Types of Heat Transfer

$$\frac{q_x}{A} = -k\frac{dT}{dx}$$ •conduction (Fourier's Law)

$\underline{v}\cdot\nabla T$ •forced convection (due to flow)

S •source terms

$$\frac{Dv_z^*}{Dt} = \left(\nabla^2 v_z\right)^* + \mathrm{Gr}T^*$$ •free convection (fluid motion due to density variations brought on by temperature differences)

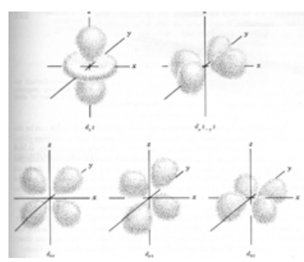
ΔH_{vap} •heat transfer with phase change (e.g. condensing fluids)

last subject in the course { **•radiation**

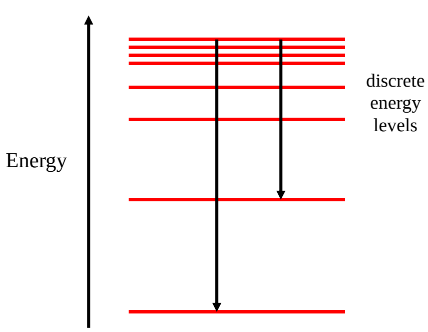
Heat transfer due to radiation

•in atoms and molecules electrons can exist in multiple, discrete energy states

•transfers between energy states are accompanied by an emission of radiation



Sienko and Plane, Chemistry: Principles and Applications, McGraw Hill, 1979



Quantum Mechanics

Radiation versus Conduction and Convection

Continuum view

- Conduction is caused by macroscopic temperature gradients
- Convection is caused by macroscopic flow
- Radiation? **NO CONTINUUM EXPLANATION**

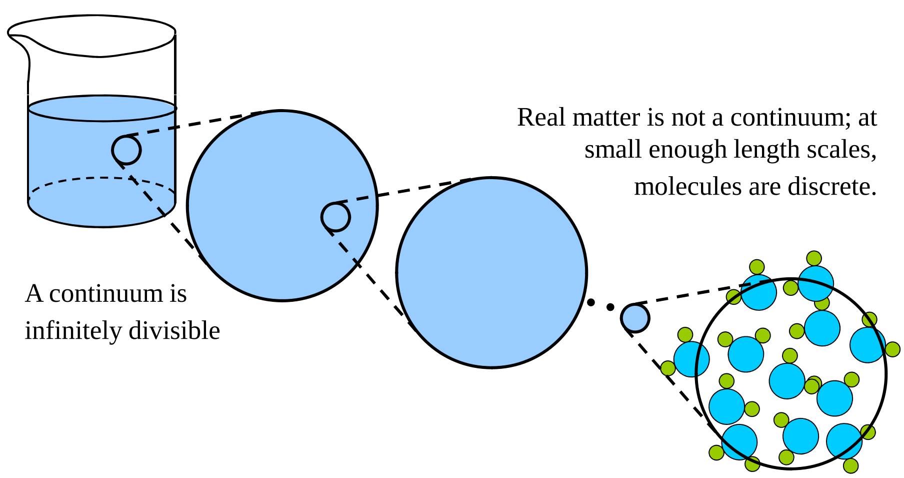
Molecular view

- Conduction?
- Convection?

There is, of course, a molecular explanation of these effects, since we know that matter is made of atoms and molecules

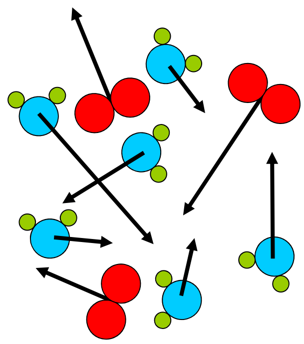
- Radiation is caused by changes in electron energy states in molecules and atoms

Continuum versus Molecular description of matter



Individual molecules carry:

- •chemical identity
- •macroscopic velocity (speed and direction)
- •internal energy (Brownian velocity)



When they undergo Brownian motion within an inhomogeneous mixture, they cause:

