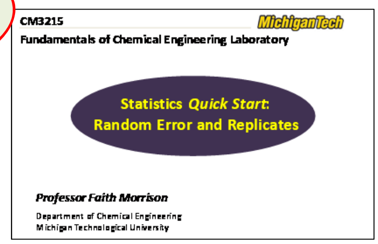
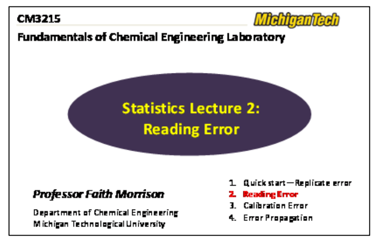
Where are we in our discussion of error analysis?

Let's revisit:





1

From Lecture 1: Quick Start, Replicate Errors:

Measurements are affected by errors

(uncertainty)

There are two general categories of errors (uncertainties) in experimental measurements:

- **Systematic errors**
- **Random errors**

2

From Lecture 1: Quick Start, Replicate Errors:

Measurements are affected by errors

Random errors (uncertainty)

1. Varies in sign and magnitude for identical conditions
2. May be due to the instrument or the process being measured
3. Must be understood and communicated with results

Sources:

Always present (need to minimize)
- Random process, instrument fluctuations
- Randomized systematic trends (e.g. operator identity, thermal drift)
- Rare events

Solutions:

Do:
- Replicate and average

Always an option
- Improve measurement methods, practices
- Isolate from rare events

3

From Lecture 1: Quick Start, Replicate Errors:

Measurements are affected by errors (uncertainty)

We have identified three sources of standard error:

- **Random errors (replicate error)**
- **Reading errors**
- **Calibration errors**

Averaging replicates allows us to calculate a standard error due to **random** error

$e_s = \frac{s}{\sqrt{n}}$	Standard error of replicates
$e_s = \frac{e_R}{\sqrt{3}}$	Standard reading error
$e_s = ?$	Standard calibration error

For all three types of errors, we write a **variance**.

4

From Lecture 2: Reading Error:

Measurements are affected by errors

(uncertainty)

Systematic errors

1. Has same sign and magnitude for identical conditions
2. Must be checked for, identified, eliminated, randomized

Sources:

- Unavoidable: Calibration of instruments
- Unavoidable: Reading error (resolution, coarse scale)
- Mistakes (need to fix):
  - Consistent operator error
  - Failure to produce experimentally conditions assumed in an analysis (e.g. steady state, isothermal, well mixed, pure component, etc.)

Solutions:

- Always an option:
  - Recalibrate
  - Improve instrument resolution
  - Apply correction for identified error
  - Improve procedures, experimental design
  - Shift to other methods
- Do: Take data in random order; rotate operators

5

From Lecture 2: Reading Error:

Measurements are affected by errors (uncertainty)

We have identified three sources of standard error:

- **Random errors (replicate error)**
- **Reading errors**
- **Calibration errors**

There are two common types of systematic error that we consider, **reading** and **calibration** error.

$e_s = \frac{s}{\sqrt{n}}$	Standard error of replicates
$e_s = \frac{e_R}{\sqrt{3}}$	Standard reading error
$e_s = ?$	Standard calibration error

We obtained **reading** error in the previous lecture.

For all three types of errors, we write a **variance**.

6

From Lecture 2: Reading Error:

Measurements are affected by errors (uncertainty)

We have identified three sources of standard error:

- **Random errors (replicate error)**
- **Reading errors**
- **Calibration errors**

$e_s = \frac{s}{\sqrt{n}}$	Standard error of replicates
$e_s = \frac{e_R}{\sqrt{3}}$	Standard reading error
$e_s = ?$	Standard calibration error

Now we consider systematic **calibration** error.

For all three types of errors, we write a **variance**.

7

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Fundamentals of Chemical Engineering Laboratory

Statistics Lecture 3:

Calibration Error

Professor Faith Morrison

Department of Chemical Engineering
Michigan Technological University

1. Quick start—Replicate error
2. Reading Error
3. **Calibration Error**
4. Error Propagation

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Obtaining a Good Estimate of Precision

What is the Standard Error of a Measurement?

