Radiation Heat Transfer

• In Unit Operations
• Heat Shields

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Summary (Part 2 thus far)

Within homogeneous phases:
- Microscopic Energy Balances
- 1D Steady solutions

rectangular:
\[
\frac{q_x}{A} = C_1 \quad T = ax + b
\]

cylindrical:
\[
\frac{q_r}{A} = \frac{C_1}{r} \quad T = a \ln r + b
\]

- Temperature and Newton's law of cooling boundary conditions
  (if h is supplied)
- Unsteady solutions (from literature)
  ✓ Carslaw and Jeager
  ✓ Heisler charts

\[ \rho c_p \left( \frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T \right) = k \nabla^2 T + S_e \]

conduction

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Across phase boundaries:

- Microscopic Energy, Momentum, and Mass Balances
  
  Micro momentum:
  \[
  \rho \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p + \mu \nabla^2 \mathbf{v} + \rho g
  \]

  Micro energy:
  \[
  \rho C_p \left( \frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T \right) = k \nabla^2 T + S_e
  \]

- Simultaneous effects (complex)
- Solutions are difficult to obtain (and often not really necessary)
  
  ⇒ use dimensional analysis and expts to obtain \( h \)

- Data correlations for:
  - forced convection
  - natural convection
  - evaporation/condensation
  - radiation

One more type of heat transfer

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**Radiation versus Conduction and Convection**

Continuum view

- Conduction is caused by macroscopic temperature gradients
- Convection is caused by macroscopic flow
- Radiation? NO CONTINUUM EXPLANATION
Continuum versus Molecular description of matter

A continuum is infinitely divisible

Real matter is not a continuum; at small enough length scales, molecules are discrete.

Radiation versus Conduction and Convection

**Continuum view**
- Conduction is caused by macroscopic temperature gradients
- Convection is caused by macroscopic flow
- Radiation? NO CONTINUUM EXPLANATION

**Molecular view**
- Conduction—Brownian motion
- Convection—flow
- Radiation is caused by changes in electron energy states in molecules and atoms

There is also, of course, a molecular explanation of these effects, since we know that matter is made of atoms and molecules.
Individual molecules carry:
- chemical identity
- macroscopic velocity (speed and direction)
- internal energy (Brownian velocity)

When they undergo Brownian motion within an inhomogeneous mixture, they cause:
- diffusion (mass transport)
- exchange of momentum (viscous transport)
- conduction (energy transport)

Kinetic Theory
J. C. Maxwell, L. Boltzmann, 1860

- Molecules are in constant motion (Brownian motion)
- Temperature is related to $E_{l, av}$ of the molecules

Simplest model
- no particle volume
- no intermolecular forces

More realistic model
- finite particle volume
- intermolecular forces
Radiation Heat Transfer is related to these non-Brownian mechanisms.

But, there is more to molecular energy than just Brownian motion...

- In atoms and molecules, electrons can exist in multiple, discrete energy states
- Transfers between energy states are accompanied by an emission of radiation

Energy levels

Molecular view

Quantum Mechanics


Molecular view

Kinetic Theory

Is based on Brownian motion (molecules in constant motion proportional to their temperature)

Predicts that properties that are carried by individual molecules (chemical identity, momentum, average kinetic energy) will be transported \textbf{DOWN} gradients in these quantities.

\Rightarrow \text{Gradient transport laws are due to Brownian motion}

Heat Transfer by Radiation

Is due to the release of energy stored in molecules that is \textbf{NOT} related to average kinetic energy (temperature), but rather to changing populations of excited states.

\Rightarrow \text{Radiation is NOT a Brownian effect}
How does this relate to chemical engineering?

Consider a furnace with an internal blower:

There is heat transfer due to convection:

\[ q_{\text{convection}} = hA(T_s - T_b) \]

There is also heat transfer due to radiation.

\[ q_{\text{total}} = q_{\text{conv}} + q_{\text{rad}} \]

Where do we get \( q_{\text{rad}} \)?

Where do we get \( q_{\text{radiation}} \)?

\[ q_{\text{total}} = q_{\text{conv}} + q_{\text{rad}} \]

Answer:

We need to look into the physics of this mode of heat transfer.
Radiation

• does not require a medium to transfer energy (works in a vacuum)
• travels at the speed of light, \( c = 3 \times 10^{10} \text{ cm/s} \)
• travels as a wave; differs from x-rays, light, only by wavelength, \( \lambda \)
• radiation is important when temperatures are high

examples:
• the sun
• home radiator
• hot walls in vacuum oven
• heat exchanger walls when \( \Delta T \) is high and a vapor film has formed

\[
\frac{q}{A} \propto T^4
\]

Note: absolute temperature units

Why is radiation flux related to temperature and not to something else?

