More Complicated Flows III: Boundary-Layer Flow

(plus Miscellaneous topics)

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More complicated flows II

Powerful:
Solving never-before-solved problems.

Left to explore:

• What is non-creeping flow like? (boundary layers)
• Viscosity dominates in creeping flow, what about the flow where inertia dominates? (potential flow)
• What about mixed flows (viscous+inertial)? (boundary layers)
• What about really complex flows (curly)? (vorticity, irrotational+circulation)
More complicated flows II

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**graphical** correlation

Steady flow of an incompressible, Newtonian fluid around a sphere

\[ \frac{24}{Re} = \text{Creeping flow} \]
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What does non-creeping flow look like?

Text, Figure 8.22, p649, from Sakamoto and Haniu, 1990

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Steady flow of an incompressible, Newtonian fluid around a sphere

McCabe et al., Unit Ops of Chem Eng, 5th edition, p147

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Nondimensional Navier-Stokes Equation:

\[
\frac{\partial \mathbf{v}^*}{\partial t^*} + (\mathbf{v} \cdot \nabla) \mathbf{v}^* = -\nabla^* P + \frac{\mu}{\rho V D^2} (\nabla^2 \mathbf{v})^* + \frac{g D}{V^2} \mathbf{g}^*
\]

With the appropriate terms in spherical coordinates

Consider the high Re limit:

\[
\frac{\partial \mathbf{v}^*}{\partial t^*} + (\mathbf{v} \cdot \nabla) \mathbf{v}^* = -\nabla^* P + \frac{1}{Re} (\nabla^2 \mathbf{v})^* + \frac{1}{Fr^2} \mathbf{g}^*
\]

\(Re \to \infty\)

Now solve for a sphere

No free surfaces
Potential flow around a Sphere (high Re, no viscosity)

\[ \nabla^* \cdot \mathbf{v}^* = 0 \]

\[ \frac{\partial \mathbf{v}^*}{\partial t^*} + (\mathbf{v} \cdot \nabla^*) \mathbf{v}^* = -\frac{\partial P^*}{\partial z^*} \]

\[ C_D = \frac{2}{\pi} \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} [-P^* \cos \theta]_{r^*=r} \sin \theta d\theta d\phi \]

Solution:

\[ \mathbf{v} = \begin{pmatrix} v_v \left(1 - \frac{R^3}{r^3}\right) \cos \theta \\ -v_v \left(1 + \frac{R^3}{r^3}\right) \sin \theta \\ 0 \end{pmatrix} \]

\[ P(r, \theta) = P_v + \frac{1}{2} \rho v_v^2 \left( 2 \left(\frac{R^3}{r^3}\right) \left(1 - \frac{3}{2} \sin^2 \theta\right) - \left(\frac{R}{r}\right)^6 \left(1 - \frac{3}{4} \sin^2 \theta\right) \right) \]

How does this compare to what we see at high Re?
Potential flow around a Sphere (high Re, no viscosity)

Solution:

How does this compare to what we see at high Re?
Potential flow around a Sphere (high Re, no viscosity)

Solution:

\[ \nabla^* \cdot v^* = 0 \]
\[ \frac{\partial v^*}{\partial t^*} + (v \cdot \nabla) v^* = -\frac{1}{\rho^*} \nabla^* P^* - \nabla \Phi^* \]
\[ C_D = \frac{2}{\pi} \int_0^\pi \int_0^{2\pi} [\frac{1}{\rho^*} \frac{1}{2} \rho^* v^2 \sin^2 \theta] \, d\theta \, d\phi \]

(equation 8.203)

Wrong!

Wrong!
**Potential flow around a Sphere** (high Re, no viscosity)

**Wrong!**

\[
\frac{d\mathbf{v}^*}{dt} = (\mathbf{v} \cdot \nabla)\mathbf{v}^* = -\frac{\partial p^*}{\partial x}
\]

Predicts:

- No drag (d’Alembert’s paradox)
- Slip at the wall
- Approximately right pressure profile (near the wall)
- Right velocity field away from the wall

\[
P(r, \theta) = P_0 + \frac{1}{2} \rho v_0^2 \left( 2 \left( \frac{R^2}{r^3} \right) \left( 1 - \frac{3}{2} \sin^2 \theta \right) - \left( \frac{R^2}{r^2} \right) \left( 1 - \frac{3}{4} \sin^2 \theta \right) \right)
\]

(equation 8.238-9)
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Prandtl’s Great Idea (1904):

