Heat Exchanger Effectiveness

To calculate \( Q \), we need both inlet and outlet temperatures:

\[
Q = U A \Delta T_m = U A (F_T \Delta T_{lm})
\]

\[
\Delta T_{lm} = \frac{(T'_1 - T_1) - (T'_2 - T_2)}{\ln \frac{(T'_1 - T_1)}{(T'_2 - T_2)}}
\]

What if the outlet temperatures are unknown? e.g. calculate the performance of a given heat exchanger.

To calculate unknown outlet temperatures:

Procedure:
1. guess outlet temperatures
2. calculate \( \Delta T_{lm} \), \( F_T \)
3. calculate \( Q \)
4. calculate \( Q \) from energy balance
5. compare, adjust, repeat.

This tedious procedure can be simplified by the definition of heat-exchanger effectiveness, \( \varepsilon \).
Heat Exchanger Effectiveness

Consider a counter-current double-pipe heat exchanger:

Energy balance cold side:

\[ Q_{in,\text{cold}} = Q = (mC_p)^{\text{cold}}(T_{co} - T_{ci}) \]

Energy balance hot side:

\[ Q_{in,\text{hot}} = -Q = (mC_p)^{\text{hot}}(T_{ho} - T_{hi}) \]

\[ \frac{(mC_p)^{\text{hot}}}{(mC_p)^{\text{cold}}} = \frac{T_{co} - T_{ci}}{-(T_{ho} - T_{hi})} = \frac{\Delta T_c}{\Delta T_h} \]

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Temperature profile in a double-pipe heat exchanger:

\[ \frac{T' - T}{T'_1 - T_1} = e^{\alpha_0 x} \]

\[ \alpha_0 = 2\pi RU \left( \frac{1}{C'_p m'} - \frac{1}{C'_p m} \right) \]

Note that the temperature curves are only approximately linear.
Case 1: \[ \begin{cases} \left( mC_p \right)_{\text{hot}} > \left( mC_p \right)_{\text{cold}} \\ \Delta T_C > \Delta T_h \end{cases} \] cold fluid = minimum fluid

We want to compare the amount of heat transferred in this case to the amount of heat transferred in a PERFECT heat exchanger.

If the heat exchanger were perfect, \( T_{hi} = T_{co} \)

cold side:
this temperature difference only depends on inlet temperatures

\[ Q_{A=\infty} = \left( mC_p \right)_{\text{cold}} (T_{hi} - T_{ci}) \]
Heat Exchanger Effectiveness, $\varepsilon$

$$\varepsilon \equiv \frac{Q}{Q_{A=\infty}}$$

$$\Rightarrow Q = \varepsilon \left( mC_p \right)_{\text{cold}} (T_{hi} - T_{ci})$$

cold fluid = minimum fluid

if $\varepsilon$ is known, we can calculate $Q$ without iterations

Case 2:

$$\begin{cases} 
\left( mC_p \right)_{\text{hot}} < \left( mC_p \right)_{\text{cold}} & \text{hot fluid = minimum fluid} \\
\Delta T_c < \Delta T_h 
\end{cases}$$

distance along the exchanger

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If the heat exchanger were perfect, $T_{hi}=T_{co}$

![Diagram showing temperature differences and heat flow]

Hot side:

- This temperature difference only depends on inlet temperatures.
- Distance along the exchanger.

\[ Q_{A=\infty} = (mC_p)_{hot} (T_{hi} - T_{ci}) \]

Heat Exchanger Effectiveness, $\epsilon$

\[ \epsilon = \frac{Q}{Q_{A=\infty}} \]

\[ \Rightarrow Q = \epsilon (mC_p)_{hot} (T_{hi} - T_{ci}) \]

Hot fluid = minimum fluid

In general,

\[ Q = \epsilon (mC_p)_{min} (T_{hi} - T_{ci}) \]

If $\epsilon$ is known, we can calculate $Q$ without iterations.

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But where do we get ε?

The same equations we use in the trial-and-error solution can be combined algebraically to give ε as a function of \( (mC_p)_\text{min}, \ (mC_p)_\text{max} \).

\[
\begin{align*}
\varepsilon &= \left(\frac{-UA}{(mC_p)_\text{min}} \left(1 - \frac{(mC_p)_{\text{min}}}{(mC_p)_{\text{max}}}\right)\right) \\
&\quad \cdot \left[1 - \left(\frac{(mC_p)_{\text{min}}}{(mC_p)_{\text{max}}}\right) e^{-\frac{-UA}{(mC_p)_{\text{min}}} \left(1 - \frac{(mC_p)_{\text{min}}}{(mC_p)_{\text{max}}}\right)}\right] \\
&\quad \cdot \left[1 - \frac{(mC_p)_{\text{min}}}{(mC_p)_{\text{max}}} e^{-\frac{-UA}{(mC_p)_{\text{min}}} \left(1 - \frac{(mC_p)_{\text{min}}}{(mC_p)_{\text{max}}}\right)}\right]
\end{align*}
\]

This relation is plotted in Geankoplis, as is the relation for co-current flow.

Heat Exchanger Effectiveness for Double-pipe or 1-1 Shell-and-Tube Heat Exchangers

counter-current

counter-current co-current

\[ C_{\text{min}} = (mC_p)_{\text{min}} \]

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3. (25 points) Water flowing at a rate of 0.723 kg/s enters the inside of a countercurrent, double-pipe heat exchanger at 300K and is heated by an oil stream that enters at 385K at a rate of 3.2 kg/s. The heat capacity of the oil is 1.89 kJ/kg K, and the average heat capacity of water over the temperature range of interest is 4.192 kJ/kg K. The overall heat-transfer coefficient of the exchanger is 300 W/m² K, and the area for heat transfer is 15.4 m². What is the total amount of heat transferred?