$e_s \equiv$ Standard Error

Part 3: Calibration Errors

Three sources:

- **Replicate errors**
- **Reading errors**
- **Calibration errors**

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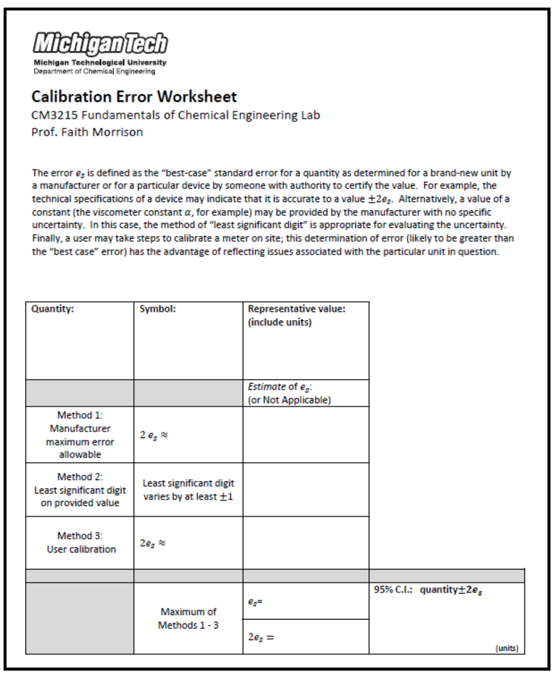

Michigan Tech
Michigan Technological University
Department of Chemical Engineering

Calibration Error Worksheet
CM3215 Fundamentals of Chemical Engineering Lab
Prof. Faith Morrison

The error e_s is defined as the "best-case" standard error for a quantity as determined for a brand-new unit by a manufacturer or for a particular device by someone with authority to certify the value. For example, the technical specifications of a device may indicate that it is accurate to a value $\pm 2e_s$. Alternatively, a value of a constant (the viscometer constant α, for example) may be provided by the manufacturer with no specific uncertainty. In this case, the method of "least significant digit" is appropriate for evaluating the uncertainty. Finally, a user may take steps to calibrate a meter on site; this determination of error (likely to be greater than the "best case" error) has the advantage of reflecting issues associated with the particular unit in question.

Quantity:	Symbol:	Representative value: (include units)	
		Estimate of e_s: (or Not Applicable)	
Method 1: Manufacturer maximum error allowable	$2\,e_s \approx$		
Method 2: Least significant digit on provided value	Least significant digit varies by at least ± 1		
Method 3: User calibration	$2e_s \approx$		
	Maximum of Methods 1 - 3	e_s=	95% C.I.: quantity$\pm 2e_s$
		$2e_s =$	(units)

www.chem.mtu.edu/~fmorriso/cm3215/CalibrationErrorWorksheet.pdf

Handy worksheet for calibration error

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Measurements are affected by errors

Systematic errors

1. Has same sign and magnitude for identical conditions
2. Must be checked for, identified, eliminated, randomized

Sources:

- Miscalibration of instruments
- Consistent operator error (e.g. parallax)
- Failure to produce experimentally conditions assumed in an analysis (e.g. steady state, isothermal, well mixed, pure component, etc.)

Solutions:

- Recalibrate
- Apply correction for identified error
- Improve procedures, experimental design
- Shift to other methods
- Take data in random order; rotate operators

11

Measurements are affected by errors

Systematic errors

1. Has same sign and magnitude for identical conditions
2. Must be checked for, identified, eliminated, randomized

Sources:

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- Consistent operator error (e.g. parallax)
- Failure to produce experimentally conditions assumed in an analysis (e.g. steady state, isothermal, well mixed, pure component, etc.)

Solutions:

- Recalibrate
- Apply correction for identified error
- Improve procedures, experimental design
- Shift to other methods
- Take data in random order; rotate operators

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Calibration

What is *calibration*?