(From kinetic theory, temperature is related to average kinetic energy)

Answer:

• As a molecule gains energy, it both speeds up (increases average kinetic energy) and increases its population of excited states.
• The increase in average kinetic energy is reflected in temperature (directly proportional), and heat transfer through conduction.
• The increase in number of electrons in excited states is reflected in increased radiation heat flux. Electrons enter excited states in proportion to absolute \( T^4 \).
What causes energy transfer by radiation?

- Energy hits surface
- Pushes some molecules into an excited state
- When the molecules/atoms relax from the excited state, they emit radiation

\[
\frac{q_{\text{emit}}}{A} \propto T^4
\]

\[
\alpha = \text{absorptivity}
\]

\[
\alpha = \frac{q_{\text{absorbed}}}{q_{\text{incident}}} < 1
\]
Absorption

\[ \alpha = \text{absorptivity} \]

\[ \alpha = \frac{q_{\text{absorbed}}}{q_{\text{incident}}} < 1 \]

In general, absorptivity \( \alpha \) is a function of wavelength:

\[ \alpha = \alpha (\lambda) \]

gray body: a body for which \( \alpha \) is constant (does not depend on \( \lambda \))
black body: a body for which \( \alpha = 1 \), i.e. absorbs all incident radiation

Kirchhoff’s Law: emissivity equals absorptivity at the same temperature:

\[ \alpha = \varepsilon \]

the fraction of energy absorbed by a material

the relative amount of energy emitted from that material compared to a black body

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**Black Bodies**

*Stefan-Boltzmann Law*: the amount of energy emitted by a black body is proportional to \( T^4 \)

\[
\frac{q_{\text{emitted}, \text{black body}}}{A} = \sigma T^4
\]

\[ \sigma = 0.1712 \times 10^{-8} \text{ BTU ft}^2 \text{R}^4 \]
\[ \sigma = 5.676 \times 10^{-8} \text{ W m}^2 \text{K}^4 \]

**Non-Black Bodies**

\[ \varepsilon = \text{emissivity} \]
\[ \varepsilon = \frac{q_{\text{emitted}}}{q_{\text{emitted, black body}}} \]

\[
\frac{q_{\text{emitted, non-black body}}}{A} = \varepsilon \frac{q_{\text{emitted, black body}}}{A} = \varepsilon \sigma T^4
\]

Energy emitted by a non-black body

\[
\frac{q_{\text{emitted, non-black body}}}{A} = \varepsilon \sigma T^4
\]
**Radiation**

**Summary:**

- Absorptivity, $\alpha$
  - gray body: $\alpha = \text{constant}$
  - black body: $\alpha = 1$
- Emissivity, $\varepsilon$
  $$q_{\text{emit}} = \varepsilon q_{\text{emit blackbody}}$$
- Kirchoff’s law: $\alpha = \varepsilon$
- Stefan-Boltzman law
  $$\frac{q_{\text{emit blackbody}}}{A} = \sigma T^4$$

$$\sigma = 0.1712 \times 10^{-8} \frac{\text{BTU}}{\text{h ft}^2 \text{R}^3}$$

$$\sigma = 5.676 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \text{K}^4}$$

---

**How does this relate to chemical engineering?**

Consider a furnace with an internal blower:

There is heat transfer due to convection:

$$q_{\text{conv}} = h A (T_s - T_b)$$

(Use correlations)

There is also heat transfer due to radiation:

$$q_{\text{rad}} = h_{\text{rad}} A (T_s - T_b)$$

$$q_{\text{total}} = q_{\text{conv}} + q_{\text{rad}}$$

$$q_{\text{total}} = (h_{\text{conv}} + h_{\text{rad}}) A (T_s - T_b)$$

Because these two types of physics do not interact, we can just add the effects.
How does this relate to chemical engineering?