- Keep the good parts of the potential flow solution: $v$ in free stream, $p(r, \theta)$ near the surface
- Throw away the bad parts: slip at the wall
- Solve a new problem near the wall with $p(r, \theta)$ from the potential-flow solution

Boundary Layer Theory

Boundary Layer Theory

- Choose simplest boundary layer (flat plate)
- Nondimensionalize Navier-Stokes
- Eliminate small terms
- Solve

$$
\rho \left( \frac{\partial v}{\partial t} + v \cdot \nabla v \right) = -\nabla p + \mu \nabla^2 v + \rho g
$$

Characteristic values:

- $U$ in principal flow direction $v_1$
- $V$ in direction perpendicular to wall, $v_2$
- $L$ length of plate for $x_1$
- $\delta$ boundary layer thickness for $x_2$

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More complicated flows III

**Boundary Layer Theory**

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Note that for this flow, two length scales and two velocities were found to be appropriate for the dimensional analysis.

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**It works!**

**Boundary Layer Theory**

- Apply to uniform flow approaching a sphere

**Boundary layer velocity profiles as you progress from the stagnation point (0°) to the top of the sphere (90°) and beyond.**
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Boundary Layer Theory

• Explains boundary-layer separation
• Golf ball problem
• BL separation caused by adverse pressure gradient

The pressure distribution is like a storage mechanism for momentum in the flow; as other momentum sources die out, the pressure drives the flow.

H. Schlichting, Boundary Layer Theory (McGraw-Hill, NY 1955.)
What did we do?

Same strategy as:

- Turbulent tube flow
- Noncircular conduits
- Drag on obstacles

1. Find a simple problem that allows us to identify the physics
2. Nondimensionalize
3. Explore that problem
4. Take data and correlate
5. Solve real problems

Solve Real Problems.
Powerful.
More complicated flows II

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See text

CM3110
Transport I
Part I: Fluid Mechanics

**Miscellaneous Topics**

*Fluidized Beds*

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ChemE Application of Ergun Equation

Fluidized beds

- ion exchange columns
- packed bed reactors
- packed distillation columns
- filtration
- flow through soil (environmental issues, enhanced oil recovery)
- fluidized bed reactors

Image from: fluidizedbed2008.webs.com

ChemE Application of Ergun Equation

Calculate the minimum superficial velocity at which a bed becomes fluidized.

In a fluidized bed reactor, the flow rate of the gas is adjusted to overcome the force of gravity and fluidize a bed of particles; in this state heat and mass transfer is good due to the chaotic motion.

The $\Delta p$ vs $Q$ relationship can come from the Ergun eqn at small $Re_p$

$\frac{150}{Re_p} + 1.75 = f_p$

note: $Re_p$ vs $Re_{pH}$

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Now we perform a force balance on the bed:

$$m_a = \sum f$$

When the forces balance, *incipient fluidization* is achieved. The net effect of gravity and buoyancy is:

$$\rho_v - \rho \frac{(1 - \varepsilon)ALg}{\rho_v}$$

**Flow Through Noncircular Conduits – All Flow Regimes**

Ergun equation:

$$\frac{100}{3} \frac{f_{DH}}{Re_{DH}} + \frac{1.75}{3} = f_{DH}$$

Note: $Re_p$ vs $Re_{DH}$

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When the forces balance, *incipient fluidization*

\[
\Delta p A = (\rho_p - \rho)(1 - \varepsilon)ALg
\]

\[
\frac{150}{Re_p} = f_p
\]

velocity at the point of *incipient fluidization*

Complete solution steps in Denn, Process Fluid Mechanics (Prentice Hall, 1980)

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Transport I

Part I: Fluid Mechanics

**Miscellaneous Topics**

**Compressible Flow**

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Compressible Fluids

• most fluids are somewhat compressible
• in chemical-engineering processes, compressibility is unimportant at most operating pressures
• even gases may be modeled as incompressible if $\Delta p < p_{\text{mean}}$

EXCEPT: When the fluid velocity approaches the speed of sound

How is pressure information transmitted in liquids and gases?

The Hydraulic Lift operates on Pascal’s principle

Pressure exerted on an enclosed liquid is transmitted equally to every part of the liquid and to the walls of the container.
The Hydraulic Lift operates on Pascal’s principle:

\[ P \text{ (pressure)} = \text{constant} \]

Pressure exerted on an enclosed liquid is transmitted equally to every part of the liquid and to the walls of the container. and essentially, instantaneously.