Calibration is a step made to establish the correctness and utility of a device.

1. A *standard* is used (a device or material whose correctness or properties are known.
2. The *unit under test* and the standard are both made to make a measurement.
3. The performance of the unit under test is assigned based on the comparison to the standard – we say that the unit under test is *calibrated* against the standard.

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Factory Calibration

- Many devices come to us calibrated by the manufacturers
- How do we know the accuracy of these devices?

$$T = 39.8 \pm ?$$

14

Factory Calibration

- Many devices come to us calibrated by the manufacturers
- How do we know the accuracy of these devices?

$$T = 39.8 \pm ?$$

We check with the manufacturer.

15

Factory Calibration

Omega HH501BJK

(HH501BJK)
Display Type: Backlit LCD
Number of Digits: 1999
Input Channel: 2
Temperature Range: -50 to 1370°C
~~(-58 to 1950°F~~
Resolution: 0.1°/1°
~~**Display:** 1~~
W/Bead Wire Type K Thermocouple: 2
Basic Accuracy: 0.1% +1°C

Sensor resolution here is what we call Sensitivity (part of reading error)—the smallest change a sensor can detect in the quantity that it is measuring (dual display, $0.1^o/1^oC$)

www.omega.com/Temperature/pdf/HH500.pdf

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Factory Calibration

Omega HH501BJK

- Standard limits of error: greater of 2.2^oC or 0.75%
- Special limits of error: greater of 1.1^oC or 0.4%

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Wire Color Codes and Limits of Error

To view the printable version, click here
(Printable version also contains international color codes)

ANSI Code	Alloy Combination + Lead	Alloy Combination − Lead	Color Coding Thermocouple Grade	Color Coding Extension Grade	Maximum Useful Temperature Range ++	Maximum Thermocouple Grade Temperature Range	EMF (mV) Over Max Temperature Range	Standard Limits of Error** (above 0°C)	Special Limits of Error** (above 0°C)	International IEC 584-3	Comments Environment – Bare Wire	ANSI Code
J	IRON Fe (magnetic)	CONSTANTAN COPPER-NICKEL Cu-Ni			Thermocouple Grade: 32 to 1382°F 0 to 750°C Extension Grade: 32 to 392°F 0 to 200°C	-346 to 2193°F -210 to 1200°C	-8.095 to 69.553	greater of 2.2°C or 0.75%	greater of 1.1°C or 0.4%		Reducing, Vacuum, Inert. Limited Use in Oxidizing at High Temperatures. Not Recommended for Low Temperatures.	J
K	CHROMEGA® NICKELCHROMIUM Ni-Cr	ALOMEGA® NICKEL-ALUMINUM Ni-Al (magnetic)			Thermocouple Grade: -328 to 2282°F -200 to 1250°C Extension Grade: 32 to 392°F 0 to 200°C	-454 to 2501°F -270 to 1372°C	-6.458 to 54.886	greater of 2.2°C or 0.75%	greater of 1.1°C or 0.4%		Clean Oxidizing and Inert. Limited Use in Vacuum or Reducing. Wide Temperature Range, Most Popular Calibration	K

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www.omega.com/techref/colorcodes.html

Factory Calibration

Omega HH501BJK

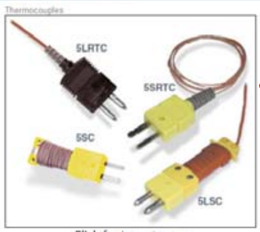
- Standard limits of error: greater of 2.2^oC or 0.75%
- Special limits of error: greater of 1.1^oC or 0.4%

Which limits apply depends on what you buy:

Ready-Made Insulated Thermocouples with Kapton®, PFA, Glass Braid Insulation and Molded Connectors

5LRTC, and 5SRTC Series

Thermocouples



Click for larger image.

