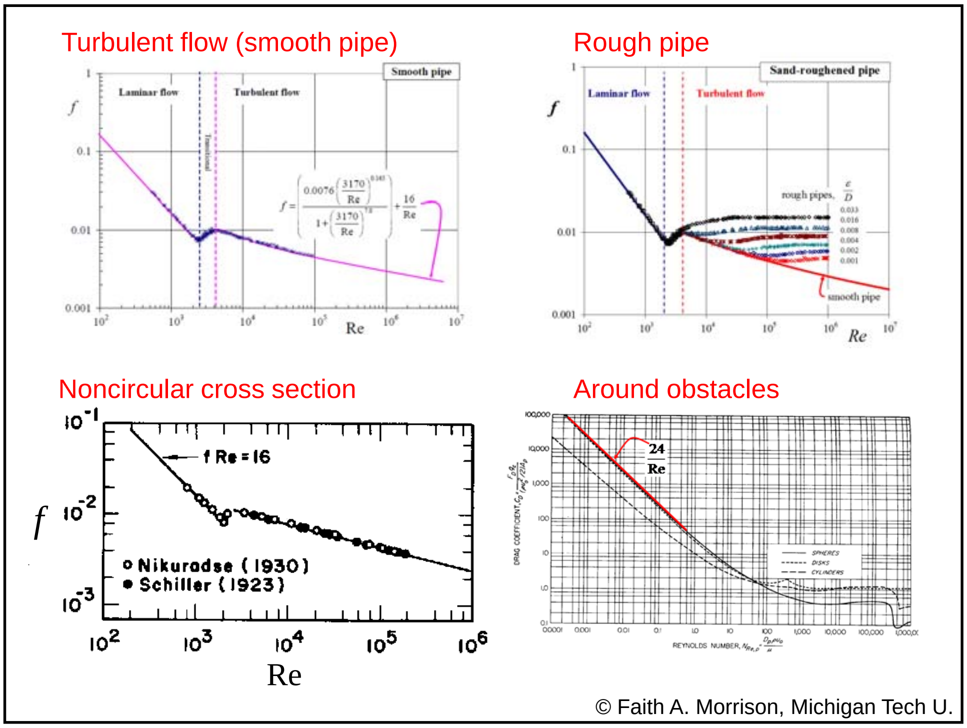
•Flow in pipes at all flow rates (laminar and turbulent)
Solution: Navier-Stokes, Re, Fr, L/D, dimensionless wall force = f; $f = f(\mathrm{Re}, L/D)$

•Rough pipes
Solution: add additional length scale; then nondimensionalize

•Non-circular conduits
Solution: Use hydraulic diameter as the length scale of the flow to nondimensionalize

•Flow around obstacles (spheres, other complex shapes
Solution: Navier-Stokes, Re, dimensionless drag $= CD$; $CD = CD(\mathrm{Re})$

•Boundary layers
Solution: Two components of velocity need independent lengthscales


Turbulent flow (smooth pipe)
Laminar flow
Turbulent flow
Smooth pipe
Re
Rough pipe
Laminar flow
Turbulent flow
Sand-roughened pipe
rough pipes
smooth pipe
Re
Noncircular cross section
f Re = 16
Nikuradse (1930)
Schiller (1923)
Re
Around obstacles
24
Re
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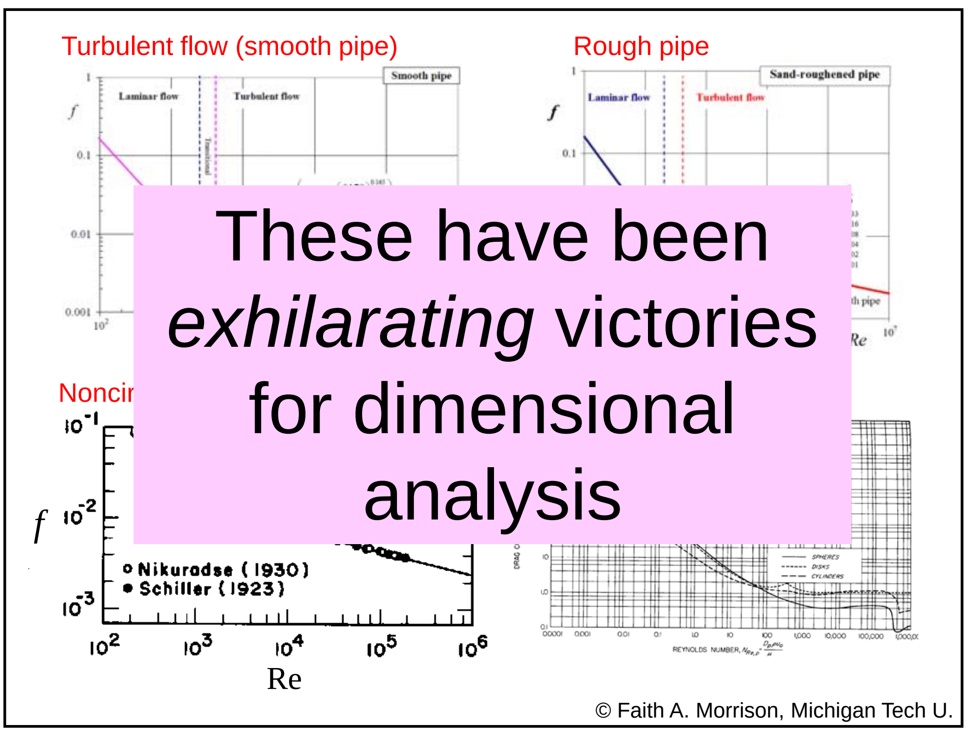
Turbulent flow (smooth pipe)
Laminar flow
Turbulent flow
Smooth pipe
Rough pipe
Laminar flow
Turbulent flow
Sand-roughened pipe
These have been *exhilarating* victories for dimensional analysis
Nikuradse (1930)
Schiller (1923)
Re
© Faith A. Morrison, Michigan Tech U.

Now, move to heat transfer:

- Forced convection heat transfer from fluid to wall
  Solution: ?
- Natural convection heat transfer from fluid to wall
  Solution: ?
- Radiation heat transfer from solid to fluid
  Solution: ?

