A Study on Vortex Shedding From Spheres in a Uniform Flow

H. Sakamoto
H. Hanui

Department of Mechanical Engineering, Kiiami Institute of Technology, Hokkaido, 090 Japan

1 Introduction

Wakes behind a sphere are encountered so frequently in engineering applications that large amounts of research have been conducted and massive amounts of data have been accumulated. Many reports which concern only the vortex shedding frequency from a sphere have been published. Möller (1938) has measured the frequency of vortex shedding from a sphere by flow-visualization in the Reynolds number range of $10^3 < \text{Re} < 10^4$. He demonstrated that there are two Strouhal numbers in this Reynolds number range, i.e., a high-mode and a low-mode Strouhal number. Cometta (1957) measured the Strouhal number in the ranges of $10^3 < \text{Re} < 4 \times 10^4$ using a hot-wire technique, and also found the same phenomenon. However, his conclusion that the two Strouhal numbers exist only when the sphere has a certain diameter is not very convincing from a hydrodynamic viewpoint. Achenback (1974) experimentally investigated on the Strouhal number by means of both the flow-visualization and hot-wire techniques, and demonstrated that the values for the higher frequency mode of Strouhal numbers showed the same results as those obtained by Möller at Reynolds numbers lower than $\text{Re} = 3 \times 10^3$. He also clarified that, when the Reynolds number is above $6 \times 10^4$, the vortices with the lower frequency mode are periodically shed from the sphere, and this phenomenon could be observed as far as the upper critical Reynolds number of $3.7 \times 10^5$. However, his measurement which used a hot-wire probe attached to the surface of the sphere failed to detect any low-mode Strouhal number in the range of the Reynolds numbers lower than $\text{Re} = 6 \times 10^4$. In other words, he did not succeed in confirming the existence of two Strouhal numbers in the ranges of $10^3 < \text{Re} < 10^4$, as was reported by Möller and Cometta. Recently, Kim and Durbin (1988) performed experiments in the wakes of spheres for the Reynolds number range of $5 \times 10^3 < \text{Re} < 6 \times 10^4$, and they found two distinct modes of unsteadiness which were associated with the small scale instability of the separating shear layer and with the large scale instability of the wake. In particular, they demonstrated that the higher frequency was detected only in the region of the wake immediately downstream of the sphere when the Reynolds number exceeded 800. Taneda (1978) examined the wake configuration formed behind a sphere in the range of $10^4 < \text{Re} < 10^6$ by means of the flow-visualization methods. He clarified that the Strouhal number which was based on the wavelength caused by the alternate fluctuations of the wake was about 0.2 in the ranges of $10^4 < \text{Re} < 3 \times 10^5$. Majumdar and Douglas (1970), Calvert (1972), Magarvey and Bishop (1961), and Pao and Kao (1977) also have measured the frequency of the vortex shedding from the sphere. However, they only measured in some limited ranges for the Reynolds number, and their results did not always coincide with one another.

Thus, many reports have been made concerning the vortex shedding from a sphere. The results, however, are fairly different from report to report. Therefore, it must be pointed out that reliable values for the Strouhal number in a wide range of the Reynolds numbers have not yet been found. This is due to the fact that the formation mechanism of the vortex shedding from the change of the Reynolds number has not been properly clarified. Consider these facts described above, the present study is intended to measure the Strouhal number of a sphere by means of the hot-wire measurement for a wide range of the Reynolds numbers, including a low Reynolds number with a very low stream velocity. In particular, it closely examines the relationship between the wake configuration behind the sphere and the frequency of vortex shedding, observing its form through the flow visualization and concentrating on the high-mode and low-mode Strouhal numbers in the range of the Reynolds number of $10^3 < \text{Re} < 10^4$. 

