Where are we in our discussion of error analysis?

Let's revisit:

CM3215 MichiganTech

Fundamentals of Chemical Engineering Laboratory

Statistics *Quick Start*: Random Error and Replicates

Professor Faith Morrison

Department of Chemical Engineering
Michigan Technological University

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From Lecture 1: Quick Start, Replicate Errors:

Measurements are affected by errors

(uncertainty)

There are two general categories of errors (uncertainties) in experimental measurements:

- **Systematic errors**
- **Random errors**

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From Lecture 1: Quick Start, Replicate Errors:

Measurements are affected by errors

Random errors **(uncertainty)**

1. Varies in sign and magnitude for identical conditions
2. May be due to the instrument or the process being measured
3. Must be understood and communicated with results

Sources:

Always present (need to minimize):
- Random process, instrument fluctuations
- Randomized systematic trends (e.g. operator identity, thermal drift)
- Rare events

Solutions:

Do:
- Replicate and average

Always an option:
- Improve measurement methods, practices
- Isolate from rare events

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From Lecture 1: Quick Start, Replicate Errors:

Solution for Random Errors:

$\bar{x}$ **is a Good Estimate of** x

Sample mean

Use $t_{0.025,n-1}$ for $n < 7$)

$E(x) = \bar{x} \pm 2e_s$ with 95% confidence

Expected value of x

Standard Error. If only random error present:

$$e_s = \frac{s}{\sqrt{n}}$$

Sample standard deviation

1. Minimize whatever is causing random errors
2. Replicate, average, construct 95% CI of mean

DONE…

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But, more than random errors are present

NOT ~~DONE~~

Systematic Errors →

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From Lecture 1: Quick Start, Replicate Errors:

Measurements are affected by errors

Systematic errors

(uncertainty)

1. Has same sign and magnitude for identical conditions
2. Must be checked for, identified, eliminated, randomized

Sources:

- Calibration of instruments
- Reading error (resolution, coarse scale)
- Consistent operator error
- Failure to produce experimentally conditions assumed in an analysis (e.g. steady state, isothermal, well mixed, pure component, etc.)

Solutions:

- Recalibrate
- Improve instrument resolution
- Apply correction for identified error
- Improve procedures, experimental design
- Shift to other methods
- Take data in random order; rotate operators

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From Lecture 1: Quick Start, Replicate Errors:

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Mistakes (need to fix) — Consistent operator error; Failure to produce experimentally conditions assumed in an analysis

Solutions:

- Recalibrate
- Improve instrument resolution
- Apply correction for identified error
- Improve procedures, experimental design
- Shift to other methods

Always an option — Recalibrate; Improve instrument resolution; Apply correction for identified error; Improve procedures, experimental design; Shift to other methods

Do: • Take data in random order; rotate operators

7

From Lecture 1: Quick Start, Replicate Errors:

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(uncertainty)

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Sources:

- Calibra
- Readin
- Consis
- Failure
  assum
  well mixed, pure component, etc.)

Mistakes (need to fix)

This seems complicated!
Is all this work necessary?

Solutions:

- Recalibrate
- Improve instrument resolution
- Apply correction for identified error
- Improve procedures, experimental design
- Shift to other methods

Always an option

Do: • Take data in random order; rotate operators

8

From Lecture 1: Quick Start, Replicate Errors:

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Sources:

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  well mixed, pure component, etc.)

Mistakes (need to fix)

Solut

Always an option

Do:

This seems complicated! Is all this work necessary?

<u>Yes,</u> if you want to (eventually) get right answers.

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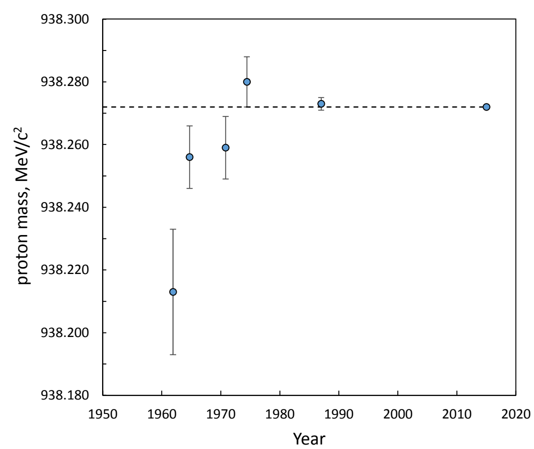
Even experts have difficulty identifying sources of uncertainty

Consider the experimentally determined mass of a proton as published between 1960 and 2015:

At every stage, expert researchers did their best to determine the correct value AND estimate the error.