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  Solution: ?
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  Solut
- Radiation heat transfer
  Solut

We have already started using the results/techniques of dimensional analysis through defining the heat transfer coefficient, h

Now, move to heat transfer:

•Forced convection heat transfer from fluid to wall

Solution: ?

•Natural convection hea

Solut

•Radiation heat transfer

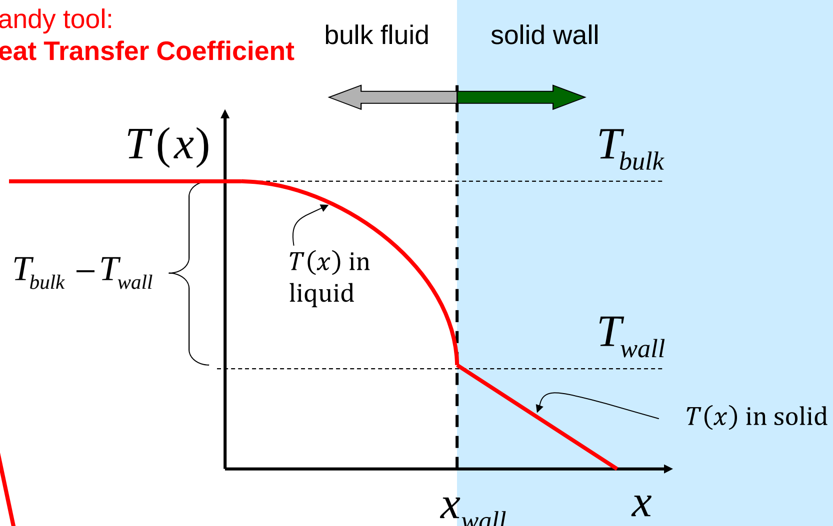
Solut

We have already started using the results/techniques of dimensional analysis through defining the heat transfer coefficient, h

(recall that we did this in fluids too: we used $f(\mathrm{Re})$ long before we knew where that all came from)

Handy tool:

Heat Transfer Coefficient



The temperature difference at the fluid-wall interface is caused by complex phenomena that are lumped together into the heat transfer coefficient, *h*

The flux at the wall is given by the empirical expression known as
Newton's Law of Cooling

This expression serves as the definition of the heat transfer coefficient.

$$\left|\frac{q_x}{A}\right| \equiv h \left|T_{bulk} - T_{wall}\right|$$

h depends on:

- geometry
- fluid velocity
- fluid properties
- temperature difference

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The flux at the wall is given by the empirical expression known as
Newton's Law of Cooling

This expression serves as the definition of the heat transfer coefficient.

$$\left|\frac{q_x}{A}\right| \equiv h \left|T_{bulk} - T_{wall}\right|$$

To get values of h for various situations, we need to measure data and create data correlations (**dimensional analysis**)

h depends on:

- geometry
- fluid velocity
- fluid properties
- temperature difference

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Complex Heat transfer Problems to Solve:

•Forced convection heat transfer from fluid to wall

Solution: ?

•Natural convection heat transfer from fluid to wall

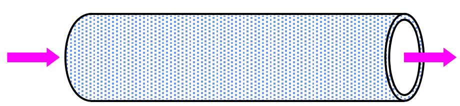
Solution: ?

•Radiation heat transfer from solid to fluid

Solution: ?

- The values of h will be different for these three situations (different physics)
- Investigate simple problems in each category, model them, take data, correlate

Chosen problem: Forced Convection Heat Transfer



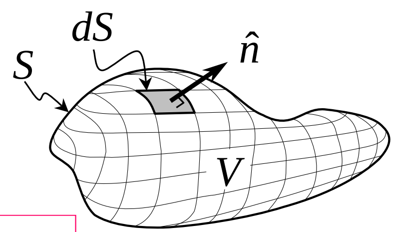
Following procedure familiar from pipe flow,

- **What are governing equations?**
- **Scale factors (dimensionless numbers)?**
- **Quantity of interest?**

Heat flux at the wall

General Energy Transport Equation
(microscopic energy balance)

As for the derivation of the microscopic momentum balance, the microscopic energy balance is derived on an arbitrary volume, *V*, enclosed by a surface, *S*.



Gibbs notation:

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S$$

see handout for component notation

General Energy Transport Equation
(microscopic energy balance)

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S$$

rate of change: $\frac{\partial T}{\partial t}$

convection: $\underline{v} \cdot \nabla T$

conduction (all directions): $k \nabla^2 T$

source (energy generated per unit volume per time): S

velocity must satisfy equation of motion, equation of continuity

see handout for component notation

Equation of energy for Newtonian fluids of constant density, ρ, and thermal conductivity, k, with source term (source could be viscous dissipation, electrical energy, chemical energy, etc., with units of energy/(volume time)).

CM310 Fall 1999 Faith Morrison

Source: R. B. Bird, W. E. Stewart, and E. N. Lightfoot, *Transport Processes*, Wiley, NY, 1960, page 319.

Note: this handout is on the web:
www.chem.mtu.edu/~fmorriso/cm310/energy2013.pdf

Gibbs notation (vector notation)

$$\left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T\right) = \frac{k}{\rho \hat{C}_p} \nabla^2 T + \frac{S}{\rho \hat{C}_p}$$

Cartesian (xyz) coordinates:

$$\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} = \frac{k}{\rho \hat{C}_p}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + \frac{S}{\rho \hat{C}_p}$$

Cylindrical (rθz) coordinates:

$$\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r}\frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} = \frac{k}{\rho \hat{C}_p}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2}\right) + \frac{S}{\rho \hat{C}_p}$$

Spherical (rθϕ) coordinates:

$$\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r}\frac{\partial T}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial T}{\partial \phi} = \frac{k}{\rho \hat{C}_p}\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right) + \frac{1}{r^2 \sin^2}\right.$$