Consider a furnace with an internal blower:

There is heat transfer due to convection:

\[ q_{\text{convection}} = h A (T_s - T_b) \]

(Use correlations)

There is also heat transfer due to radiation:

\[ q_{\text{radiation}} = h_{\text{rad}} A (T_s - T_b) \]

Because these two types of physics do not interact, we can just add the effects.

\[ q_{\text{total}} = q_{\text{conv}} + q_{\text{rad}} \]

\[ q_{\text{total}} = (h_{\text{conv}} + h_{\text{rad}}) A (T_s - T_b) \]

Where do we get \( h_{\text{rad}} \)?

Where do we get \( h_{\text{rad}} \)?

\[ \frac{q_{\text{emit.body}}}{A} = \varepsilon \sigma T_{\text{body}}^4 \]

\[ \frac{q_{\text{emit.walls}}}{A} = \sigma T_s^4 \]

black body, \( \varepsilon_{\text{walls}} \approx 1 \)

net flux to object:

\[ \frac{q_{\text{net}}}{A} = \frac{q_{\text{absorbed}}}{A} - \frac{q_{\text{emit}}}{A} \]

Net flux to the body = Heat/area absorbed - Heat/area emitted
Heat Lectures 10-11 CM3110

Where do we get $h_{\text{rad}}$?

\[
\frac{q_{\text{emit.body}}}{A} = \varepsilon T_{\text{body}}^4
\]

net flux to object:

\[
\frac{q_{\text{net}}}{A} = \frac{q_{\text{absorbed}}}{A} - \frac{q_{\text{emit}}}{A}
\]

fraction absorbed

frac{q_{\text{absorbed.body}}}{A} = \alpha

Incident energy

\[
\frac{q_{\text{emit.body}}}{A} = \varepsilon \frac{T_{\text{body}}^4}{T_s}
\]

energy emitted by walls, which are acting as a black body

\[
\frac{q_{\text{emit.body}}}{A} = \alpha \frac{T_{\text{body}}^4}{T_s}
\]

\[
\frac{q_{\text{emit.body}}}{A} = \varepsilon \left( T_{\text{body}}^4 - T_s^4 \right)
\]

Object is a gray body,

$\varepsilon_{\text{body}} = \varepsilon$

assuming: $\varepsilon_{T_s} = \varepsilon_{T_{\text{body}}}$
Where do we get $h_{rad}$?

Net flux to object:

$$\frac{q_{net}}{A} = \frac{q_{absorbed}}{A} - \frac{q_{emit}}{A}$$

Net flux to the body = Heat/area absorbed - Heat/area emitted

Finally, calculate $h_{rad}$

Net energy absorbed:

$$q_{net} = A\varepsilon \bigg|_{T_s} \sigma (T_s^4 - T_{body}^4)$$

assuming: $\varepsilon|_{T_s} \approx \varepsilon|_{T_{body}}$

Equating with expression for $h$:

$$h_{rad}A(T_s - T_b) = A\varepsilon \bigg|_{T_s} \sigma (T_s^4 - T_{body}^4)$$

$$h_{rad} = \frac{\varepsilon |_{T_s} \sigma (T_s^4 - T_{body}^4)}{(T_s - T_{body})}$$

Geankoplis 4th ed., eqn 4.10-10 p304
Example: Geankoplis 4.10-3

A horizontal oxidized steel pipe carrying steam and having an OD of 0.1683m has a surface temperature of 374.9 K and is exposed to air at 297.1 K in a large enclosure. Calculate the heat loss for 0.305 m of pipe.

\[ \varepsilon_{steel} = 0.79 \]

Answers:
\[ h_{\text{radiation}} = 6.9W/m^2K \]
\[ h_{\text{convection}} = 6.1W/m^2K \]
\[ Q = 163W \]
One final topic: **Radiation Heat Transfer Between Two Infinite Plates**

Consider a quantity of radiation energy that is emitted from surface 1.

**Left plate at** $T_1$
1. emit
2. reflect
3. absorb
4. emit

**Right plate at** $T_2$
5. absorb
6. reflect
7. emit

Reflected = not absorbed

**See:** Geankoplis, section 4.11B
**Also:** Bird, Stewart, and Lightfoot, "Transport Phenomena" 1960 Wiley PP446-448

Quantity of energy **incident** at surface 2:

$$\frac{q_{1-2}}{A} = \varepsilon_1 \sigma T_1^4$$

Quantity of energy **absorbed** at surface 2:

$$\alpha_2 \left( \frac{q_{1-2}}{A} \right) A = \varepsilon_2 \left( \varepsilon_1 \sigma T_1^4 \right) A$$

$$\alpha_2 = \varepsilon_2$$

Quantity of energy **reflected** from surface 2:

$$\left( 1 - \varepsilon_2 \right) \left( \varepsilon_1 A \sigma T_1^4 \right)$$

This energy goes back to surface 1.
Radiation Heat Transfer
Between Two Infinite Plates