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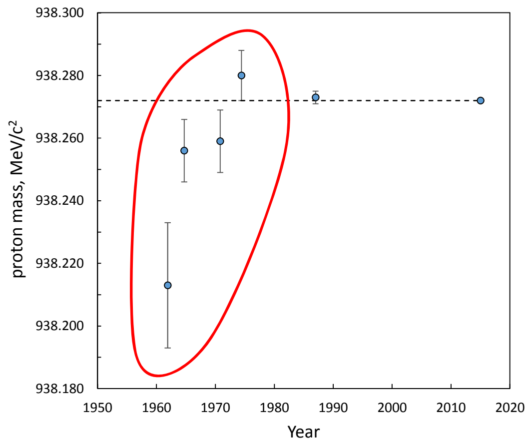
Even experts have difficulty identifying sources of uncertainty

Consider the experimentally determined mass of a proton as published between 1960 and 2015:

At every stage, expert researchers did their best to determine the correct value AND estimate the error.

Except for *the two most recent data points*, ***none*** of these error bars encompass the true value of the proton mass.



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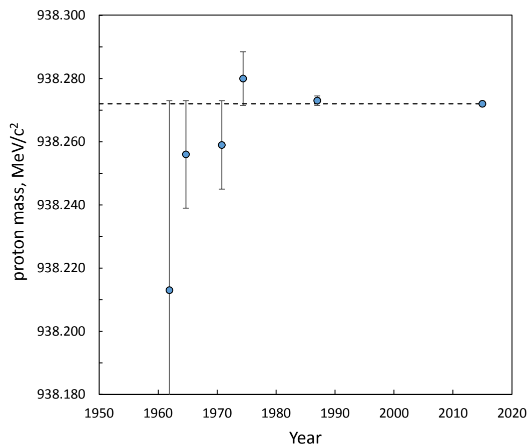
Even experts have difficulty identifying sources of uncertainty

If the error bars had been correctly determined, it would have looked more like this:

All of *these* error bars encompass the true value of the proton mass.

The analysis that went into the error bars tells us what are the ***largest sources of error***; the largest errors show us how to improve the process.



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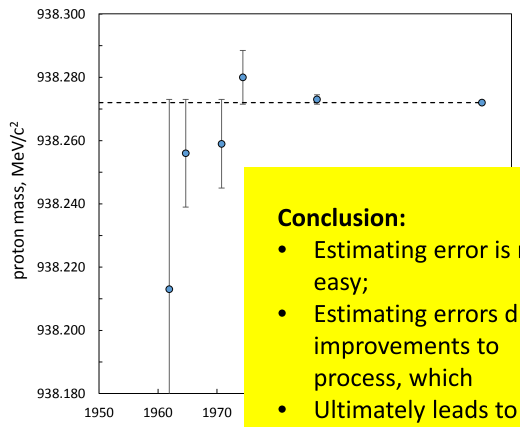

Conclusion:

- Estimating error is not easy;
- Estimating errors drives improvements to process, which
- Ultimately leads to better results.

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From Lecture 1: Quick Start, Replicate Errors:

Measurements are affected by errors

(uncertainty)

Systematic errors

1. Has same sign and magnitude for identical conditions
2. Must be checked for, identified, eliminated, randomized

Sources:

- Unavoidable: Calibration of instruments
- Unavoidable: Reading error (resolution, coarse scale)
- Mistakes (need to fix):
  - Consistent operator error
  - Failure to produce experimentally conditions assumed in an analysis (e.g. steady state, isothermal, well mixed, pure component, etc.)