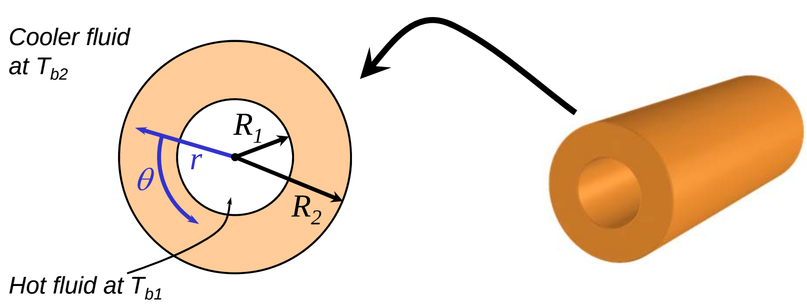
** REVIEW ** REVIEW **

Forced-convection heat transfer

Example: Heat flux in a cylindrical shell

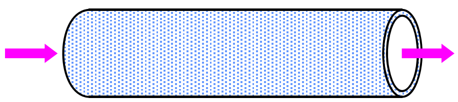
Assumptions:
- long pipe
- steady state
- k = thermal conductivity of wall
- h_1, h_2 = heat transfer coefficients

What is the steady state temperature profile in a cylindrical shell (pipe) if the ***fluid on the inside*** *is at T_{b1} and the* ***fluid on the outside*** *is at T_{b2}? (T_{b1}>T_{b2})*



Now: How do develop correlations for *h*?

Consider: Heat-transfer to from flowing fluid inside of a tube – forced-convection heat transfer



T_1= core bulk temperature
T_o= wall temperature
$T(r,\theta,z)$ = temp distribution in the fluid

In principle, with the right math/computer tools, we could calculate the complete temperature and velocity profiles in the moving fluid.

What are governing equations?

Microscopic energy balance plus Navier-Stokes, continuity

Scale factors?

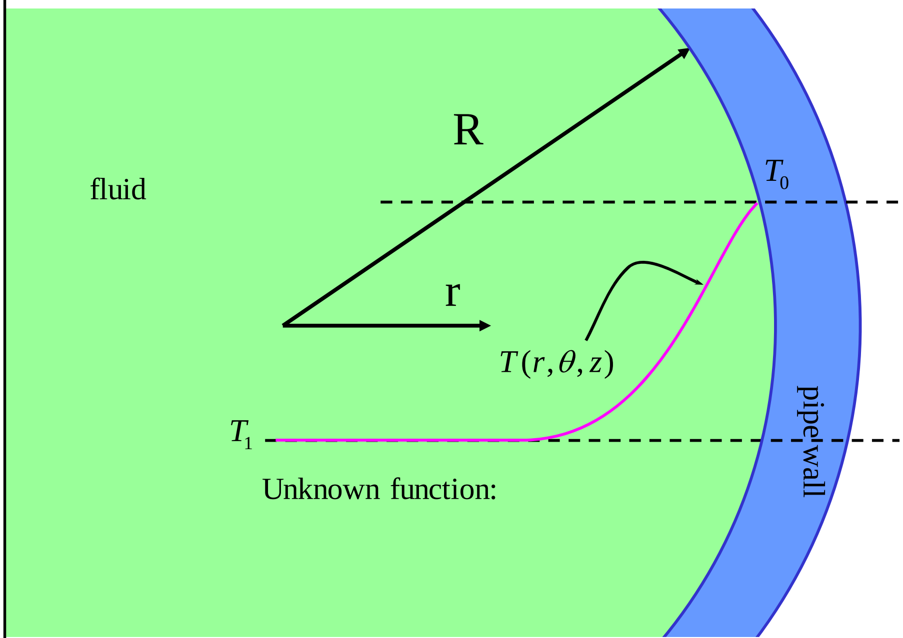
Re, Fr, L/D plus whatever comes from the rest of the analysis

Quantity of interest (like wall force, drag)?

Heat transfer coefficient

The quantity of interest in forced-convection heat transfer is *h*

How is the heat transfer coefficient related to the full solution for $T(r,\theta,z)$?



At the wall (r = R), we can relate *T* profile to *h* through the total heat flow through the wall, Q:

$$Q = \iint_S \left[\hat{n} \cdot \underset{\sim}{q}\right]_{surface} dS$$

may vary with θ, z

$$\int_0^{2\pi}\int_0^{L} -k \left.\frac{\partial T}{\partial r}\right|_{r=R} R\,dz\,d\theta = Q = h(2\pi RL)(T_1 - T_o)$$

Total heat conducted to the wall from the fluid

Total heat flow through the wall in terms of *h*

Now, non-dimensionalize this expression

Non-dimensionalize

non-dimensional variables:

position:

$$r^* \equiv \frac{r}{D}$$

$$z^* \equiv \frac{z}{D}$$

temperature:

$$T^* \equiv \frac{T - T_o}{(T_1 - T_o)}$$

$$h(\pi D L)(T_1 - T_o) = \int_0^{2\pi}\int_0^{L/D} -k \left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1/2} \frac{(T_1 - T_o)}{D}\frac{D^2}{2} dz^* d\theta$$

$$2\pi\left(\frac{hD}{k}\right)\left(\frac{L}{D}\right) = \int_0^{2\pi}\int_0^{L/D} -\left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1/2} dz^* d\theta$$

Nusselt number, Nu
(dimensionless heat-transfer coefficient)

$$Nu = Nu\left(T^*, \frac{L}{D}\right)$$

one additional dimensionless group

$$h(\pi DL)(T_1 - T_o) = \int_0^{2\pi}\int_0^{L/D} -k \left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1/2} \frac{(T_1 - T_o)}{D}\frac{D^2}{2} dz^* d\theta$$

This is a function of Re through $\underline{v}$

$$2\pi\left(\frac{hD}{k}\right)\left(\frac{L}{D}\right) = \int_0^{2\pi}\int_0^{L/D} -\left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1/2} dz^* d\theta$$

Nusselt number, Nu
(dimensionless heat-transfer coefficient)

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Non-dimensional Energy Equation

$$\frac{\partial T^*}{\partial t^*} + v_x^* \frac{\partial T^*}{\partial x^*} + v_y^* \frac{\partial T^*}{\partial y^*} + v_z^* \frac{\partial T^*}{\partial z^*} = \frac{1}{\mathrm{Pe}}\left(\frac{\partial^2 T^*}{\partial x^{*2}} + \frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\partial^2 T^*}{\partial z^{*2}}\right) + S^*$$

$$\mathrm{Pe} \equiv \mathrm{Pr}\,\mathrm{Re} = \frac{\hat{C}_p \mu}{k}\frac{\rho V D}{\mu}$$