Vortex shedding from spheres at Reynolds numbers from $3 \times 10^2$ to $4 \times 10^4$ in a uniform flow was investigated experimentally. Standard hot-wire technique were used to measure the vortex shedding frequency from spheres in a low-speed wind tunnel. Flow-visualization experiments were carried out in a water channel. Important results from the investigation were that (i) the variation of the Strouhal number $St(= fD/U_\infty)$, $U_\infty$: freestream velocity, $D$: diameter of the sphere, $f$: vortex shedding frequency) with the Reynolds number $(= U_\infty D/x)$, $x$: kinematic viscosity) can be classified into four regions, (ii) the Reynolds number at which the hairpin-shaped vortices begin to change from laminar to turbulent vortices so that the wake structure behind the sphere is not shown clearly when a Reynolds number of about 800 is reached, and (vi) at Reynolds numbers ranging from $8 \times 10^2$ to $1.5 \times 10^4$, the higher and lower frequency modes of the Strouhal number coexist.
The rod was fixed horizontally on the centerline of the test section by four fine cross-wires, 0.2mm in diameter, and its rear by a small steel rod 3mm in diameter and 0.6m in length. The turbulence level in the freestream was below 0.2 percent at the wind tunnel and 0.1 percent at the water channel, and they did not vary noticeably with the change of the free-stream velocity.

2.2 Measurement of Freestream Velocity. In the wind tunnel test, it was difficult to measure the freestream velocity using a pitot tube or hot-wire anemometer when the velocity was lower than about 3 m/s. Therefore, when the free-stream velocity was lower than 3 m/s, it was estimated by using Roshko's formula (Roshko, 1956) based on the vortex shedding frequency of the regular mode from a two-dimensional circular cylinder mounted at the upstream of the test section. Since the vortex shedding of the regular mode from a two-dimensional circular cylinder is generated only in the limited range of the Reynolds numbers of $40 < \text{Re} < 160$, thirteen cylinders whose diameters ranged from 0.3mm to 8mm were employed in accordance with the change of the freestream velocity. Also, in order to avoid the effect of the boundary layer along the tunnel wall on the frequency of vortex shedding from the circular cylinders, disks which were 7 to 10 times larger than the diameter of the cylinder were attached on both ends. Figure 2 shows a calibration curve for the velocity estimated from the vortex shedding frequency of the cylinder for each diameter. The calibration curve was produced using the following method:

First, a relation between the reference velocity, $U_0$, measured by a pitot tube and $U_i$ obtained using the cylinder with a diameter of 0.308mm was found, secondly, a relation between the reference velocity, $U_0$, using the cylinder with the diameter of 0.308mm and $U_i$ using the cylinder with a diameter of 0.506mm was found, and then the relations between the reference velocity, $U_0$, and $U_i$ obtained using each cylinder were successively found in the same manner. Also, the circular cylinder which had been placed upstream of the sphere was removed from the test section during the measurement for the vortex shedding frequency of the sphere to avoid any influence on the flow characteristics. Furthermore, the freestream velocity in the test section of the water channel was obtained by measuring the time of travel for hydrogen bubbles passing a distance of 1m.

2.3 Flow Visualization. Flow around the spheres in the water channel were visualized using uranine dye. The injection of the uranine dye into the water channel was performed using a stainless tube with a diameter of 1mm. After the uranine dye was injected for specified period of time, the stainless tube was removed from the test section in order to avoid any disturbance generated by it.

---

**Nomenclature**

- $D =$ diameter of sphere  
- $F =$ nondimensional frequency of vortex shedding $= fD^2/\nu$  
- $\text{Re} =$ Reynolds number $= U_oD/\nu$  
- $\text{St} =$ Strouhal number $= fD/U_o$  
- $T =$ taking time for shedding of one hairpin-shaped vortex  
- $U_o =$ free stream velocity  
- $f =$ frequency of vortex shedding  
- $\nu =$ kinematic viscosity of fluid

---

Journal of Fluids Engineering  
DECEMBER 1990, Vol. 112 / 387
2.4 Measurement of Frequency. The frequency of vortex shedding from the sphere placed in the wind tunnel was determined on the basis of the power spectrum analysis of the fluctuating velocity detected by a hot-wire probe mounted in the wake behind the sphere. The position of the hot-wire probe (which was somewhat dependent on the Reynolds number) was determined by observing visualized patterns in the wake; it was in the range of (3 to 4)D from the rear surface of the sphere. In order to avoid any influence from the hot-wire probe on the wake characteristics of the sphere, the prongs were made of pino-wire with a tip diameter of about 0.1mm and a length of 60mm; they were bent to a 135 degree angle and both ends of the hot wire were soldered of them. Also, the vortex shedding frequency of the sphere mounted in the water channel was determined by measuring the time necessary for a sequence of fifty vortices to be shed.