Solutions:

- Always an option:
  - Recalibrate
  - Improve instrument resolution
  - Apply correction for identified error
  - Improve procedures, experimental design
  - Shift to other methods
- Do: Take data in random order; rotate operators

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Measurements are affected by errors (uncertainty)

We have identified three sources of error:

- **Random errors (replicate error)**
- **Reading errors**
- **Calibration errors**

$e_s = \frac{s}{\sqrt{n}}$	Standard error of replicates
$e_s = ?$	Standard reading error
$e_s = ?$	Standard calibration error

We *standardize* the individual errors so that we can combine them (apples to apples)

The techniques developed to understand and report **replicate error** can be the template that we use to account for the other two sources of uncertainty.

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Measurements are affected by errors (uncertainty)

We have identified three sources of error:

- **Random errors (replicate error)**
- **Reading errors**
- **Calibration errors**

Now:

$e_s = \frac{s}{\sqrt{n}}$	Standard error of replicates
$e_s = ?$	Standard reading error
$e_s = ?$	Standard calibration error

The techniques developed to understand and report **replicate error** can be the template that we use to account for the other two sources of uncertainty.

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Fundamentals of Chemical Engineering Laboratory

Statistics Lecture 2: Reading Error

Professor Faith Morrison

Department of Chemical Engineering
Michigan Technological University

1. Quick start—Replicate error
2. **Reading Error**
3. Calibration Error
4. Error Propagation
5. Least Squares Curve Fitting

Obtaining a Good Estimate of Precision

What is the Total Error of a Measurement?

$e_s \equiv$ Standard Error

Part 2: Reading Errors

Three sources:
- **Replicate errors**
- **Reading errors**
- **Calibration errors**

We *standardize* the individual errors so that we can combine them (apples to apples)

Second source of standard error…

Obtaining a Good Estimate of Reading Error

Sometimes a measurement is very reproducible (negligible replicate error) but there is still error/uncertainty inherent in how the reading is taken.

Example: Digital Multimeter reading a 4-20mA instrument signal

Image from: appliancerepairstartup.com/wp-content/uploads/2010/12/220px-Digital_Multimeter_Aka.jpg

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Second source of standard error…

Obtaining a Good Estimate of Reading Error

Systematic errors due to **Reading errors**, include:

1. Limits of instrument sensitivity (i.e. the magnitude of change required for the instrument to respond)
2. Limits of the degree of subdivision of the scale or display
3. Fluctuations of an instrument reading

We take each in turn→

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Reading Error Worksheet
CM3215 Fundamentals of Chemical Engineering Lab
Prof. Faith Morrison

Michigan Technological University
Department of Chemical Engineering

This worksheet guides the user through the calculation of the standard error and 95% confidence scale or off a digital readout (yielding value x and subject to reading error). The reading-error-related standard error e_s may subsequently be used in propagation of error calculations of derived quantities.

	Device name:			
	Measured Quantity: (give symbol)			
	Representative value:	(include units)		Quantity or Not Applicable
	issue	**contribution to error**		
Reading error, e_R:	Sensitivity (manufac. or estimated)	How much signal does it take to cause the reading to change?	1	
	Resolution: limitation on marked scale or digital readout	Half smallest division or decimal place	2	
	Fluctuations with time of observation	(max-min)/2	3	
		Maximum of 1, 2, & 3:	$e_R =$	
	Standard error based on reading error:	$e_s = e_R/\sqrt{3}$	$e_s =$	(units)
		95% Confidence Interval on the reading: $\pm 2e_s$		

Note: If a value is supplied by, for example, a manufacturer, with no indication of the uncertainty, we do not use this worksheet. Instead, see the Calibration Error worksheet.

13-Jan-16

Handy worksheet for reading error

www.chem.mtu.edu/~fmorriso/cm3215/ReadingErrorWorksheet.pdf

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Second source of standard error…

Obtaining a Good Estimate of Reading Error

Systematic errors due to **Reading errors**, include:

1. Limits of instrument sensitivity (i.e. the magnitude of change required for the instrument to respond)

How to determine?

At every range of operation, test how much signal must be received in order for the reading to change.

Related concept: *Limit of detection* (more on this later)

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Second source of standard error…

Obtaining a Good Estimate of Reading Error

Systematic errors due to **Reading errors**, include:

2. Limits of the degree of subdivision of the scale or display

How to determine?

Estimate as
- *½ the smallest subdivision on the scale or*
- *½ the smallest digit on a digital readout*

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Second source of standard error…

Obtaining a Good Estimate of Reading Error

Systematic errors due to **Reading errors**, include:

3. Fluctuations of an instrument reading

How to determine?