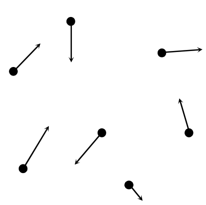
- •diffusion (mass transport)
- •exchange of momentum (momentum transport)
- •conduction (energy transport)

Kinetic Theory J. C. Maxwell, L. Boltzmann, 1860

- •Molecules are in constant motion (Brownian motion)
- •Temperature is related to $E_{k,av}$ of the molecules

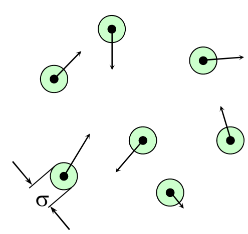
Simplest model

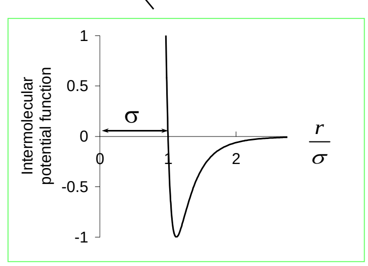
- •no particle volume
- •no intermolecular forces



More realistic model

- •finite particle volume
- •intermolecular forces





Kinetic Theory

Is based on Brownian motion (molecules in constant motion proportional to their temperature)

Predicts that properties that are carried by individual molecules (chemical identity, momentum, average kinetic energy) will be transported **DOWN** gradients in these quantities.

==> Transport laws are due to Brownian motion

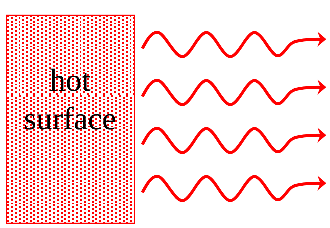
Heat Transfer by Radiation

Is due to the release of energy stored in molecules that is **NOT** related to average kinetic energy (temperature), but rather to the population of excited states.

==> Radiation is NOT a Brownian effect

Radiation

•does not require a medium to transfer energy (works in a vacuum)
•travels at the speed of light, c = 3 x 10^{10} cm/s
•travels as a *wave*; differs from x-rays, light, only by wavelength, λ
•radiation is important when temperatures are high



examples:
•the sun
•home radiator
•hot walls in vacuum oven
•heat exchanger walls when ΔT is high and a vapor film has formed

$$\frac{q}{A} \propto T^4$$

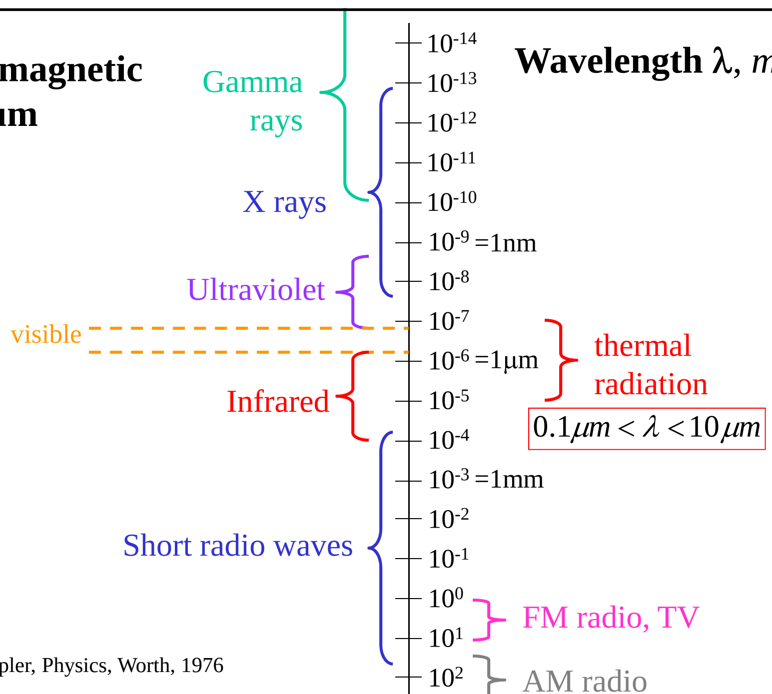
Why does radiation flux scale with temperature, which is related to average kinetic energy?

As a molecule gains energy, it both speeds up (increases average kinetic energy) and increases its population of excited states.

The increase in average kinetic energy is reflected in temperature (directly proportional).