3 Results and Discussions

Figure 3 shows the main results that have been reported concerning the Strouhal number of a sphere $St = fD/U_0$, where $f$ is the frequency of the vortex shedding, and $D$ the diameter of the sphere. The hairpin-shaped vortices begin to be periodically shed when the Reynolds number reaches about $Re = 350 \sim 400$ (about $Re = 300$ in the present experiment). The values of these Strouhal numbers up to $Re = 10^3$ are almost the same as those previously recorded except for those obtained by Magarvey and Bishop (1961). In the range of the Reynolds numbers between $10^3$ and $10^4$, two Strouhal numbers exist, namely, the high-mode and low-mode Strouhal number. The high-mode Strouhal number obtained by Möller (1938) exists up to $Re = 10^4$, but Cometta's result (Cometta, 1957) exists only up to $Re = 7.4 \times 10^3$; these values are somewhat lower than those given by Möller. On the other hand, results obtained by Kim and Durbin (1988) show clearly that the two distinct modes exist simultaneously in the Reynolds number ranges of $8 \times 10^2 < Re < 6 \times 10^4$. In particular, it is noticeable that the high-mode $St$ exists up to $Re = 6 \times 10^4$. However, those values are somewhat lower than those obtained by Möller (1938) in the Reynolds number range of $10^3 < Re < 10^4$. Also, the value for the low-mode $St$ obtained by Möller is a constant value of 0.4 in this range. When the Reynolds number is higher than about $10^4$, Mujumdar and Douglas (1970), Taneda (1978), and Calvert (1972) have reported that there exists only the low-mode $St$ and it is a constant value of approximately $St = 0.19 \sim 0.2$, although the results of Kim and Durbin (1988), and Achenback (1974) are a little different. Thus, in a range where $Re$ is comparatively small ($Re < 10^3$) or large ($Re > 10^4$), the Strouhal number obtained by the above researchers are mostly similar, while in the intermediate range of $10^3 < Re < 10^4$, the values differ considerably from researcher to researcher.

Figure 4 shows the values for the Strouhal number, $St$, obtained by the flow-visualization and hot-wire measurements in the present experiment at the range of the Reynolds numbers lower than $Re = 1.5 \times 10^4$, in which there exist two Strouhal numbers, namely, the high-mode and low-mode $St$. When a $Re$ of 300 is reached, the hairpin-shaped vortices begin to be periodically shed from the sphere, forming a laminar wake as shown in Fig. 8(d). When the $Re$ is further increased to 800, the wake flow becomes turbulent with alternate fluctuations so that the configuration of the vortices is not clearly discernible as is shown in Fig. 8(d). When the $Re$ is further increased to 800, the wake flow becomes turbulent with alternate fluctuations so that the configuration of the vortices is not clearly discernible as is shown in Fig. 8(d). When the $Re$ is further increased to 800, the wake flow becomes turbulent with alternate fluctuations so that the configuration of the vortices is not clearly discernible as is shown in Fig. 8(d).
the vortex sheet separated from the sphere surface, and in accordance with its pulsation, the cylindrical vortices (defined as vortex tube in the present study) begin to be periodically shed, covering the vortex formation region as shown in Fig. 6(a). The values for the Strouhal number determined based on the frequency of the vortex tube shedding estimated from the visual observation are very similar to the high-mode St obtained by Möller (1938) and Achenbach (1974). This suggests that the high-mode St is based on the shedding of the vortex tube generated by the pulsations of the vortex sheet. Also, when a hot-wire is placed in the extreme neighborhood of the sphere, the frequency of the vortex tubes shed from the sphere could be detected by the spectrum analysis of the fluctuating velocity as shown in Fig. 5. On this spectrum analysis of the fluctuating velocity, the frequency of the large-scale vortex shedding which is considerably lower compared with that of the vortex tube is cut off by an analogue high-pass filter in order to appear only as a prevailing frequency based on the vortex tube. However, when the Re is larger than $1.5 \times 10^4$ in the present experiment, no clear peak is observed by the spectrum analysis because the fluctuation energy becomes extremely small. Furthermore, the energy level of the fluctuating velocity based on the shedding of the vortex tubes is much smaller than that based on wave motion of the wake with alternate fluctuations as shown in Fig. 6(b). Accordingly, it would be safe to consider that the unsteady flow around the sphere is mainly caused by the wave motion of the wake with alternate fluctuations when the Re is larger than about 800. On the other hand, Kim and Durbin (1988) have found that the high-mode St exists in the wide range of the Reynolds numbers between $8 \times 10^2$ and $6 \times 10^4$, and then it is associated with the small scale instability of the separating shear layer. However, they did not make clarifications concerning the structure of the small scale instability of the separating shear layer. Perhaps, it could be concluded that the vortex tube is formed by the small scale instability of the separating shear layer. Figure 7 shows the Strouhal number based on the hairpin-shaped vortex and the large-scale vortex shedding obtained by the hot-wire measurement, namely the low-mode St, with the Reynolds number. We classified these results into regions I - VI and transitional regions A - C, taking into account the change in the Strouhal number corresponding to the Reynolds number. The following is a rough description of each region in terms of the relationship between its St and the configuration of the wake.

