Example 5: Heat Conduction with Generation
What is the steady state temperature profile in a wire if heat is generated uniformly throughout the wire at a rate of \( S_e \) W/m³ and the outer radius is held at \( T_w \)?

\[ R \]

\[ T_w \]

\( S_e = \) energy production per unit volume

**Energy Balance**

\[
\begin{pmatrix}
\text{rate of energy accumulation} \\
\text{in}
\end{pmatrix}
\begin{pmatrix}
\text{rate of energy out} \\
\text{production}
\end{pmatrix}
= \begin{pmatrix}
\text{rate of energy in} \\
\text{out}
\end{pmatrix}
+ \begin{pmatrix}
\text{rate of energy production}
\end{pmatrix}
\]

*Conductive and convection terms - energy that passes through boundaries*

\[ e.g. \]
chemical reaction, electrical current

**conduction** - Fourier’s law

**convection** - due to flow
Example 5: Heat Conduction with Generation

Shell Balance: choose control volume

contributions to energy transport:
* generation
* radial conduction

Compare solutions

\[ \frac{T - T_e}{T_i - T_e} = \ln \frac{r}{R_i} \]

\[ \frac{T - T_w}{S_i R^2 / 4k} = 1 - \left( \frac{r}{R} \right)^2 \]