The increase in number of electrons in excited states is reflected in increased radiation flux. Electrons enter excited states in proportion to T^4.

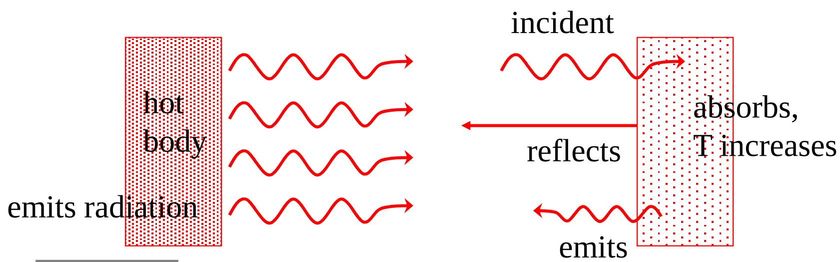
Electromagnetic Spectrum



from P. A. Tipler, Physics, Worth, 1976

What causes energy transfer by radiation?

- energy hits surface
- pushes some molecules into an excited state
- when the molecules/atoms relax from the excited state, they emit radiation



$$\frac{q_{emit}}{A} \propto T^4$$

$$\alpha = absorptivity$$

$$\alpha \equiv \frac{q_{absorbed}}{q_{incident}} < 1$$

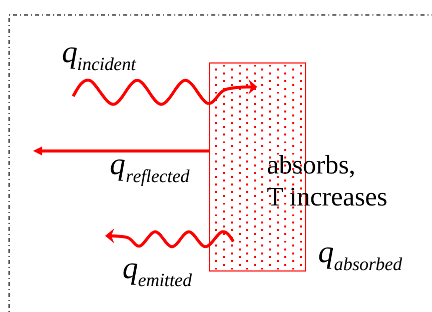
Absorption

$$\alpha = absorptivity$$

$$\alpha \equiv \frac{q_{absorbed}}{q_{incident}} < 1$$

In general, α is a function of wavelength

$$\alpha = \alpha(\lambda)$$



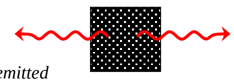
gray body: a body for which α is constant (does not depend on λ)
black body: a body for which α = 1, i.e. absorbs all incident radiation

Emission

$\varepsilon = emissivity$

$$\varepsilon \equiv \frac{q_{emitted}}{q_{emitted,black\,body}} < 1$$

gray body: a body for which α is constant
black body: a body for which α = 1



$q_{emitted}$

$\alpha = absorptivity$

$$\alpha \equiv \frac{q_{absorbed}}{q_{incident}} < 1$$

Kirchhoff's Law: emissivity equals absorptivity at the same temperature

$$\alpha = \varepsilon$$

true for black and non-black solid surfaces

the fraction of energy absorbed by a material = the relative amount of energy emitted from that material compared to a black body

$\varepsilon = emissivity$

$$\varepsilon \equiv \frac{q_{emitted}}{q_{emitted,black\,body}} < 1$$

Black Bodies

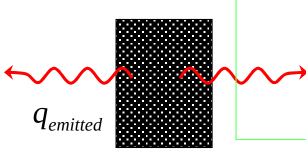
Stefan-Boltzmann Law: the amount of energy emitted by a black body is proportional to T^4

$$\frac{q_{emitted,black\,body}}{A} = \sigma T^4$$

absolute temperature

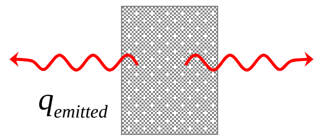
$$\sigma = 0.1712 \times 10^{-8} \frac{BTU}{h\ ft^2 R^4}$$

$$= 5.676 \times 10^{-8} \frac{W}{m^2 K^4}$$



$q_{emitted}$

Non-Black Bodies



$\varepsilon = emissivity$

$$\varepsilon \equiv \frac{q_{emitted}}{q_{emitted,black\,body}}$$

$$\frac{q_{emitted,non-black\,body}}{A} = \varepsilon\, q_{emitted,black\,body}$$

$$= \varepsilon \sigma T^4$$

Stefan-Boltzmann:

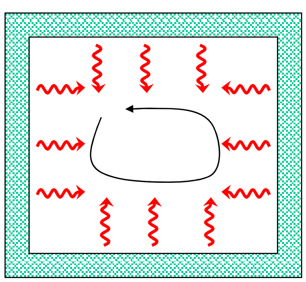
$$\frac{q_{emitted,black\,body}}{A} = \sigma T^4$$

Energy emitted by a non-black body

$$\frac{q_{emitted,non-black\,body}}{A} = \varepsilon \sigma T^4$$

How does this relate to chemical engineering?