Estimate as: $\frac{1}{2}(x_{max} - x_{min})$ *over an interval*

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One source of standard error…

Obtaining a Good Estimate of **Reading Error**

Possible reading errors:

- Determine the limits of instrument sensitivity (magnitude of change required for instrument to respond)
- Determine ½ the smallest subdivision of the scale or display
- Determine $\frac{1}{2}(x_{max} - x_{min})$ for time-fluctuating data
- Designate the reading error:

e_R = maximum of the possible reading errors

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One source of standard error…

Obtaining a Good Estimate of **Reading Error**

Possible reading errors:

- Determine the limits of instrument sensitivity (magnitude of change required for instrument to respond)
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- Determine $\frac{1}{2}(x_{max} - x_{min})$ for time-fluctuating data
- Designate the reading error:

e_R = maximum of the possible reading errors

Question: is this way of thinking about error the same method as we used for random error?

$$\mathbf{e}_{,replicate} = \frac{s}{\sqrt{n}}$$

One source of standard error…

Obtaining a Good Estimate of **Reading Error**

Possible reading errors:

- Determine the limits of instrument sensitivity (magnitude of change required for instrument to respond)
- Determine ½ the smallest subdivision of the scale or display
- Determine $\frac{1}{2}(x_{max} - x_{min})$ for time-fluctuating data
- Designate the reading error:

e_R = maximum of the possible reading errors

Question: is this way of thinking about error the same method as we used for random error?

No. We need to ***standardize*** the individual errors so that we can combine them (apples to apples)

$e_s = \frac{e_R}{\sqrt{3}}$ **Where does that come from?**

We have identified three sources of standard error:

$e_s = \frac{s}{\sqrt{n}}$	Standard error of replicates
$e_s = ?$	Standard reading error
$e_s = ?$	Standard calibration error

We ***standardize*** the individual errors so that we can combine them (apples to apples)

We seek to write each in an equivalent form, so that we can combine them into a total error, taking all sources into account.

How to proceed?

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$e_s = \frac{e_R}{\sqrt{3}}$ **Where does that come from?**

Consider:

$e_s = \frac{s}{\sqrt{n}}$ Standard error of replicates

$e_s^2 = \frac{s^2}{n}$ **Variance** associated with a sample set of n measurements subject to random error

The **variance** is a well defined statistic, designed to measure the *spread* of individual outcomes around the *mean* outcome.

The **standard error^2** is the variance of the sampling distribution of the error.

The **sampling distribution** of replicate error is the Students t probability distribution.

$$x_1, x_2, x_3, x_4, x_5 \ldots x_n$$

$$\bar{x} \equiv \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$s^2 \equiv \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

We can determine the **sampling distribution** of reading error by considering the probability of obtaining individual readings around the true reading.

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$e_s = \frac{e_R}{\sqrt{3}}$ **Where does that come from?**

Consider the reading on a digital multimeter (DMM):

For a DMM meter reading (mA) of :

6.7 mA

Image from: appliancerepairstartup.com/wp-content/uploads/2010/12/220px-Digital_Multimeter_Aka.jpg

6.65
6.66
6.67
6.68
6.69
6.70
6.71
6.72
6.73
6.74

A reading of 6.7 may correspond to any of these more precise numbers with equal probability.

½ the smallest subdivision of the scale or display:

$e_R = 0.05$

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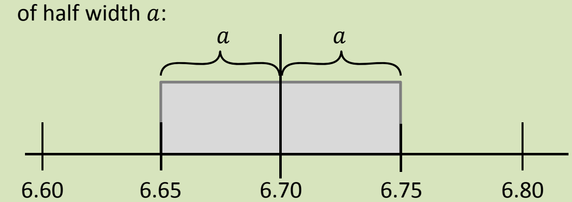
$e_s = \frac{e_R}{\sqrt{3}}$ **Where does that come from?**

6.7 mA

$e_R = 0.05 = a$

Rectangular probability distribution of half width a:



A reading of 6.7 may correspond to any of these more precise numbers with equal probability.