Quantity of energy absorbed at surface 1 (second round):
\[
\varepsilon_1 \left( 1 - \varepsilon_2 \right) \left( \varepsilon_1 A \sigma T_1^4 \right)
\]
fraction absorbed
incident energy

Quantity of energy reflected from surface 1 (second round):
\[
(1 - \varepsilon_1) \left[ (1 - \varepsilon_2) \left( \varepsilon_1 A \sigma T_1^4 \right) \right]
\]
fraction reflected (not absorbed)
incident energy

Third round – surface 2

Quantity of energy absorbed at surface 2 (third round):
\[
\varepsilon_2 \left( 1 - \varepsilon_1 \right) \left( 1 - \varepsilon_2 \right) \left( \varepsilon_1 A \sigma T_1^4 \right)
\]
fraction absorbed
incident energy

Quantity of energy reflected from surface 2 (third round):
\[
(1 - \varepsilon_2) \left[ (1 - \varepsilon_1) \left( 1 - \varepsilon_2 \right) \left( \varepsilon_1 A \sigma T_1^4 \right) \right]
\]
fraction reflected (not absorbed)
incident energy

There is a pattern.
Now, calculate the radiation energy going from surface 1 to surface 2:

\[ q_{1-2} = \sum (\text{energy absorbed at surface 2}) \]

\[ = \varepsilon_2 (\varepsilon_1 A \sigma T_1^4) \]

\[ + \varepsilon_2 (1 - \varepsilon_1)(1 - \varepsilon_2)(\varepsilon_1 A \sigma T_1^4) \]

\[ + \varepsilon_2 (1 - \varepsilon_1)^2 (1 - \varepsilon_2)^2 (\varepsilon_1 A \sigma T_1^4) \]

\[ \ldots + \varepsilon_2 (1 - \varepsilon_1)^n (1 - \varepsilon_2)^n (\varepsilon_1 A \sigma T_1^4) + \ldots \]

Later, calculate energy from 2 to 1; then subtract to obtain net energy transferred.

Radiation energy going from surface 1 to surface 2:

\[ q_{1-2} = \varepsilon_1 \varepsilon_2 A \sigma T_1^4 \sum_{n=0}^{\infty} (1 - \varepsilon_1)^n (1 - \varepsilon_2)^n \]

How can we calculate \( \sum_{n=0}^{\infty} x^n \)?

Answer: \( 1 / (1 - x) \)

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### Radiation Heat Transfer Between Two Infinite Plates

Radiation energy going from surface 1 to surface 2:

\[ q_{1-2} = \frac{\varepsilon_1\varepsilon_2 A \sigma T_1^4}{1 - [ (1 - \varepsilon_1)(1 - \varepsilon_2) ]} \]

\[ = \frac{\varepsilon_1\varepsilon_2 A \sigma T_1^4}{1 - [ 1 - \varepsilon_1 - \varepsilon_2 + \varepsilon_1\varepsilon_2 ]} = \frac{\varepsilon_1\varepsilon_2 A \sigma T_1^4}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1\varepsilon_2} \]

Final Result

\[ q_{1-2} = \frac{\sigma T_1^4}{A} \cdot \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} \]

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Radiation energy going from surface 1 to surface 2:

\[ q_{1-2} = \frac{\sigma T_1^4}{A} \cdot \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} \]

Radiation energy going from surface 2 to surface 1:

\[ q_{2-1} = \frac{\sigma T_2^4}{A} \cdot \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} \]

NET Radiation energy going from surface 1 to surface 2:

\[ q_{1-2} - q_{2-1} = \frac{\sigma (T_1^4 - T_2^4)}{A} \cdot \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} \]

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Purpose of Heat Shields:
To reduce the amount of energy transfer from (hotter) plate at $T_1$ to second (cooler) plate at $T_3$.