(i) Region I ($300 < Re < 420$)

According to Taneda's flow-visualization experiment in the range of $5 < Re < 300$ (Taneda, 1956), when the Re is over 130, faint periodic pulsative motion with a very long period occurs at the rear of the vortex-ring formed behind the sphere, forming...
a wavelike wake as shown in Fig. 8(a). Taneda also has reported that this phenomenon continues until a Reynolds number of about 300 is reached. Achenbach (1974), on the other hand, has reported that the wavelike wake turns into a so-called hairpin-shaped vortex when a Reynolds number of about 400 is reached. But, the present results which were measured by means of visual observation and spectrum analysis of the fluctuating velocity, as shown in Fig. 9 show that the hairpin-shaped vortex begins to be shed when the Re exceeds 300 (as shown in Fig. 9(a), there is no spectral peak in the power spectrum distribution when the Re is lower than 300). Taneda’s report that a wavelike wake continues to be seen as far as the range of Re = 300 justifies the present results in which the hairpin-shaped vortices begin to be shed at Re = 300.

Figure 10 shows the waveform of the fluctuating velocity which was detected by the four hot-wire probes (CH 1 ~ 4) set at intervals of 90° on the circumference of a circular facet at a right angle in the wake behind the sphere. As the figure clearly shows, the amplitude of the waveform of the fluctuating velocity decreases with increasing Reynolds number. The waveform at Re = 321 (Fig. 10(a)) shows a single frequency component, while at Re = 503 (Fig. 10(b)) and Re = 8605 (Fig. 10(c)) the waveform becomes more complex with multiple frequency components. The uncertainty in bias error is less than ±3 percent.

Fig. 8 Patterns of vortex shedding in wake at each region. For further information, see the caption of Fig. 8.

Fig. 9 Power spectrum of fluctuating velocity based on hairpin-shaped vortex and large-scale vortex shedding at each region. For further information, see the caption of Fig. 5.

Fig. 10 Waveform of fluctuating velocity detected by four hot-wires equally spaced on circumference of a circle. (Uncertainty in bias error: less than ±3 percent.)
velocity detected by CH 4 is much larger than the others; this suggests that the hairpin-shaped vortices flow out in one direction from the point near where CH 4 is set. Moreover, the power spectrum as shown in Fig. 9(b) and the waveform of the fluctuating velocity as shown in Fig. 10(a) suggest that the hairpin-shaped vortices are periodically shed in what is called a regular mode with regularity in strength and frequency. This is perhaps because the energy entraining into the vortex formation region is very regularly supplied, stored and emitted, as is clearly shown by the shedding system of the vortex as shown in Fig. 11 (where T is the time when one vortex is shed). This also explains why the vortices shed in succession are of the same strength and frequency.