The **variance** of the rectangular probability distribution is $\sigma^2 = a^2/3$

(can show mathematically; see literature)

$$e_s = \frac{e_R}{\sqrt{3}}$$

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Measurements are affected by errors (uncertainty)

We have identified three sources of standard error:

- **Random errors (replicate error)**
- **Reading errors**
- **Calibration errors**

$e_s = \frac{s}{\sqrt{n}}$	Standard error of replicates
$e_s = \frac{e_R}{\sqrt{3}}$	Standard reading error
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Now we know how to compute two out of three measurement steandard errors.

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$e_s = ?$	Standard calibration error

Now we know how to compute two out of three measurement steandard errors.

For all three types of errors, we write a **variance** of the sampling distribution.

Why? Because we know how to combine variances (see literature):

$$\sigma_{total}^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \cdots$$

They add in quadrature.

Obtaining a Good Estimate of Reading Error

Possible reading errors:

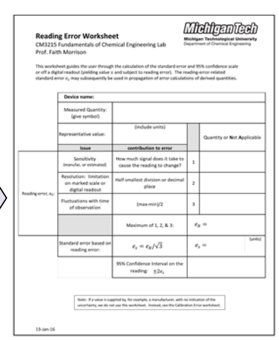
- Determine the limits of instrument sensitivity (magnitude of change required for instrument to respond)
- Determine ½ the smallest subdivision of the scale or display
- Determine $\frac{1}{2}(x_{max} - x_{min})$ for time-fluctuating data
- Designate the reading error:

  $e_R =$ maximum of the possible reading errors

- Calculate the standard reading error,

  $e_{s,reading} = \frac{e_R}{\sqrt{3}}$

These steps are summarized (and you are guided through them) on the ***Reading Error Worksheet***:



Reading Error Worksheet
CM3215 Fundamentals of Chemical Engineering Lab
Prof. Faith Morrison

Michigan Tech
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This worksheet guides the user through the calculation of the standard error and 95% confidence scale or off a digital readout (yielding value x and subject to reading error). The reading-error-related standard error e_s may subsequently be used in propagation of error calculations of derived quantities.

	Device name:			
	Measured Quantity: (give symbol)			
	Representative value:	(include units)		Quantity or Not Applicable
	issue	contribution to error		
Reading error, e_R:	Sensitivity (manufac. or estimated)	How much signal does it take to cause the reading to change?	1	
	Resolution: limitation on marked scale or digital readout	Half smallest division or decimal place	2	
	Fluctuations with time of observation	(max-min)/2	3	
		Maximum of 1, 2, & 3:	$e_R =$	
	Standard error based on reading error:	$e_s = e_R/\sqrt{3}$	$e_s =$	(units)
		95% Confidence Interval on the reading: $\pm 2e_s$		

Note: If a value is supplied by, for example, a manufacturer, with no indication of the uncertainty, we do not use this worksheet. Instead, see the Calibration Error worksheet.

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www.chem.mtu.edu/~fmorriso/cm3215/ReadingErrorWorksheet.pdf 34

EXAMPLE 1 For a 50ml beaker weighed with the CM3215 laboratory analytical balance, what is the weight and the 95% confidence interval on the weight based on reading error?

You try.

34.4081 g

Image from:
www.coleparmer.com

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	Device name:	Mettler analytical balance		
	Measured Quantity: (give symbol)	Mass, M		
	Representative value:	(include units) 32.4081 g		Quantity or Not Applicable
	issue	**contribution to error**		
Reading error, e_R:	Sensitivity (manufac. or estimated)	How much signal does it take to cause the reading to change?	1	
	Resolution: limitation on marked scale or digital readout	Half smallest division or decimal place	2	
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	Representative value:	(include units) 32.4081 g		Quantity or Not Applicable
	issue	contribution to error		
Reading error, e_R:	Sensitivity (manufac. or estimated)	How much signal does it take to cause the reading to change?	1	
	Resolution: limitation on marked scale or digital readout	Half smallest division or decimal place	2	
	Fluctuations with time of observation	(max-min)/2	3	
		Maximum of 1, 2, & 3:	$e_R =$	
	Standard error based on reading error:	$e_s = e_R/\sqrt{3}$	$e_s =$	
		95% Confidence Interval on the reading: $\pm 2e_s$		

