What is Pumping Head?

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There is often considerable confusion among students when it comes to the definition and use of pumping head. This is understandable since head is a concept that was developed many years ago as a short-cut concept for field engineers when American engineering units (then called the English system) was the only set of units presented to students. American engineering units are still the dominant units used in the U.S. chemical engineering field, but for the most part, the science education of engineers in the U.S. has moved over to the metric system. The concept of head can be a stumbling block in a student's efforts to navigate the waters between these two systems.

The concept of head is related to the mechanical energy balance[1]:

$$\frac{\Delta P}{\rho} + \frac{\Delta(v^2)}{2} + g\Delta z + F = \frac{W_s}{m} \tag{1}$$

where $\Delta P = P_2 - P_1$ is the pressure difference between two points, ρ is the density of the fluid being pumped, $\Delta(v^2) = v_2^2 - v_1^2$ is the difference of the square of velocity at two points, g is the acceleration due to gravity, $\Delta z = z_2 - z_1$ is the height difference between two points, F is the friction in the systems (expressed in consistent units), W_s is the shaft work done *on* the system (again, expressed in consistent units), and m is the mass flow rate of the fluid being pumped. Note also that point 1 is an upstream point and point 2 is a downstream point.

Let us begin by examining the units of terms in equation 1 using the American engineering system ([=] means, has units of).

$$\frac{\Delta(v^2)}{2} \quad [=] \quad \frac{ft^2}{s^2} \tag{2}$$

$$g\Delta z \quad [=] \quad \frac{ft^2}{s^2} \tag{3}$$

$$\frac{\Delta P}{\rho} \quad [=] \quad \left(\frac{lb_f}{ft^2}\right)\left(\frac{ft^3}{lb_m}\right) = \frac{ft\ lb_f}{lb_m} \tag{4}$$

The units of expressions 2 and 3 are not the same as expression 4, and the units must be reconciled before equation 1 may be used. The solution is to apply the definition of pounds force to the last expression.

$$lb_f \equiv 32.174 \frac{lb_m ft}{s^2} \tag{5}$$

This is the equivalent of dividing the velocity and height terms by g_c[1]. It is not necessary to remember which terms to divide by what; the units tell you where this conversion factor goes.

If we convert terms above using the lb_f definition, we obtain three terms with common units of $\frac{ft\ lb_f}{lb_m}$. These are **not** units of head and it is **not** permissible to "*cancel*" lb_f with lb_m.

What, then, is head? Head is mechanical energy per unit *weight*. What is weight? Weight is the force due to a mass in a gravity field, and it has units of force which are newtons (N) in the metric or SI system, or lb_f in the American engineering system.

$$\text{weight} \ = \ mg \ [=] \ N \text{ or } \ lb_f \tag{6}$$

The distinction between weight and mass is often forgotten since we all live in the Earth's gravitational field. We say, "That box weighs 90 kilograms," even though kilograms are units of mass rather than weight. The correct unit of weight in the metric system, the newton, is not used in common language (Even in countries where they use the metric system they do not use the newton; they say "I weigh 50 kilos.").

In the American engineering system the situation is at once simpler and more complicated. In this system the units of both mass and weight are the pound. The lb_f, in fact, is defined with the Earth's gravity field in mind since 1 lb_m weighs 1 lb_f on Earth. It becomes tempting, however, to confuse the two types of pounds with each other and to cancel them in equations, and this can lead to errors.

If we examine equation 1 above, we see that it is written in terms of mechanical energy per unit *mass*. To rewrite it in terms of mechanical energy per unit *weight*, or head, we must divide by the acceleration due to gravity, g

$$\frac{\Delta P}{\rho g} + \frac{\Delta(v^2)}{2g} + \Delta z + \frac{F}{g} = \frac{W_s}{mg} \tag{7}$$

Notice the result. The units on the term above which accounts for the height-difference contribution to the mechanical energy (the elevation head) now clearly has units of length (ft or m). Examining the other two terms we see

$$\frac{\Delta(v^2)}{2g} \quad [=] \quad \frac{ft^2}{s^2}\frac{s^2}{ft} = ft \tag{8}$$

$$\frac{\Delta P}{\rho g} \quad [=] \quad \left(\frac{lb_f}{ft^2}\right)\left(\frac{ft^3}{lb_m}\right)\left(\frac{s^2}{ft}\right) = \frac{lb_f s^2}{lb_m} \tag{9}$$

The last expression still needs to be converted using the definition of lb_f:

$$\frac{\Delta P}{\rho g}[=]\left(\frac{lb_f s^2}{lb_m}\right)\left(\frac{32.174 ft\ lb_m}{s^2\, lb_f}\right) = \ ft \tag{10}$$

We thus see that head, mechanical energy per unit weight, has units of *ft* which is a particularly easy unit to remember.

