Mini-Exam II
CM 3110 8 October 2009

Note:
Significant figures count.
Please box your final answers.
Please be neat.

1. (50 points) For the apparatus shown in the figure below, an incompressible, Newtonian fluid is made to flow under an axial pressure gradient in the gap between two concentric cylinders. The flow is steady, and the inlet pressure is $P_0$ and the outlet pressure is $P_L$. The tube is of length $L$ and is horizontal. You may neglect gravity. The microscopic mass balance (continuity equation) and the microscopic momentum balance (Navier-Stokes equations) are shown on the next page. For each term in all four equations, cross out the terms that are zero and give a reason for each decision.

Side view: steady flow in annulus between cylinders

See next page
2.

\[ P_1 = 14.5 \text{ psig} \]
\[ \langle V \rangle_1 = 12.23 \text{ ft/s} \]
\[ z_1 = 0 \]

\[ P_2 = ? \]
\[ \frac{\Delta V}{2 \alpha} = 3.41 \text{ ft/s} \]
\[ z_2 = 0.5 \text{ ft} \]

Mechanical energy balance:

\[ \frac{\Delta P}{P} + \frac{\Delta V^2}{2 \alpha} + g \Delta z + \mathcal{F} = -\frac{W_{s,by}}{m} \]

\[ \alpha = 1 \]

\[ \frac{P_2 - P_1}{P} + \frac{\langle V \rangle_2^2 - \langle V \rangle_1^2}{2} + g (z_2 - z_1) = 0 \]

\[ P_2 = P \left[ (z_1 - z_2) g + \frac{\langle V \rangle_1^2 - \langle V \rangle_2^2}{2} \right] + P_1 \]
\[ P_2 = \left(62.25 \frac{lbm}{ft^3}\right) \left[ -(0.5 \text{ ft}) \left( \frac{32.174 \text{ ft}}{s^2} \right) \right. \\
\left. + \left( \frac{(12.23 \text{ ft})^2}{149.5729} - \left( \frac{3.61 \text{ ft}}{13.0321} \right)^2 \right)^{\frac{1}{2}} + 14.5 \text{ psi} \right] \\
\]

\[ P_2 = \left(62.25 \frac{lbm}{ft^3}\right) \left[ -16.087 \frac{ft^2}{s^2} + 68.2704 \left( \frac{ft^2}{s^2} \right) \right] + 14.5 \frac{lf}{in^2} \\
= 3248.8524 \frac{lbm}{ft \cdot s^2} \left( \frac{8^2 \text{ lb}_f}{32.174 \text{ ft} \cdot lbm} \right) \left( \frac{ft^2}{(12 \text{ in})^2} \right) + 14.5 \frac{lf}{in^2} \\
= 0.70113 \\
\]

\[ P_2 = 15.2 \text{ psi} \]