Consider a furnace with an internal blower:



There is heat transfer due to convection:

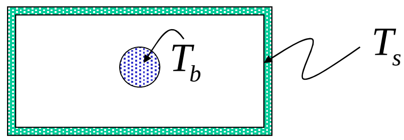
$$q_{convection} = h_{conv} A(T_s - T_b)$$

There is also heat transfer due to radiation:

$$q_{radiation} = h_{rad} A(T_s - T_b)$$

$$q_{total} = q_{conv} + q_{rad}$$

Where do we get h_{rad}?



object in furnace:

$$q_{emitted,non-black\,body} = A\varepsilon\big|_{T_b}\,\sigma T_b^4$$

using Kirchhoff's law

$$q_{absorbed} = \alpha\big|_{T_s}\,A\sigma T_s^4 = A\varepsilon\big|_{T_s}\,\sigma T_s^4$$

energy emitted by walls, which are acting as a black body

emissivity at T_s

net energy absorbed:

$$q_{\substack{transfered \\ to\,body}} = A\varepsilon\big|_{T_s}\,\sigma\left(T_s^4 - T_b^4\right)$$

$$assuming \quad \varepsilon\big|_{T_s} \approx \varepsilon\big|_{T_b}$$

Finally, calculate *h_{rad}*

net energy absorbed:

$$q_{\substack{transfered \\ to\,body}} = A\varepsilon\big|_{T_s}\,\sigma\left(T_s^4 - T_b^4\right)$$

$$assuming \quad \varepsilon\big|_{T_s} \approx \varepsilon\big|_{T_b}$$

equating with expression for h:

$$A\varepsilon\big|_{T_s}\,\sigma\left(T_s^4 - T_b^4\right) = h_{rad}A\left(T_s - T_b\right)$$

$$h_{rad} = \frac{\sigma\,\varepsilon\big|_{T_s}\left(T_s^4 - T_b^4\right)}{T_s - T_b}$$

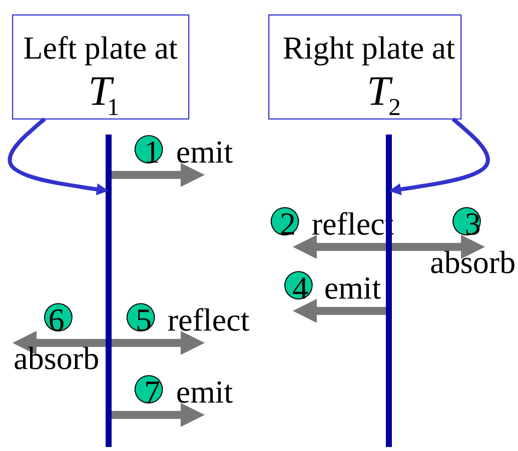
Geankoplis 4th ed., eqn 4.10-10 p304

Example: Geankoplis 4.10-3

A horizontal oxidized steel pipe carrying steam and having an OD of 0.1683m has a surface temperature of 374.9 K and is exposed to air at 297.1 K in a large enclosure. Calculate the heat loss for 0.305 m of pipe from natural convection plus radiation. For the steel pipe, use an emissivity of 0.79.

Radiation Heat Transfer Between Two Infinite Plates

Consider a quantity of radiation energy that is emitted from surface 1.



See: Geankoplis, section 4.11B

Also: Bird, Stewart, and Lightfoot, "Transport Phenomena" 1960 Wiley PP446-448

First round – surface 2

Radiation Heat Transfer Between Two Infinite Plates

Quantity of energy **incident** at surface 2:

$$\frac{q_{1-2}}{A} = \varepsilon_1 \sigma T_1^4$$

Quantity of energy **absorbed** at surface 2:

$$\alpha_2 \left(\frac{q_{1-2}}{A}\right) A = \varepsilon_2 \left(\varepsilon_1 \sigma T_1^4\right) A$$

$$\alpha_2 = \varepsilon_2$$

Quantity of energy **reflected** from surface 2:

$$\underbrace{\left(1-\varepsilon_2\right)}_{\text{fraction reflected}} \underbrace{\left(\varepsilon_1 A \sigma T_1^4\right)}_{\text{incident energy}}$$

This energy goes back to surface 1.