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	Device name:	Mettler analytical balance		
	Measured Quantity: (give symbol)	Mass, M		
	Representative value:	(include units) 32.4081 g		Quantity or Not Applicable
	issue	contribution to error		
Reading error, e_R:	Sensitivity (manufac. or estimated)	How much signal does it take to cause the reading to change?	1	1×10^{-4}g
	Resolution: limitation on marked scale or digital readout	Half smallest division or decimal place	2	0.5×10^{-4}g
	Fluctuations with time of observation	(max-min)/2	3	0.5×10^{-4}g
		Maximum of 1, 2, & 3:		$e_R = 1 \times 10^{-4}$g
	Standard error based on reading error:	$e_s = e_R/\sqrt{3}$		(units) $e_s = 5.8 \times 10^{-5}$ g
		95% Confidence Interval on the reading: $\pm 2e_s$		1.2×10^{-4} g

Note: If a value is supplied by, for example, a manufacturer, with no indication of the uncertainty, we do not use this worksheet. Instead, see the Calibration Error worksheet.

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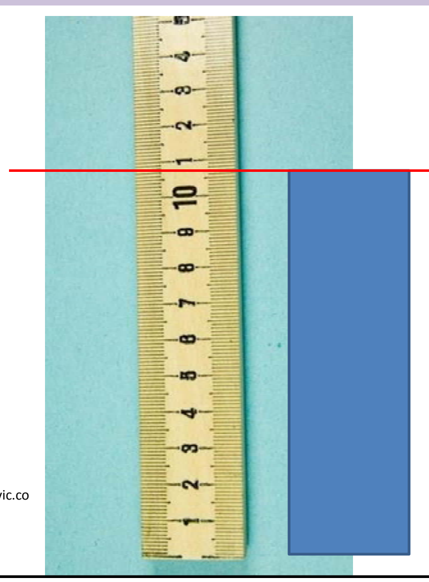
www.chem.mtu.edu/~fmorriso/cm3215/ReadingErrorWorksheet.pdf

Answer:

$34.4081 \pm 0.0001\ g$

(reading error only)

EXAMPLE 2 For height of an object measured with a meter stick as shown, what is the value and a 95% confidence interval on the height based on reading error?



You try.

Image from : www.martinaknezevic.com/events/full-sail-university-course-overview/

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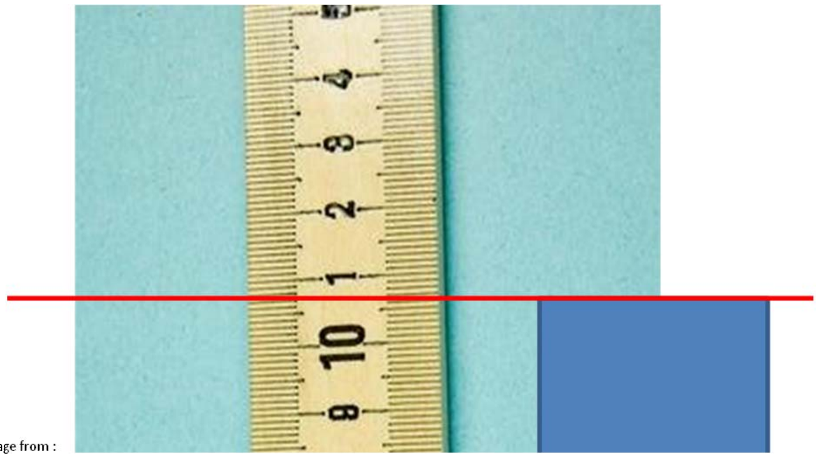

Image from : www.martinaknezevic.com/events/full-sail-university-course-overview/

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	Device name:	Laboratory wooden meter stick		
	Measured Quantity: (give symbol)	Height, h		
	Representative value:	(include units) $106.3\ mm$		Quantity or Not Applicable
	issue	contribution to error		
Reading error, e_R:	Sensitivity (manufac. or estimated)	How much signal does it take to cause the reading to change?	1	
	Resolution: limitation on marked scale or digital readout	Half smallest division or decimal place	2	
	Fluctuations with time of observation	(max-min)/2	3	
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	95% Confidence Interval on the reading: $\pm 2e_s$			

Reading error, e_R:

Note: If a value is supplied by, for example, a manufacturer, with no indication of the uncertainty, we do not use this worksheet. Instead, see the Calibration Error worksheet.