Non-dimensional Navier-Stokes Equation

$$\frac{Dv_z^*}{Dt} = -\frac{\partial P^*}{\partial z^*} + \frac{1}{\mathrm{Re}}\left(\nabla^2 v_z\right)^* + \frac{1}{Fr} g^*$$

$$\mathrm{Pr} \equiv \frac{\hat{C}_p \mu}{k}$$

Non-dimensional Continuity Equation

$$\frac{\partial v_x^*}{\partial x^*} + \frac{\partial v_y^*}{\partial y^*} + \frac{\partial v_z^*}{\partial z^*} + = 0$$

Quantity of interest

$$Nu = \frac{1}{2\pi L/D}\int_0^{2\pi}\int_0^{L/D} -\left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1/2} dz^* d\theta$$

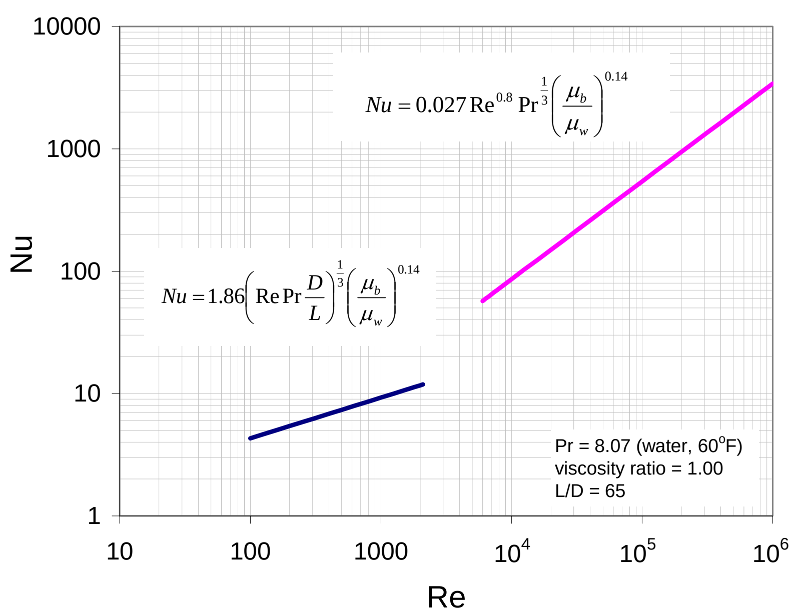
According to our dimensional analysis calculations, the dimensionless heat transfer coefficient should be found to be a function of four dimensionless groups:

no free surfaces

$$Nu = Nu\left(\mathrm{Re}, \mathrm{Pr}, \cancel{\mathrm{Fr}}, \frac{L}{D}\right)$$

Now, do the experiments.

Correlations for Forced Convection Heat Transfer Coefficients



Heat Transfer in Laminar flow in pipes: data correlation for *h*

$$Nu_a = \frac{h_a D}{k} = 1.86\left(\mathrm{Re}\,\mathrm{Pr}\,\frac{D}{L}\right)^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

the subscript "*a*" refers to the type of average temperature used in reporting the correlation

$$q = h_a A \Delta T_a$$

$$\Delta T_a = \frac{(T_w - T_{bi}) + (T_w - T_{bo})}{2}$$

Geankoplis, 4th ed. eqn 4.5-4, page 260

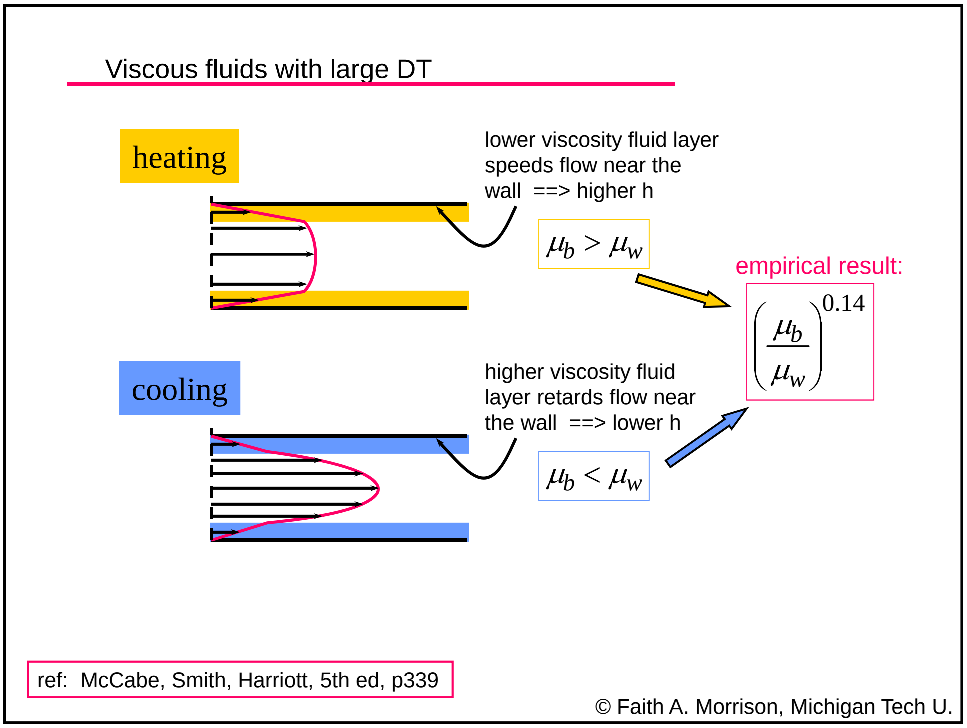
Re<2100, (RePrD/L)>100, horizontal pipes; all properties evaluated at the temperature of the bulk fluid except μ_w which is evaluated at the wall temperature.

?