Is that the whole point? Why bother with all of this mess just to use a simple unit? Engineers deal with many types of units all the time and could certainly learn to deal with other units such as $\frac{ft\ lb_f}{lb_m}$, or N, or whatever.

The desirability of using head in the mechanical energy expression is obscured in the calculations shown above by the fact that no numbers have been used. Let us examine the head produced by a pressure difference of $50\frac{lb_f}{ft^2}$ in a system pumping water.

$$\left\{\begin{array}{c}\text{mechanical energy}\\ \text{per unit } \textit{mass}\\ \text{due to pressure}\end{array}\right\} = \frac{\Delta P}{\rho} \tag{11}$$

$$= \left(50\frac{lb_f}{ft^2}\right)\left(\frac{ft^3}{62.43 lb_m}\right) \tag{12}$$

$$= 0.80\frac{ft\ lb_f}{lb_m} \tag{13}$$

$$\left\{\begin{array}{c}\text{mechanical energy}\\ \text{per unit } \textit{weight}\\ \text{due to pressure}\end{array}\right\} = \frac{\Delta P}{\rho g} = \left(\frac{\Delta P}{\rho}\right)\left(\frac{1}{g}\right) \tag{14}$$

$$= \left(0.80\frac{ft\ lb_f}{lb_m}\right)\left(\frac{s^2}{32.174 ft}\right)\left(\frac{32.174 ft\ lb_m}{s^2\, lb_f}\right) \tag{15}$$

$$= 0.80 ft \tag{16}$$

As we see in the final equation, the number 32.174 appears in two places, once in the numerator as part of the unit conversion from lb_f to $\frac{ft\ lb_m}{s^2}$, and secondly in the denominator as g, the acceleration due to gravity. Because these two factors are numerically the same, they cancel (the numbers cancel, not the units), and expressions having units of $\frac{ft\ lb_f}{lb_m}$ and *ft* of head are <u>numerically</u> the same.

Thinking in terms of head allows an engineer to have some intuition about the amount of mechanical energy per unit weight under consideration. Let us examine elevation head. The value in *ft* of the elevation head is the height difference between two reference points. Now if we express the velocity term in head units (velocity head $\equiv \frac{\Delta(v^2)}{2g}$) we can understand what, for example, 10 *ft* of velocity head means. It is the equivalent amount of mechanical energy per unit weight to that represented by 10 *ft* of water. Likewise if the pressure term is expressed in head units (pressure head $\equiv \frac{\Delta P}{\rho g}$), a similar understanding is obtained. In pumping this is particularly useful since we can get a handle on the characteristics of a pump by taking about the developed head of the pump. The developed head of a pump represents the height to which a fluid may be lifted by the pump in a frictionless piping system. If the pump is not used to lift fluid, we still can understand that the pump can use its ability to do work (produce mechanical energy per unit weight) to produce elevation, velocity, or pressure head or any combination of these.

In summary, *ft of head* is a unit of length. It expresses the mechanical energy per unit weight of a flowing system. It may be converted to other systems by using the conversion factors for length. When you do this you must remember that you are keeping track of a quantity which expresses mechanical energy per unit weight. In the SI system, m of head has exactly that meaning. Note that in the SI system $\frac{N\ m}{kg}$ (the analogous expression to $\frac{ft\ lb_f}{lb_m}$) is **not** numerically equal to m of head. This is because the conversion factor for $\frac{kg\ \ m}{s^2}$ to N is one, rather than being numerically equal to g in the SI unit system (check this to convince yourself).

References

[1] Felder, Richard M. and Ronald W. Rousseau, *Elementary Principles of Chemical Processes*, 2nd Edition (John Wiley, & Sons: New York, 1986).