

# Homework 3

## CM3110

### Fall 2011

**Exam 2 Tuesday Oct 4, Dow 641, 6:30pm-8:00pm (topics on HW2)**

**Exam 3 Tuesday Oct 25, Dow 641, 6:30pm-8:00pm (topics on HW3)**

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1. Tangential annular flow of a Newtonian fluid. Problem 7.40 in Morrison "An Introduction to Fluid Mechanics," Chapter 7, on the web at [www.chem.mtu.edu/~fmorriso/cm310/MorrisonChapter7%28c%292011.pdf](http://www.chem.mtu.edu/~fmorriso/cm310/MorrisonChapter7%28c%292011.pdf)
2. Geankoplis 2.7-9. Water having a density of  $998 \text{ kg/m}^3$  is flowing at the rate of  $1.676 \text{ m/s}$  in a  $3.068\text{-in.}$ -diameter horizontal pipe at a pressure  $p_1$  of  $68.9 \text{ kPa abs}$ . It then passes to a pipe having an inside diameter of  $2.067\text{-in.}$ 
  - a. Calculate the new pressure  $p_2$  in the  $2.067\text{-in.}$  pipe. Assume no friction losses (**Answer  $p_2=64\text{kPa}$** )
  - b. If the piping is vertical and the flow is upward, calculate the new pressure  $p_2$ . The pressure tap for  $p_2$  is  $0.457 \text{ m}$  above the tap for  $p_1$  (**Answer  $p_2=59\text{kPa}$** )
3. Geankoplis 2.6-4. Bulk velocity for flow between parallel plates. A fluid flowing in laminar flow in the x-direction between two parallel plates has a velocity profile given by the following:

$$v_x = v_{x \max} \left[ 1 - \left( \frac{y}{y_0} \right)^2 \right]$$

where  $2y_0$  is the distance between the plates,  $y$  is the distance from the center line, and  $v_x$  is the velocity in the x direction at position  $y$ . Calculate the relationship between  $\langle v_x \rangle$  (bulk or average velocity) to  $v_{x,\max}$  (**Answer:  $\langle v_x \rangle = \frac{2}{3} v_{x,\max}$** )

4. Geankoplis 3.2-1 Flow measurement using a pitot tube. A pitot tube is used to measure the flow rate of water at  $20^\circ\text{C}$  in the center of a pipe having an inside diameter of  $102.3 \text{ mm}$ . The manometer reading is  $78 \text{ mm}$  of carbon tetrachloride at  $20^\circ\text{C}$ . The pitot tube coefficient is  $0.98$ .
  - a. Calculate the velocity at the center of the tube and the average velocity (**Answer:  $v_{\max}=0.94 \text{ m/s}$ ,  $v_{\text{av}}=0.77 \text{ m/s}$** ) and
  - b. Calculate the volumetric flow rate of water (**Answer:  $6.4 \times 10^{-3} \text{ m}^3/\text{s}$** ).
5. What is the average flow rate for the steady flow of a power-law fluid in pressure-driven flow in a tube? The fluid is a power-law fluid and the viscosity follows the power law  $\eta = m\dot{\gamma}^{n-1}$ , where

$\eta$  is the viscosity in Pa s,  $\dot{\gamma} = -\frac{dv_z}{dr}$  is the shear rate in  $s^{-1}$ . You may neglect the effect of gravity

(Answer: see lecture 7, p15).

6. Geankoplis 3.5-1 Pressure drop of a power-law fluid, banana puree. A power-law biological fluid, banana puree, is flowing at 23.9°C, with a velocity of 1.018 m/s, through a smooth tube 6.10 m long having an inside diameter of 12.67 mm. The fluid is a power-law fluid and the viscosity follows the power law  $\eta = m\dot{\gamma}^{n-1}$ , where  $\eta$  is the viscosity in Pa s,  $\dot{\gamma} = -\frac{dv_z}{dr}$  is the shear rate in  $s^{-1}$ , and for banana puree  $m=K=6.00$  (in the appropriate SI units) and  $n=0.454$ . The density of banana puree is 976  $kg/m^3$ . Calculate the pressure drop for the flow in the pipe.  
(Answer: 250  $kN/m^2$ )
7. Geankoplis 2.8-5 Force of water stream on a wall. Water at 298 K discharges from a nozzle and travels horizontally, hitting a flat wall inclined 45° to the vertical. The nozzle has a diameter of 12mm, and the water leaves the nozzle with a flat velocity profile at a velocity of 6.0 m/s. The flow is frictionless and you may assume no loss of energy. The amount of fluid splitting in each direction along the plate can be determined by using the continuity equation and a macroscopic momentum balance. Calculate this flow division and the force vector on the wall. **Answer:  $m_2=0.58$  kg/s,  $m_3=0.099$  kg/s,  $\underline{R}(\text{force on wall})=-2.0N e_x+2.0 N e_y$**
8. A chromatography column is being designed, and we desire to calculate the expected pressure drop as a function of flow rate with water as the mobile phase. The fluid packing material has a surface area/volume of  $5.0 \times 10^3$   $m^2/m^3$ , and after packing the column, the void fraction of the column was measured to be 0.18. The column is 1.0 cm inner diameter and 1.0 meter long. Calculate the pressure drop for water flow rates of 1ml/min, 5ml/min, and 10 ml/min. **Answer: at 10 ml/min  $\Delta p=2.6 \times 10^4$  Pa (3.8psi).**
9. What is the hydraulic diameter of a rectangular duct of height 5.0 in and width 12 in? For room temperature air flowing at a Reynolds number of  $5.0 \times 10^4$  through this duct, what is the average velocity? What is the friction factor? What is the pressure drop for 1000 feet of duct in psi?  
**Answer: 0.04 psi**
10. Use a macroscopic momentum balance to relate pressure drop to vector fluid force on the walls in a straight pipe. **Answer:  $\underline{F} = \Delta p \pi R^2 \hat{e}_z$ , see lecture 8, page 15**
11. The terminal velocity of a sphere falling in a liquid is the velocity at steady state when the downward force due to gravity on the sphere is exactly balanced by the upward retarding force due to the fluid. What is the terminal velocity of a sphere of density  $\rho_s$  falling in a fluid of density  $\rho$  in the Stokes-flow regime? **Answer: terminal velocity =  $(\rho_s - \rho)(2/9)gR^2/\mu$**