We assumed constant ρ, k, m, etc. Therefore we did not predict a viscosity-temperature dependence. If viscosity is not assumed constant, the dimensionless group shown below is predicted to appear in correlations.

$$Nu_a = \frac{h_a D}{k} = 1.86\left(\mathrm{Re}\,\mathrm{Pr}\,\frac{D}{L}\right)^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

(reminiscent of pipe wall roughness; needed to modify dimensional analysis to correlate on roughness)


Viscous fluids with large DT
heating
lower viscosity fluid layer speeds flow near the wall ==> higher h
$\mu_b > \mu_w$
empirical result:
$\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$
cooling
higher viscosity fluid layer retards flow near the wall ==> lower h
$\mu_b < \mu_w$
ref: McCabe, Smith, Harriott, 5th ed, p339
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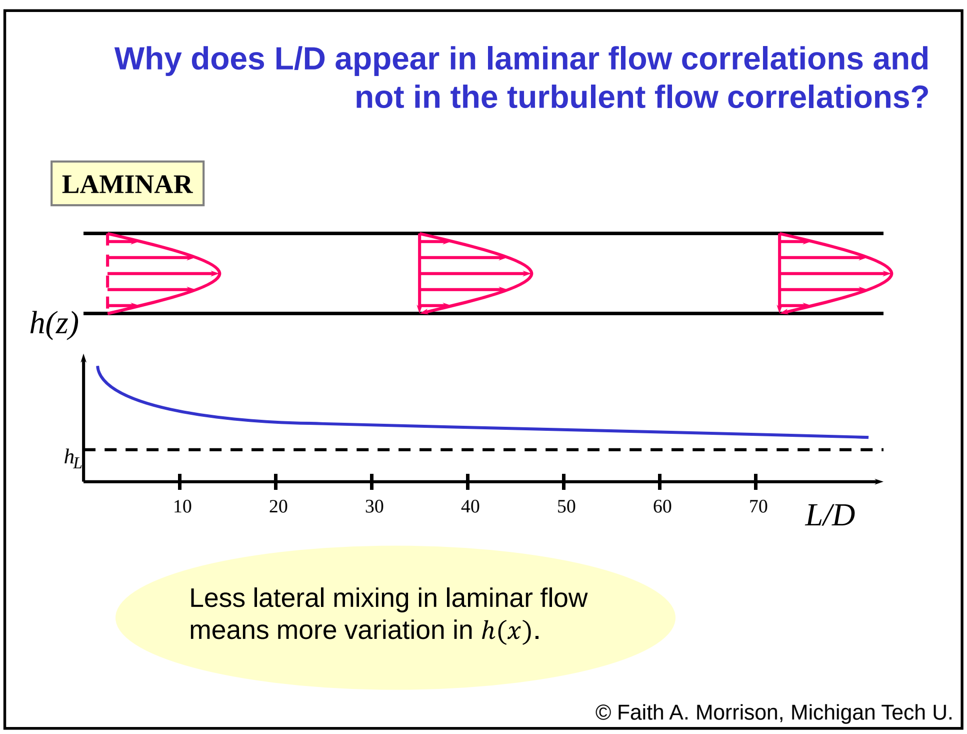
Why does L/D appear in laminar flow correlations and not in the turbulent flow correlations?
LAMINAR
$h(z)$
h_L
10
20
30
40
50
60
70
L/D
Less lateral mixing in laminar flow means more variation in $h(x)$.
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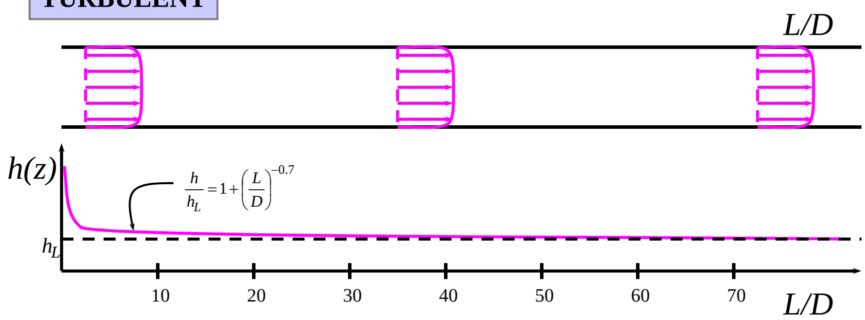
TURBULENT



In turbulent flow, good lateral mixing reduces the variation in h along the pipe length.

Example of *partial* solution to Homework

laminar flow in pipes	$Nu_a = \frac{h_a D}{k} = 1.86\left(\mathrm{Re}\,\mathrm{Pr}\frac{D}{L}\right)^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$	Re<2100, (RePrD/L)>100, horizontal pipes, eqn 4.5-4, page 238; all properties evaluated at the temperature of the bulk fluid except μ_w which is evaluated at the wall temperature.
turbulent flow in smooth tubes	$Nu_{lm} = \frac{h_{lm} D}{k} = 0.027\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$	Re>6000, 0.7 <Pr <16,000, L/D>60 , eqn 4.5-8, page 239; all properties evaluated at the mean temperature of the bulk fluid except μ_w which is evaluated at the wall temperature. The mean is the average of the inlet and outlet bulk temperatures; not valid for liquid metals.
air at 1atm in turbulent flow in pipes	$h_{lm} = \frac{3.52V(m/s)^{0.8}}{D(m)^{0.2}}$ $h_{lm} = \frac{0.5V(ft/s)^{0.8}}{D(ft)^{0.2}}$	equation 4.5-9, page 239
water in turbulent flow in pipes	$h_{lm} = 1429\left(1 + 0.0146T(^{\circ}C)\right)\frac{V(m/s)^{0.8}}{D(m)^{0.2}}$ $h_{lm} = 150\left(1 + 0.011T(^{\circ}F)\right)\frac{V(ft/s)^{0.8}}{D(ft)^{0.2}}$	4 < T(°C)<105, equation 4.5-10, page 239

Complex Heat transfer Problems to Solve:

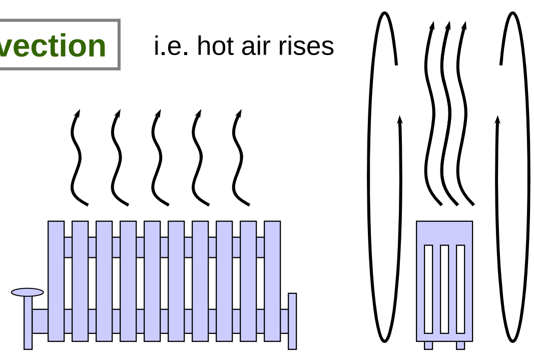
✓ •Forced convection heat transfer from fluid to wall
Solution: ?