(ii) Transitional Region A (420 < Re < 480)

When the Re is greater than 420, the amplitude and waveform of the fluctuating velocity based on the shedding of the hairpin-shaped vortices begin to be generated with irregularity, and their shedding direction oscillates intermittently from left to right. The vortex shedding of the regular mode which existed in the previous region is also seen intermittently. Perhaps, regular and irregular modes appear alternately and last for a certain period of time in this region.

(iii) Region II (480 < Re < 650)

When the Re exceeds 480, the waveform of the fluctuating velocity based on the shedding of the hairpin-shaped vortices becomes irregular as shown in Fig. 10(b). In other words, the shedding pattern of the vortex is always in the irregular mode. This is perhaps because the supply, storage, and emission of energy within the vortex formation region becomes imbalanced so that the strength of the vortices shed from the formation region differs from one another. Furthermore, as is shown in Fig. 10(b), the amplitude of the fluctuating velocity differs from CH 1 ~ 4 as time passes; this suggests a change in the shedding point of the hairpin-shaped vortices. Accordingly, as Taneda (1978) has pointed out, the vortices are shed as if the facet incising the vortexes are rotating slowly and irregularly about an axis through the center of the sphere in the direction of the undisturbed flow.

Figure 12 shows the nondimensional frequency \( F = f d^2 / u \) in terms of the frequency, \( f \), of the hairpin-shaped vortex shedding. According to Roshko (1974), Karman's vortex streets begin to be shed with regularity from a two-dimensional circular cylinder when a Re of about 40 is reached, and the shedding pattern becomes irregular when the Re approaches 160. Roshko has also reported that the non-dimensional frequency \( F = f d^2 / u \), where \( d \) is the diameter of the cylinder) is distributed on different straight lines in the regular and irregular modes with the Reynolds number.

The non-dimensional frequency, \( F \), of the vortices shed from the sphere shows a similar property, i.e., it is regular in region I and irregular in region II, while the transitional region A is somewhere between the two regions. Furthermore, when the Re exceeds 650, the shedding pattern of the vortices is fairly different from that region II so that the non-dimensional frequency is distributed on a different straight line as shown in Fig. 12.

(iv) Transitional Region B (650 < Re < 800)

As has already been previously described, when the Re exceeds 650, the cylindrical vortex sheet separated from the sphere surface shows pulsation, and the vortex tubes begin to be periodically shed in accordance with the pulsation as if covering the vortex formation region. These vortices diffuse near the sphere without retaining a cylindrical form. Also, the large-scale vortices flow out from the formation region begin to change from laminar to turbulent vortices, thus obscuring their form.

(v) Region III (800 < Re < 3000)

When the Re exceeds 800, some of the vortex tubes formed by the vortex sheet separating from the sphere surface flow into the vortex formation region at its rear end, while others are shed in small vortex loops. Some of the small vortex loops move downstream, intricately intertwining with the large-scale vortices which alternatingly flow out from the formation region as is shown in Fig. 6(b). As is clearly shown by the fluctuating waveform of Fig. 10(c), the point where the large-scale vortices are shed rotates slowly and irregularly since the amplitude detected by each hot-wire probe CH differs as the time elapses. And the large-scale vortices move away from the sphere rotating at random about an axis parallel to the flow through the center of the sphere; this has already been pointed out by Taneda (1978). The shape of the large-scale vortex has not yet been clarified. However, Perry et al. (1981) investigated the structure of turbulent wake behind an axis-symmetrical blunt body using a flying hot-wire apparatus, and clarified that it has the same structure as a hairpin-shaped vortex that is formed behind the blunt body in the low Reynolds numbers. Therefore, the large-scale vortex that flows out with alternate fluctuations away from the sphere may have the same structure as the
hairpin-shaped vortex observed in the low Reynolds number region \((Re < 800)\) whose model is shown in Fig. 8(d). Its structure has not yet been sufficiently clarified through detailed measurement concerning the structure of the turbulent wake behind the sphere. Therefore, it would be desirable to continue the investigation of the structure of the turbulent wake behind the sphere.