Second round – surface 1

Radiation Heat Transfer Between Two Infinite Plates

Quantity of energy **absorbed** at surface 1 (second round):

$$\underbrace{\varepsilon_1}_{\text{fraction absorbed}} \underbrace{\left[\left(1-\varepsilon_2\right)\left(\varepsilon_1 A \sigma T_1^4\right)\right]}_{\text{incident energy}}$$

Quantity of energy **reflected** from surface 1 (second round):

$$\underbrace{\left(1-\varepsilon_1\right)}_{\text{fraction reflected}} \underbrace{\left[\left(1-\varepsilon_2\right)\left(\varepsilon_1 A \sigma T_1^4\right)\right]}_{\text{incident energy}}$$

Third round – surface 2

Quantity of energy **absorbed** at surface 2 (third round):

$$\underbrace{\varepsilon_2}_{\text{fraction absorbed}} \underbrace{\left[(1-\varepsilon_1)(1-\varepsilon_2)\left(\varepsilon_1 A\sigma T_1^4\right)\right]}_{\text{incident energy}}$$

Quantity of energy **reflected** from surface 2 (third round):

$$\underbrace{(1-\varepsilon_2)}_{\text{fraction reflected}} \underbrace{\left[(1-\varepsilon_1)(1-\varepsilon_2)\left(\varepsilon_1 A\sigma T_1^4\right)\right]}_{\text{incident energy}}$$

There is a pattern.

Now, calculate the radiation energy going from surface 1 to surface 2:

Later, calculate energy from 2 to 1; then subtract to obtain **net energy transferred**.

$$q_{1-2} = \begin{pmatrix} energy\ \ from \\ 1 \rightarrow 2 \end{pmatrix} = \sum \begin{pmatrix} energy\ absorbed \\ at\ surface\ 2 \end{pmatrix}$$

$$= \varepsilon_2\left(\varepsilon_1 A\sigma T_1^4\right)$$

$$+ \varepsilon_2(1-\varepsilon_1)(1-\varepsilon_2)\left(\varepsilon_1 A\sigma T_1^4\right)$$

$$+ \varepsilon_2(1-\varepsilon_1)^2(1-\varepsilon_2)^2\left(\varepsilon_1 A\sigma T_1^4\right)$$

$$\ldots + \varepsilon_2(1-\varepsilon_1)^n(1-\varepsilon_2)^n\left(\varepsilon_1 A\sigma T_1^4\right) + \ldots$$

Radiation Heat Transfer Between Two Infinite Plates

Radiation energy going from surface 1 to surface 2:

$$q_{1-2} = \varepsilon_1 \varepsilon_2 A \sigma T_1^4 \sum_{n=0}^{\infty} (1-\varepsilon_1)^n (1-\varepsilon_2)^n$$

How can we calculate $\sum_{n=0}^{\infty} x^n$?

Answer: $S = \frac{1}{1-x}$

Radiation Heat Transfer Between Two Infinite Plates

Radiation energy going from surface 1 to surface 2:

$$q_{1-2} = \frac{\varepsilon_1 \varepsilon_2 A \sigma T_1^4}{1-[(1-\varepsilon_1)(1-\varepsilon_2)]}$$

$$= \frac{\varepsilon_1 \varepsilon_2 A \sigma T_1^4}{1-[1-\varepsilon_1-\varepsilon_2+\varepsilon_1\varepsilon_2]} = \frac{\varepsilon_1 \varepsilon_2 A \sigma T_1^4}{\varepsilon_1+\varepsilon_2-\varepsilon_1\varepsilon_2}$$

$$\frac{q_{1-2}}{A} = \frac{\sigma T_1^4}{\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1}$$

Final Result

Radiation Heat Transfer Between Two Infinite Plates

Radiation energy going from surface 1 to surface 2:

$$\frac{q_{1-2}}{A} = \frac{\sigma T_1^4}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