➡ •Natural convection heat transfer from fluid to wall
Solution: ?

•Radiation heat transfer from solid to fluid
Solution: ?

Free Convection

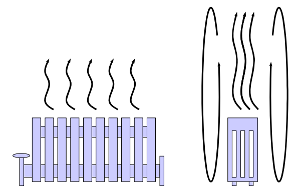
i.e. hot air rises



•heat moves from hot surface to cold air (fluid) by radiation and conduction
•increase in fluid temperature decreases fluid density
•recirculation flow begins
•recirculation adds to the heat-transfer from conduction and radiation

$\Rightarrow$ coupled heat and momentum transport

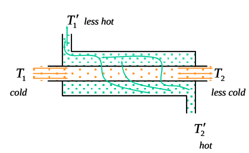
Free Convection   i.e. hot air rises

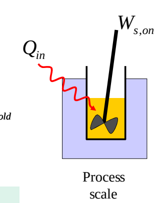


How can we solve **real** problems involving free (natural) convection?

We'll try this: Let's review how we approached solving real problems in *earlier* cases, i.e. in fluid mechanics, forced convection.

Engineering Modeling





- Choose an idealized problem and solve it
- From insight obtained from **ideal** problem, identify governing equations of **real** problem
- Nondimensionalize the governing equations; deduce dimensionless scale factors (e.g. Re, Fr for fluids)
- Design experiments to test modeling thus far
- Revise modeling (structure of dimensional analysis, identity of scale factors, e.g. add roughness lengthscale)
- Design additional experiments
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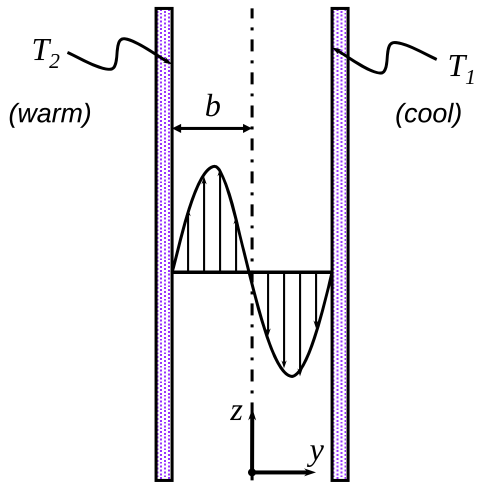
Example: Free convection between long parallel plates *or* heat transfer through double-pane glass windows



$$T_2 > T_1$$

assumptions:
long, wide slit
steady state
no source terms
viscosity constant
density varies with T

Calculate: T, $\underline{v}$ profiles

In our analyses of momentum transport so far, we have assumed constant density

$\Longrightarrow$ use Navier-Stokes equation

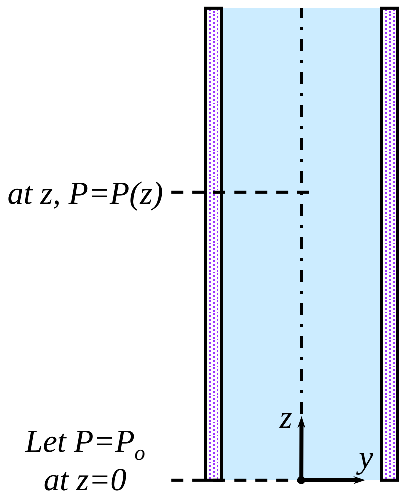
Actually, we can use the Navier-Stokes equation for any problem for which the following equation holds:

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$

Is Pressure a function of z?

YES, there should be hydrostatic pressure $(\rho g h)$

"Pressure at the bottom of a column of fluid = pressure at top $+\rho g h$."



average density

$$p_0 = p(z) + \bar{\rho} g z$$
$$p(z) = p_0 - \bar{\rho} g z$$

$$\Rightarrow \frac{dP}{dz} = -\bar{\rho} g$$

(look up the physics in the literature)

To account for the temperature variation of ρ:

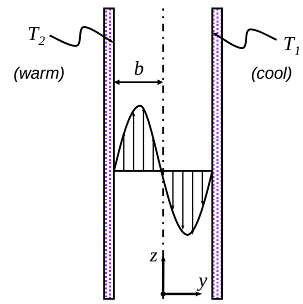
$$\rho = \bar{\rho} - \bar{\rho}\bar{\beta}\left(T - \bar{T}\right)$$

$\bar{\rho} =$ mean density

$\bar{\beta} =$ volumetric coefficient of expansion at $\bar{T}$

$$\bar{T} = \frac{T_1 + T_2}{2}$$

Example : Natural convection between vertical plates



$T_2 > T_1$

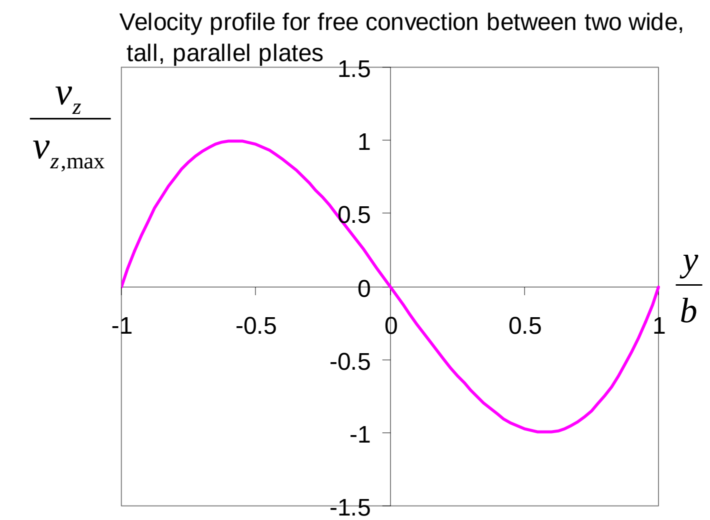
You try.

