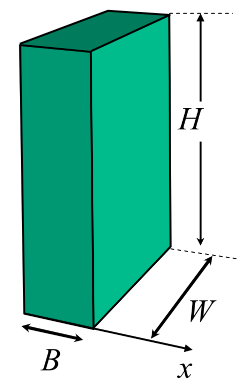
Example 2: Heat flux in a rectangular solid

Assumptions:
- wide, tall slab
- steady state
- *h_1 and h_2 are the heat transfer coefficients of the left and right walls*

What is the steady state temperature profile in a rectangular slab if the <u>fluid on one side</u> is held at T_{b1} and the <u>fluid on the other side</u> is held at T_{b2}?

Bulk temperature on left T_{b1}



T_{b2} Bulk temperature on right

$T_{b1} > T_{b2}$

Newton's law of cooling boundary conditions

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Problem-Solving Procedure - heat-transfer problems

1. sketch system
2. choose coordinate system
3. choose a control volume - small dimension in the direction of flux
4. perform an energy balance (will contain energy flux)
5. substitute in *Fourier's law of heat conduction,* e.g. $\frac{q_x}{A} = -k\left(\frac{dT}{dx}\right)$
6. solve the <u>differential</u> equation for temperature profile
7. apply boundary conditions

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Example 2: Heat flux in a slab

Solution:

$$\frac{q_x}{A} = c_1$$

Constant

$$T = \frac{-c_1}{k} x + c_2$$

Boundary conditions?

Rectangular slab with Newton's law of cooling BCs

This is the same as Example 1, EXCEPT there are different boundary conditions.

With Newton's law of cooling boundary condition, we know the flux at the boundary in terms of the heat transfer coefficient, h:

$$\left.\frac{q_x}{A}\right|_{x=0} = h_1 \left(T_{b1} - T_{w1}\right) > 0$$

$$\left.\frac{q_x}{A}\right|_{x=B} = h_2 \left(T_{w2} - T_{b2}\right) > 0$$

The flux is **positive** (heat flows in the $+x$-direction)

but, we do not know these temps

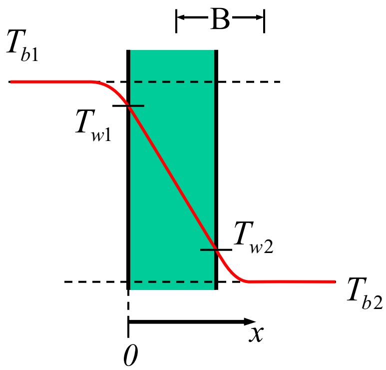
How do we apply these boundary conditions?

Soln from Example 1:

$$\frac{q_x}{A} = c_1$$

$$T = \frac{-c_1}{k} x + c_2$$

2 unknown constants to solve for, c_1, c_2.



We can eliminate the wall temps from the BC by using the solution for *T(x)*.

then solve for c_1, c_2.

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Example 4: Heat flux in a slab

After some algebra,

$$c_1 = \frac{(T_{b1} - T_{b2})}{\left(\frac{1}{h_1} + \frac{B}{k} + \frac{1}{h_2}\right)}$$

$$c_2 = \frac{T_{b1}\left(\frac{1}{h_2} + \frac{B}{k}\right) + \frac{1}{h_1} T_{b2}}{\left(\frac{1}{h_1} + \frac{B}{k} + \frac{1}{h_2}\right)}$$

Substituting back into the solution, we obtain the final result.

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Example 2: Heat flux in a slab with Newton's law of cooling boundary conditions (heat transfer coefficients h_1, h_2)

Solution: (temp profile, flux)

Temperature profile:

$$\frac{T_{b1}-T}{T_{b1}-T_{b2}} = \frac{\frac{x}{k}+\frac{1}{h_1}}{\left(\frac{1}{h_1}+\frac{B}{k}+\frac{1}{h_2}\right)}$$

Flux:

$$\frac{q_x}{A} = \frac{T_{b1}-T_{b2}}{\frac{1}{h_1}+\frac{B}{k}+\frac{1}{h_2}}$$

Rectangular slab with Newton's law of cooling BCs 7

Example 4: Heat flux in a slab

Example:

What is the temperature in the middle of a slab (thickness = B, thermal conductivity = k=26 BTU/h ft ^{o}F) if the left side is exposed to a fluid of temperature 120^{o}F and the right side is exposed to a fluid of temperature 50^{o}F? The heat transfer coefficients at the two faces are the same and are equal to 2 BTU/h ft2 ^{o}F

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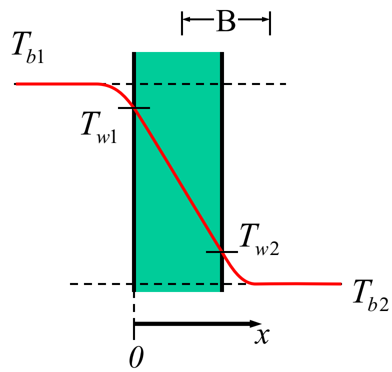
Example 4: Heat flux in a slab

Example:

For heat conduction in a slab with Newton's law of cooling boundary conditions, we sketched the solution as shown. If the heat transfer coefficients became infinitely large, how would the sketch change? What are the predictions for *T(x)* and the flux for this case?