- Sold in Packs of 5 Economically Priced
- New Kapton® Insulation Available Now (All models)!
- J, K, T and E Calibrations Available
- Made from Special Limits of Error Wire
- 20, 24, 30 and 36 AWG Available (SRTC Series)
- 30 and 36 AWG Wires with PFA Insulation. Max. Service Temp 260°C (500°F)
- 30 AWG Wires with Glass Braid Insulation. Maximum Service Temp 480°C (900°F)
- 1 and 2 meter (36 and 72") Lengths Standard
- Convenient 5-Pack
- Molded Subminiature Connector with Integral Strain Relief/Spool
- Max Service Temp for Connector Body 220°C (425°F)
- Available from Stock in Convenient 5-Packs
- PFA, Kapton®, or Glass Braid Insulation

View related products - Temperature Sensors and Instruments

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www.omega.com/pptst/5LSC_5SRTC.html

Factory Calibration

- Many devices come to us calibrated by the manufacturers
- How do we know the accuracy of these devices?

According to the manufacturer:

$$T = 39.8 \pm 1.1^o C$$
$$38.7 \leq T \leq 40.9^o C$$

(note that at higher temperatures the uncertainty is even higher, 0.4%)

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Factory Calibration

- Many devices come to us calibrated by the manufacturers
- How do we know the accuracy of these devices?

According to the manufacturer:

$$T = 39.8 \pm 1.1^o C$$
$$38.7 \leq T \leq 40.9^o C$$

Let's check.

20

EXAMPLE 3: For the temperature indicators in the lab, what is the standard error and 95% confidence interval for the measurement? Consider replicate error, reading error, and calibration error.

Let's try.

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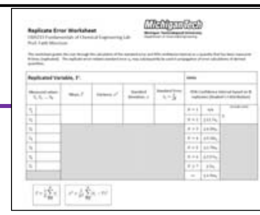
Replicate Error

- Test 11 meters
- Calculate standard error $e_s = \frac{s}{\sqrt{n}}$

Replicate Error



- 11 thermocouples, 11 temperature indicators
- Indicators in service for various amounts of time
- Factory calibrated
- All measuring the temperature of a bath thermostated to 40.0^oC
- Record temperatures

Y=			mean	variance	std dev	std error		95% CI	
	T	N		s^2	s	e_s	$2e_s$	lower	upper
	°C	11	39.2	1.1	1.1	0.3	0.6	38.6	39.9
Y_1	39.3		°C	°C	°C	°C	°C	°C	°C
Y_2	40.0								
Y_3	36.7								
Y_4	39.9								
Y_5	39.8								
Y_6	39								
Y_7	38								
Y_8	39.9								
Y_9	39								
Y_{10}	40.1								
Y_{11}	40.0								

$$e_s = \frac{s}{\sqrt{n}} = \frac{1.1}{\sqrt{11}} = 0.3^oC$$

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Reading Error

Issues:

- Sensitivity
- Resolution
- Fluctuations

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Reading Error

Issues:

- Sensitivity
- Resolution
- Fluctuations

You try.

Reading Error Worksheet
CM3215 Fundamentals of Chemical Engineering Lab
Prof. Faith Morrison

MichiganTech
Michigan Technological University
Department of Chemical Engineering

This worksheet guides the user through the calculation of the standard error and 95% confidence scale or off a digital readout (yielding value X and subject to reading error). The reading-error-related standard error e_s may subsequently be used in propagation of error calculations of derived quantities.

	Reading error				
	Measured Quantity: (give symbol)				
	Representative value:	(include units)		Quantity or Not Applicable	
	issue	contribution to error			
Reading error, e_R:	Resolution	How much signal does it take to cause the reading to change?	1		
	Limitation on marked scale or digital readout	Half smallest division or decimal place	2		
	Fluctuations with time of observation	(max-min)/2	3		
		Maximum of 1, 2 & 3:	e_R=		(units)
	Standard error based on reading error:	$e_s=e_R/\sqrt{3}$	e_s=		
		95% Confidence Interval on the reading: +/-1.96e_s			