45

Final Result: (free convection between two slabs)

$$v_z(y) = \frac{\bar{\rho}\bar{\beta}g\left(T_2 - T_1\right)b^2}{12\mu}\left[\left(\frac{y}{b}\right)^3 - \left(\frac{y}{b}\right)\right]$$

(see next slide for plot)



Nondimensionalize the governing equations; deduce dimensionless scale factors

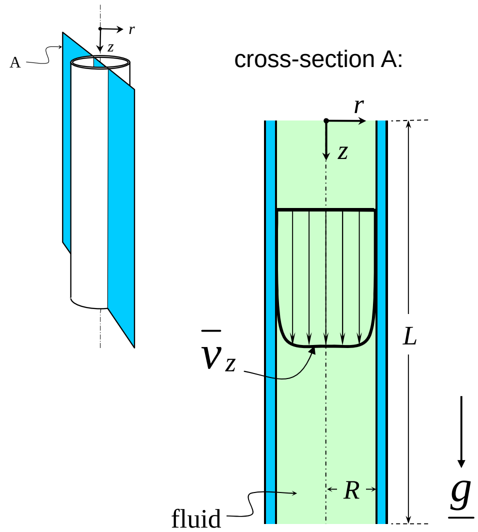
To nondimensionalized the Navier-Stokes for free convection problems, we follow the simple problem we just completed *(ρ=ρ(T),* v_{av}*=0)*.

$$\rho\left(\frac{\partial \underline{v}}{\partial t}+\underline{v}\cdot\nabla\underline{v}\right)=-\nabla P+\mu\nabla^2\underline{v}+\rho\underline{g}$$

? ? ?

Following the previous example, how do we handle the various densities?

How did we nondimensionalized the Navier-Stokes before?



FORCED CONVECTION

EXAMPLE I: Pressure-driven flow of a Newtonian fluid in a tube:

- •steady state
- •well developed
- •long tube

There was an average velocity used as the characteristic velocity

FORCED CONVECTION ***FORCED CONVECTION*** ***FORCED CONVECTION***

z-component of the Navier-Stokes Equation:

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)$$
$$=-\frac{\partial P}{\partial z}+\mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right)+\rho g_z$$

Choose:

D = characteristic length
V = characteristic velocity
D/V = characteristic time
ρV^2 = characteristic pressure

This velocity is an imposed (forced) average velocity

FORCED CONVECTION **FORCED CONVECTION** **FORCED CONVECTION**

non-dimensional variables:

time:

$$t^* \equiv \frac{tV}{D}$$

position:

$$r^* \equiv \frac{r}{D}$$

$$z^* \equiv \frac{z}{D}$$

velocity:

$$v_z^* \equiv \frac{v_z}{V}$$

$$v_r^* \equiv \frac{v_r}{V}$$

$$v_\theta^* \equiv \frac{v_\theta}{V}$$

driving force:

$$P^* \equiv \frac{P}{\rho V^2}$$

$$g_z^* \equiv \frac{g_z}{g}$$

FORCED CONVECTION **FORCED CONVECTION** **FORCED CONVECTION**

z-component of the nondimensional Navier-Stokes Equation:

$$\frac{1}{\mathrm{Re}} \qquad \frac{1}{Fr}$$

$$\frac{Dv_z^*}{Dt} = -\frac{\partial P^*}{\partial z^*} + \frac{\mu}{\rho V D}\left(\nabla^2 v_z\right)^* + \frac{gD}{V^2} g^*$$

$$\left(\nabla^2 v_z\right)^* \equiv \frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^* \frac{\partial v_z^*}{\partial r^*}\right) + \frac{1}{r^{*2}}\frac{\partial^2 v_z^*}{\partial \theta^2} + \frac{\partial^2 v_z^*}{\partial z^{*2}}$$

$$\frac{Dv_z^*}{Dt} \equiv \left(\frac{\partial v_z^*}{\partial t^*} + v_r^* \frac{\partial v_z^*}{\partial r^*} + \frac{v_\theta^*}{r^*}\frac{\partial v_z^*}{\partial \theta} + v_z \frac{\partial v_z^*}{\partial z^*}\right)$$

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FREE CONVECTION **FREE CONVECTION** **FREE CONVECTION**

For free convection, what is the average velocity?
Answer: zero!

for forced convection we used: $v_z^* = \frac{v_z}{V}$ $\quad V \equiv \langle v \rangle$

for free convection $\langle v \rangle = 0$; what V should we use for free convection?

Solution: use a Reynolds-number type expression so that no characteristic velocity imposes itself (as we'll see now how that works):

$$v_z^* = \frac{\bar{\rho} v_z D}{\mu} \quad \Rightarrow V = \frac{\mu}{D\bar{\rho}}$$

FREE CONVECTION **FREE CONVECTION** **FREE CONVECTION**

When non-dimensionalizing the Navier-Stokes, what do I use for ρ? (answer from idealized problem)

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z}\right) = -\frac{\partial P}{\partial z} + \mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right) + \rho g_z$$

here we use $\bar{\rho}$ because the issue is volumetric flow rate

as before, for pressure gradient we use $-\bar{\rho} g$

here we use $\rho(T)$ because the issue is driving the flow by density differences affected by gravity

FREE CONVECTION ***FREE CONVECTION*** ***FREE CONVECTION***

non-dimensional variables:

time:

$$t^* \equiv \frac{t\mu}{D^2\overline{\rho}}$$

position:

$$r^* \equiv \frac{r}{D}$$

$$z^* \equiv \frac{z}{D}$$

velocity:

$$v_z^* \equiv \frac{v_z D\overline{\rho}}{\mu}$$

$$v_r^* \equiv \frac{\overline{\rho} v_r D}{\mu}$$

$$v_\theta^* \equiv \frac{\overline{\rho} v_\theta D}{\mu}$$

driving force:

$$T^* = \frac{T - \overline{T}}{T_2 - \overline{T}}$$

FREE CONVECTION ***FREE CONVECTION*** ***FREE CONVECTION***

SOLUTION: *z*-component of the **nondimensional** Navier-Stokes Equation (free convection):

$$\frac{Dv_z^*}{Dt} = \left(\nabla^2 v_z\right)^* + \left[\frac{gD^3\overline{\rho}^2\overline{\beta}\left(T_2 - \overline{T}\right)}{\mu^2}\right]T^*$$