Note:
$q_{net,1\rightarrow 2} = q_{net,2\rightarrow 3} = q$

Analysis of Radiation Shields

We will assume that the emissivity is the same for all surfaces.

\[
\frac{q_{net,1\rightarrow 2}}{A} = \frac{\sigma \left( T_1^4 - T_2^4 \right)}{\left( \frac{1}{\varepsilon} + \frac{1}{\varepsilon} - 1 \right)}
\]

\[
\frac{q_{net,2\rightarrow 3}}{A} = \frac{\sigma \left( T_2^4 - T_3^4 \right)}{\left( \frac{1}{\varepsilon} + \frac{1}{\varepsilon} - 1 \right)}
\]

Now we eliminate $T_2$ between these equations.

Note:
$q_{net,1\rightarrow 2} = q_{net,2\rightarrow 3} = q$
Analysis of Radiation Shields

\[ \frac{q}{A} = \frac{\sigma}{\varepsilon} \left( T_1^4 - T_2^4 \right) \]

\[ \frac{q}{A} = \frac{\sigma}{\varepsilon} \left( T_2^4 - T_3^4 \right) \]

\[ T_2^4 = \frac{q}{\sigma A} \left( \frac{2}{\varepsilon} - 1 \right) + T_3^4 \]

\[ \frac{q}{\sigma A} \left( \frac{2}{\varepsilon} - 1 \right) = T_1^4 - \frac{q}{\sigma A} \left( \frac{2}{\varepsilon} - 1 \right) - T_3^4 \]

\[ \frac{2q}{\sigma A} \left( \frac{2}{\varepsilon} - 1 \right) = T_1^4 - T_3^4 \]

\[ q = \left( \frac{1}{2} \right) \sigma \left( T_1^4 - T_3^4 \right) \]

With one heat shield present, \( q \) falls by half compared to no heat shield.

by the same analysis,

\[ \frac{q}{A} = \left( \frac{1}{N + 1} \right) \sigma \left( T_1^4 - T_3^4 \right) \]

With \( N \) heat shields present, \( q \) falls by a factor of \( 1/N \) compared to no heat shield.
**Radiation**

**Summary:**

\[
\begin{align*}
\sigma &= 0.1712 \times 10^{-8} \text{ RTU} \\
\sigma &= 5.676 \times 10^{-8} \text{ W m}^{-2}\text{K}^{-4}
\end{align*}
\]

- Absorptivity, \( \alpha \)
  - gray body: \( \alpha = \text{constant} \)
  - black body: \( \alpha = 1 \)
- Emissivity, \( \varepsilon \)
  \[ q_{\text{emit}} = \varepsilon q_{\text{emit,blackbody}} \]
- Kirchoff's law: \( \alpha = \varepsilon \)
- Stefan-Boltzman law
  \[ q_{\text{emit,blackbody}} = \sigma T^4 \]

**General properties:**

- **Heat transfer coefficient:**
  \[ h_{\text{Rad}} = \varepsilon T_{\text{source}} \sigma \left( T_{\text{source}}^4 - T_{\text{body}}^4 \right) \]

**Heat shields:**

\[
q = \left( \frac{1}{N+1} \right) \sigma \left( T_{1}^4 - T_{3}^4 \right) \left( \frac{2}{\varepsilon - 1} \right)
\]

Always use absolute temperature (Kelvin) in radiation calculations.

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**CM3110**

Transport Processes and Unit Operations I

**Part 2:**

**Professor Faith Morrison**

Department of Chemical Engineering
Michigan Technological University

CM3110 - Momentum and Heat Transport
CM3120 – Heat and Mass Transport

www.chem.mtu.edu/~fmorriso/cm310/cm310.html

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Summary

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  - rectangular: \[ \frac{q_x}{A} = \frac{C_1}{T} = aT + b \]
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- Temperature and Newton's law of cooling boundary conditions
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Micro energy:
\[ \rho \mathcal{C}_p \left( \frac{\partial T}{\partial t} + v \cdot \nabla T \right) = k \nabla^2 T + S_e \]

Across phase boundaries:

- Microscopic Energy, Momentum, and Mass Balances
- Simultaneous effects (complex)
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  - use dimensional analysis to obtain \( h \)
- \( h \) Data correlations for:
  - forced convection
  - natural convection
  - evaporation/condensation
  - radiation

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Summary

Heat Transfer Unit Operations

- Macroscopic energy balances
- Heat Exchangers
  - double pipe ($\Delta T_{in}$)
  - Shell-and-tube
  - Heat exchanger effectiveness
- Evaporators/Condensers
- Ovens
- Heat Shields