For static incompressible liquids, the law of conservation of mass states:

\[ \frac{V^2}{p} = \text{constant} \]

For static compressible fluids (gases), pressure causes volume change:

\[ \frac{N}{p} = \text{constant} \]

\[ \frac{N}{p + \Delta p} = \text{constant} \]

\[ \frac{N}{p + \Delta p} = \frac{V}{V + \Delta V} \]
For moving \textit{incompressible} liquids and gases,

The presence of the obstacle is felt by the upstream fluid (pressure) and that information is transmitted very rapidly throughout the fluid. The streamlines adjust according to momentum conservation.

For \textit{compressible fluids} moving near sonic speeds, information (pressure) and the gas itself are moving at comparable speeds.

Pressure piles up at the shock wave.
**Compressible Fluids**

Velocity of a fluid = variable = supersonic, sonic, subsonic

Velocity of a pressure wave = constant = speed of sound

A shock forms where the pressure waves from the obstacle stack up, and the speed of the pressure wave traveling upstream equals the speed of the fluid traveling downstream.

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**Relief Valves (Safety Valves)**

The rapid flows in relief valves can become sonic.

For supersonic flow, the flow rate is constant no matter what the pressure drop is.

(pressure waves pile up)

Choked flow can be understood from basic equations of compressible fluid mechanics.

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Momentum and Energy in Compressible Fluids

Microscopic momentum balance:

$$\rho \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p + \nabla \cdot \mathbf{f} + \rho \mathbf{g}$$

Mechanical energy balance:

$$\frac{\Delta p}{\rho} + \frac{\Delta (\mathbf{v}^2)}{2\alpha} + g\delta z + F = \frac{W_{s,on}}{m}$$
Mechanical energy balance (compressible)

Back up one step in the derivation and reintegrate without constant $\rho$ assumption.

$$\frac{dp}{\rho} + VdV + gdz + dF = \frac{dW_{s,out}}{m}$$

Assume:
• constant cross section
• constant mass flow $\rho VA = GA$
• neglect gravity
• no shaft work

$G \equiv \rho V = \text{mass velocity}$

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Ideal Gas Law

$$pV = NRT$$
$$\frac{V}{N} = \frac{RT}{p}$$
$$\frac{V}{MN} = \frac{RT}{pM}$$
$$\frac{1}{\rho} = \frac{RT}{pM}$$

For isothermal flow:

$$p_1V_1 = NRT$$
$$p_2V_2 = NRT$$

$$\frac{p_1}{p_2} = \frac{V_2}{V_1}$$

Also,

$$\frac{\rho_{av}}{p_{av}} = \frac{M}{RT}$$

$$\frac{2\rho_{av}}{p_1 + p_2} = \frac{M}{RT}$$
The compressible MEB predicts that there is a maximum velocity at

\[ V_{\text{max}} = \sqrt{\frac{p_2}{\rho_2}} = \sqrt{\frac{RT}{M}} = \text{isothermal speed of sound} \]

A better assumption than isothermal flow is adiabatic flow (no heat transferred). For this case,

\[ V_{\text{max}} = \sqrt{\frac{\gamma p_2}{\rho_2}} = \sqrt{\frac{\gamma RT}{M}} = \text{adiabatic speed of sound} \]

\[ \gamma = \frac{C_p}{C_v} \]

(see book)
Numerical PDE Solving with Comsol 5.1

www.comsol.com

Finite-element numerical differential equation solver. Applications include fluid mechanics and heat transfer.

1. Choose the physics (2D, 2D axisymmetric, laminar, steady/unsteady, etc.)
2. Choose flow geometry and fluid (shape of the flow domain)
3. Define boundary conditions
4. Design and generate mesh
5. Solve the problem
6. Calculate and plot engineering quantities of interest.
Launch the program

Choose the physics

(Actually, these screen shots are from Comsol 4.2)
Choose flow geometry and fluid

Comsol Multiphysics 5.1

(Actually, these screen shots are from Comsol 4.2)

Define boundary conditions

Comsol Multiphysics 5.1

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Comsol Multiphysics 5.1

Design and generate mesh

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Comsol Multiphysics 5.1

Solve the problem

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View the solution

Comsol Multiphysics 5.1

Calculate engineering problems of interest

(actually, these screen shots are from Comsol 4.2)
Comsol project:
- Due last day of classes
- Individual work
- 2 points for part 1 (instructions given)
- 3 points for part 2 (no instructions)
- Coming soon

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Transport I
Part II: Heat Transfer

Introduction to Heat Transfer

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