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www.chem.mtu.edu/~fmorriso/cm3215/ReadingErrorWorksheet.pdf

Reading Error Worksheet
CM3215 Fundamentals of Chemical Engineering Lab
Prof. Faith Morrison

Michigan Tech
Michigan Technological University
Department of Chemical Engineering

This worksheet guides the user through the calculation of the standard error and 95% confidence scale or off a digital readout (yielding value x and subject to reading error). The reading-error-related standard error e_s may subsequently be used in propagation of error calculations of derived quantities.

Device name:	Laboratory wooden meter stick			
Measured Quantity: (give symbol)	Height, h			
Representative value:	(include units) 106.3 mm			Quantity or Not Applicable
issue	contribution to error			
Sensitivity (manufac. or estimated)	How much signal does it take to cause the reading to change?	1	0.3 mm	← (at the most optimistic)
Resolution: limitation on marked scale or digital readout	Half smallest division or decimal place	2	0.5 mm	
Fluctuations with time of observation	(max-min)/2	3	n/a	
	Maximum of 1, 2, & 3:	$e_R =$	0.5 mm	
Standard error based on reading error:	$e_s = e_R/\sqrt{3}$	$e_s =$	0.29 (units) mm	
	95% Confidence Interval on the reading: $\pm 2e_s$		0.58 mm	

Reading error, e_R:

Note: If a value is supplied by, for example, a manufacturer, with no indication of the uncertainty, we do not use this worksheet. Instead, see the Calibration Error worksheet.

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EXAMPLE 2 For height of an object measured with a meter stick as shown, what is the value and a 95% confidence interval on the height based on reading error?

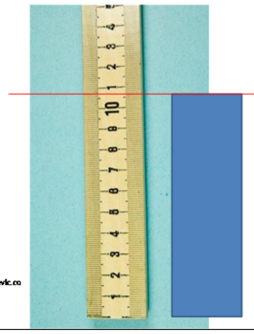

Image from: www.martinaknezevic.com/events/full-sail-university-course-overview/

You try.

Answer:

$106.3 \pm 0.6\ mm$

(at the most optimistic; reading error only)

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Obtaining a Good Estimate of a Quantity

Summary:

Replicate (random) error:

- Measure the quantity several times – replicates
- The average value is a good estimate of the quantity we are measuring if only random errors are present
- The 95% confidence interval comes from $\pm(**)e_s$
- $(**) = 2$ if the number of replicates is 7 or higher
- $(**)$ comes from the Student's t distribution if $N < 7$ (=t.inv.2t(0.05,n-1))
- Report one sig fig on error limits (unless that digit is 1 or 2)

Reading error:

- Determine signal needed to change reading
- Determine half smallest division or decimal place
- Determine average of fluctuations
- Max of those $/\sqrt{3}$ =reading standard error
- use $\pm 2e_s$ for 95% confidence interval

Combining Errors:

$$e^2_{s,combined} = e^2_{s,replicate} + e^2_{s,reading} + e^2_{s,calibration}$$

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Measurements are affected by errors (uncertainty)

We have identified three sources of standard error:

- **Random errors (replicate error)**
- **Reading errors**
- **Calibration errors**

Next:

$e_s = \frac{s}{\sqrt{n}}$	Standard error of replicates
$e_s = \frac{e_R}{\sqrt{3}}$	Standard reading error
$e_s = ?$	Standard calibration error

For all three types of errors, we write a **variance**.

(the variance of the sampling distribution; combine in quadrature)

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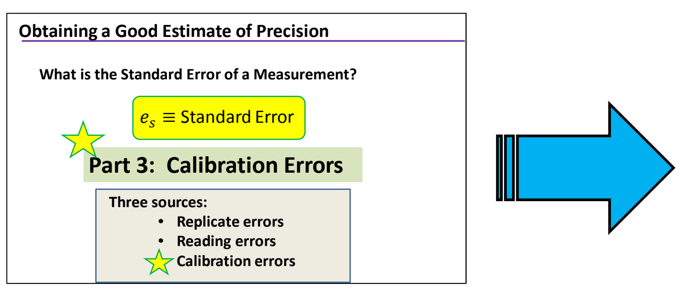
Next: Calibration Errors



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