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Boundary Conditions on Heat-Transfer Problems

- wall temperature specified $\left. T \right|_{boundary} = 50^o C$
- boundary wall flux specified
  - particular value given $\left. \frac{q_x}{A} \right|_{boundary} = 1.24 \times 10^5 \frac{W}{m^2}$
  - insulating boundary $\left. \frac{q_x}{A} \right|_{boundary} = 0$
  - Newton's law of cooling $\left. \frac{q_x}{A} \right|_{boundary} = h(T_{wall} - T_{bulk})$
- temperature/flux continuity along boundary of two different materials $\left. \frac{q_x}{A} \right|_{boundary\,1} = \left. \frac{q_x}{A} \right|_{boundary\,2}$ $\left. T \right|_{boundary1} = \left. T \right|_{boundary2}$

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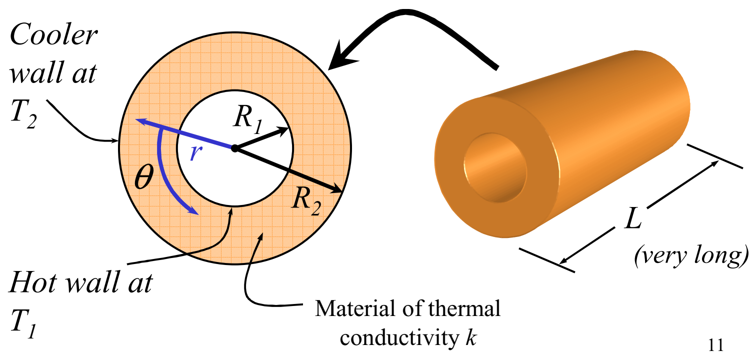
Example 3: Heat flux in a cylindrical shell

Assumptions:
- long pipe
- steady state
- k = thermal conductivity of wall

*What is the steady state temperature profile in a cylindrical shell (pipe) if the **inner wall** is at T_1 and the **outer wall** is at T_2? ($T_1 > T_2$)*



Example 3: Heat flux in a cylindrical shell

Solution:

$$\frac{q_r}{A} = \frac{c_1}{r}$$

$$T = -\frac{c_1}{k}\ln r + c_2$$

Not constant

Boundary conditions?

Example 3: Heat flux in an annulus with temperature boundary conditions

Solutions:

$$\frac{q_r}{A} = \frac{T_1 - T_2}{\ln \frac{R_2}{R_1}} \left(\frac{k}{r} \right)$$

The heat flux q_r/A DOES depend on, *k;* also q_r/A decreases as $1/r$

$$\frac{T_2 - T}{T_2 - T_1} = \frac{\ln \frac{R_2}{r}}{\ln \frac{R_2}{R_1}}$$

Note that $T(r)$ does not depend on the thermal conductivity, *k* (steady state)

Pipe with temperature BCs

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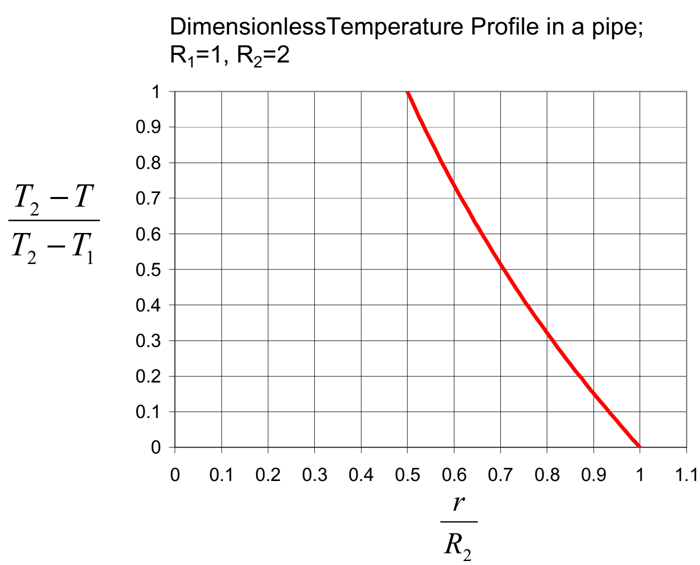

Pipe with temperature BCs

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