Or any appropriate characteristic ΔT

Grashof number

$$\left(\nabla^2 v_z\right)^* \equiv \frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial v_z^*}{\partial r^*}\right) + \frac{1}{r^{*2}}\frac{\partial^2 v_z^*}{\partial \theta^2} + \frac{\partial^2 v_z^*}{\partial z^{*2}}$$

$$\frac{Dv_z^*}{Dt} \equiv \left(\frac{\partial v_z^*}{\partial t^*} + v_r^*\frac{\partial v_z^*}{\partial r^*} + \frac{v_\theta^*}{r^*}\frac{\partial v_z^*}{\partial \theta} + v_z\frac{\partial v_z^*}{\partial z^*}\right)$$

FREE CONVECTION FREE CONVECTION FREE CONVECTION

Dimensionless Equation of Motion (free convection)

$$\frac{Dv_z^*}{Dt^*} = \left(\nabla^2 v_z\right)^* + \text{Gr}T^*$$

Dimensionless Energy Equation (free convection; Re = 1)

$$\left(\frac{\partial T^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* T^*\right) = \frac{1}{\text{Pr}} \nabla^{*2} T^*$$

$$\text{Nu} = \text{Nu}\left(T^*, \frac{L}{D}\right) \Rightarrow \text{Nu} = \text{Nu}\left(\text{Pr}, \text{Gr}, \frac{L}{D}\right)$$

FREE CONVECTION FREE CONVECTION FREE CONVECTION

Example: Natural convection from vertical planes and cylinders

$$\text{Nu} = \frac{hL}{k} = a\text{Gr}^m \text{Pr}^m$$

- a,m are given in Table 4.7-1, page 255 Geankoplis for several cases
- L is the height of the plate
- all physical properties evaluated at the film temperature, T_f

$$T_f = \frac{T_w + T_b}{2}$$

Experience with Dimensional Analysis thus far:

✓ •Flow in pipes at all flow rates (laminar and turbulent)

Solution: Navier-Stokes, Re, Fr, L/D, dimensionless wall force = f; f=f(Re, L/D)

✓ •Flow around obstacles (spheres, other complex shapes

Solution: Navier-Stokes, Re, dimensionless drag= C_D; $C_D = C_D(Re)$

✓ •Forced convection heat transfer from fluid to wall

Solution: Microscopic energy, Navier-Stokes, Re, Pr, L/D, heat transfer coefficient=h; h = h(Re,Pr,L/D)

✓ •Natural convection heat transfer from fluid to wall

Solution: Microscopic energy, Navier-Stokes, Gr, Pr, L/D, heat transfer coefficient=h; h = h(Gr,Pr,L/D)

Experience with Dimensional Analysis thus far:

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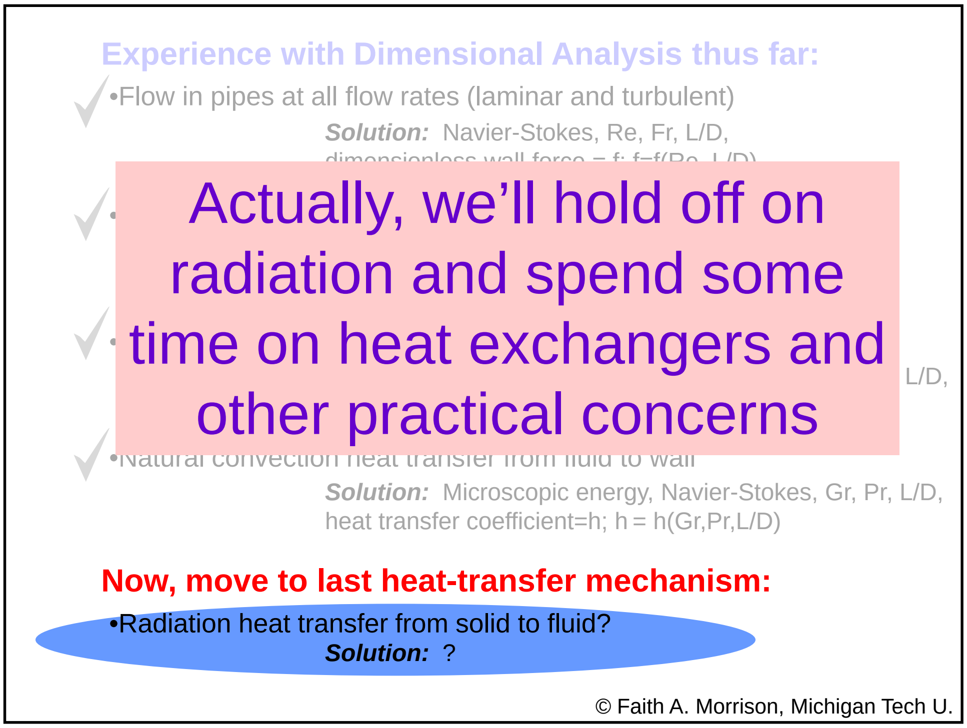
✓ •Natural convection heat transfer from fluid to wall

Solution: Microscopic energy, Navier-Stokes, Gr, Pr, L/D, heat transfer coefficient=h; h = h(Gr,Pr,L/D)

Now, move to last heat-transfer mechanism:

•Radiation heat transfer from solid to fluid?

Solution: ?


Experience with Dimensional Analysis thus far:
•Flow in pipes at all flow rates (laminar and turbulent)
Solution: Navier-Stokes, Re, Fr, L/D,
Actually, we'll hold off on radiation and spend some time on heat exchangers and other practical concerns
L/D,
•Natural convection heat transfer from fluid to wall
Solution: Microscopic energy, Navier-Stokes, Gr, Pr, L/D, heat transfer coefficient=h; h = h(Gr,Pr,L/D)
Now, move to last heat-transfer mechanism:
•Radiation heat transfer from solid to fluid?
Solution: ?
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