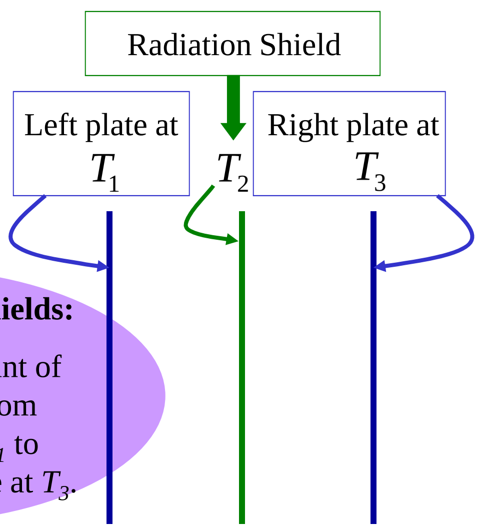
Radiation energy going from surface 2 to surface 1:

$$\frac{q_{2-1}}{A} = \frac{\sigma T_2^4}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

NET Radiation energy going from surface 1 to surface 2:

$$\frac{q_{1-2} - q_{2-1}}{A} = \frac{\sigma \left(T_1^4 - T_2^4\right)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right)}$$

Radiation Shields



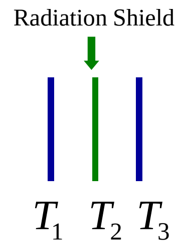
Purpose of Heat Shields:

To reduce the amount of energy transfer from (hotter) plate at T_1 to second (cooler) plate at T_3.

Note:

$$q_{net,1\rightarrow 2} = q_{net,2\rightarrow 3} = q$$

Analysis of Radiation Shields



We will assume that the emissivity is the same for all surfaces.

$$\frac{q_{net,1\rightarrow 2}}{A} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\left(\frac{1}{\varepsilon} + \frac{1}{\varepsilon} - 1\right)}$$

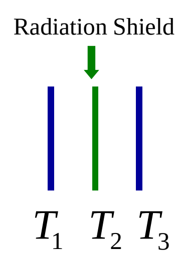
$$\frac{q_{net,2\rightarrow 3}}{A} = \frac{\sigma\left(T_2^4 - T_3^4\right)}{\left(\frac{1}{\varepsilon} + \frac{1}{\varepsilon} - 1\right)}$$

Now we eliminate T_2 between these equations.

Note:

$$q_{net,1\rightarrow 2} = q_{net,2\rightarrow 3} = q$$

Analysis of Radiation Shields



$$\frac{q}{A} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\left(\frac{2}{\varepsilon} - 1\right)} \qquad \frac{q}{A} = \frac{\sigma\left(T_2^4 - T_3^4\right)}{\left(\frac{2}{\varepsilon} - 1\right)}$$

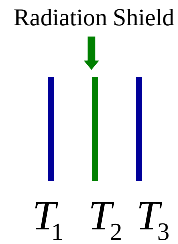
$$T_2^4 = \frac{q}{\sigma A}\left(\frac{2}{\varepsilon} - 1\right) + T_3^4$$

$$\frac{q}{\sigma A}\left(\frac{2}{\varepsilon} - 1\right) = T_1^4 - \frac{q}{\sigma A}\left(\frac{2}{\varepsilon} - 1\right) - T_3^4$$

$$\frac{2q}{\sigma A}\left(\frac{2}{\varepsilon} - 1\right) = T_1^4 - T_3^4$$

$$\frac{q}{A} = \left(\frac{1}{2}\right)\frac{\sigma\left(T_1^4 - T_3^4\right)}{\left(2/\varepsilon - 1\right)}$$

Analysis of Radiation Shields



1 Heat Shield

$$\frac{q}{A}=\left(\frac{1}{2}\right)\frac{\sigma\left(T_1^4-T_3^4\right)}{\left(2/\varepsilon-1\right)}$$

With one heat shield present, q falls by half compared to no heat shield.

by the same analysis,

N Heat Shields

$$\frac{q}{A}=\left(\frac{1}{N+1}\right)\frac{\sigma\left(T_1^4-T_3^4\right)}{\left(2/\varepsilon-1\right)}$$

With N heat shields present, q falls by a factor of $1/N$ compared to no heat shield.