(vi) Transitional Region C \((3 \times 10^3 < Re < 6 \times 10^3)\)

As is shown in Fig. 7, the Strouhal number decreases rapidly when the Re exceeds \(3 \times 10^3\). In this region no evident peak is seen in the power spectrum distribution which has widely scattered values as shown in Fig. 9(e). This is perhaps because the vortex sheet which separates from the sphere surface changes from laminar to turbulent flow. The Strouhal number becomes smaller perhaps because it takes a longer time to store the necessary energy for the shedding of large-scale vortices since the dissipation of the energy increases in the turbulent vortex sheet so that the energy entraining into the vortex formation region decreases.

(vii) Region IV \((6 \times 10^3 < Re < 3.7 \times 10^4)\)

When the Re exceeds \(6 \times 10^3\), the vortex sheet separating from the surface of the sphere becomes completely turbulent. As is the case with the Karman vortex streets formed behind a two-dimensional body, the vortices shed from the formation region become stabilized when transition from laminar to turbulent flow occurs in the separated shear layer. Therefore, the alternate shedding of the large-scale vortices from the formation region also shows the pseudoregularity as is seen in the power spectrum distribution of Fig. 9(f). The Strouhal number based on the frequency of vortex shedding increases again with an increase in the Reynolds number until it becomes constant at \(0.19 - 0.195\) when the Re exceeds \(2 \times 10^4\). For the Strouhal number in the region where the Re is over the upper value of \(4 \times 10^4\) studied in the present experiment, Achenbach has already reported that the Strouhal number which has a constant value of 0.19 exists as far as the region of \(Re = 3.7 \times 10^3\); this is called the upper critical Reynolds number. Further, Taneda (1978) has clarified that when the Re exceeds the upper critical Reynolds number, the vortex sheets separating from the sphere flow out and roll up to form a pair of streamwise vortices without the periodic vortex shedding.

Table 1 shows the results of the present experiment for Reynolds numbers ranging from \(10^3\) to \(4 \times 10^4\), together with those obtained by Achenbach (1974), Kim and Durbin (1988), and Taneda (1978) at Reynolds numbers higher than \(Re = 4 \times 10^5\). The table shows the change in the shape of vortices formed behind a sphere, the change in the direction of vortex shedding, and the existence or non-existence of the periodic vortex shedding in accordance with the change in the Reynolds number.

4 Conclusions

The present study has described an experimental investigation of the Strouhal number of the spheres by means of both the flow-visualization and hot-wire measurements in a wide range of the Reynolds number including the very low freestream velocity. The results led to be the following conclusions.

(1) The variation of the Strouhal number with the Reynolds number can be classified into four regions, and the relationship between the Strouhal number and the configuration of the wake in each region is clarified.

(2) When a Reynolds number exceeds about 300, the hairpin-shaped vortices begin to be periodically shed with regularity in its strength and frequency, forming laminar vortices until a Reynolds number is about 800.

(3) When a Reynolds number exceeds about 800, the hairpin-shaped vortices begin to change from the laminar to turbulent vortices with alternate fluctuations, this pattern of vortex shedding continues as far as the region of \(Re = 3.7 \times 10^3\), which is called the upper critical Reynolds number.

(4) When a Reynolds number exceeds 800, the vortex tubes begin to be periodically shed covering the vortex formation region, in accordance with the pulsation of the vortex sheet separated from the surface of the sphere.

(5) At the Reynolds number ranging from \(8 \times 10^2\) to \(1.5 \times 10^6\) (which is some lower than that obtained by Kim and Durbin, 1988), the higher and lower frequency modes of the Strouhal number, which are caused by the periodic fluctuation in the vortex tube formed by the pulsation of the vortex sheet separated from the surface of the sphere and in the turbulent wake with progressive wave motion respectively, are coexisted.

Acknowledgment

The authors express their sincere thanks to Mr. Y. Obata, Department of Mechanical Engineering, Kitami Institute of Technology, for his assistance in the construction of the experimental apparatus.

References