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	Reading error			
	Measured Quantity: (give symbol)	$T, {}^oC$		
	Representative value:	(include units)		Quantity or Not Applicable
	issue	contribution to error		
Reading error, e_R:	Resolution	How much signal does it take to cause the reading to change?	1	
	Limitation on marked scale or digital readout	Half smallest division or decimal place	2	
	Fluctuations with time of observation	(max-min)/2	3	
		Maximum of 1, 2 & 3:	e_R=	
	Standard error based on reading error:	$e_s=e_R/\sqrt{3}$	e_s=	
		95% Confidence Interval on the reading: +/-1.96e_s		

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Summary of Errors

Standard Errors, e_x:

- Replicate standard error is $\frac{s}{\sqrt{n}} = 0.3^o C$
- Reading standard error is $\frac{e_R}{\sqrt{3}} = 0.06^o C$
- Calibration error?

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Calibration Error

Issues:

- Manufacturer maximum error allowable
- Least significant digit on provided value
- User calibration

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Calibration Error

Issues:

- Manufacturer maximum error allowable
- Least significant digit on provided value
- User calibration

You try.

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Calibration Error Worksheet
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Quantity:	Symbol:	Representative value: (include units)	
		Estimate of e_s: (or Not Applicable)	
Method 1: Manufacturer maximum error allowable	$2\,e_s \approx$		
Method 2: Least significant digit on provided value	Least significant digit varies by at least ± 1		
Method 3: User calibration	$2e_s \approx$		
	Maximum of Methods 1 - 3	$e_s =$	95% C.I.: quantity$\pm 2e_s$ (units)
		$2e_s =$	

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Calibration Error

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The error e_s is defined as the "best-case" standard error for a quantity as determined for a brand-new unit by a manufacturer or for a particular device by someone with authority to certify the value. For example, the technical specifications of a device may indicate that it is accurate to a value $\pm 2e_s$. Alternatively, a value of a constant (the viscometer constant α, for example) may be provided by the manufacturer with no specific uncertainty. In this case, the method of "least significant digit" is appropriate for evaluating the uncertainty. Finally, a user may take steps to calibrate a meter on site; this determination of error (likely to be greater than the "best case" error) has the advantage of reflecting issues associated with the particular unit in question.

Quantity:	Symbol:	Representative value: (include units)
		Estimate of e_s: (or Not Applicable)
Method 1: Manufacturer maximum error allowable	$2\,e_s \approx 1.1^oC$	
Method 2: Least significant digit on provided value	Least significant digit varies by at least ± 1	
Method 3: User calibration	$2e_s \approx$	
	Maximum of Methods 1 - 3	

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Summary of Errors

Standard Errors, e_x:

- Replicate standard error is $\frac{s}{\sqrt{n}} = 0.3^o C$
- Reading standard error is $\frac{e_R}{\sqrt{3}} = 0.06^o C$
- Calibration error is $e_s = 0.55^o C$

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Summary of Errors

Standard Errors, e_x:

- Replicate standard error is $\frac{s}{\sqrt{n}} = 0.3^o C$
- Reading standard error is $\frac{e_R}{\sqrt{3}} = 0.06^o C$
- Calibration error is $e_s = 0.55^o C$

Calibration error dominates

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Factory Calibration

- Many devices come to us calibrated by the manufacturers
- How do we know the accuracy of these devices?

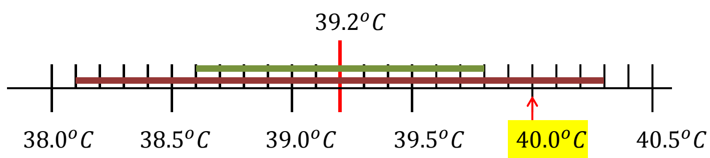
According to the manufacturer:

$$T = 39.2 \pm 1.1^o C$$
$$38.1 \leq T \leq 40.3^o C$$

Performance in our laboratory:

(ignoring calibration error)

$$T = 39.2 \pm 0.6^o C$$
$$38.6 \leq T \leq 39.8^o C$$



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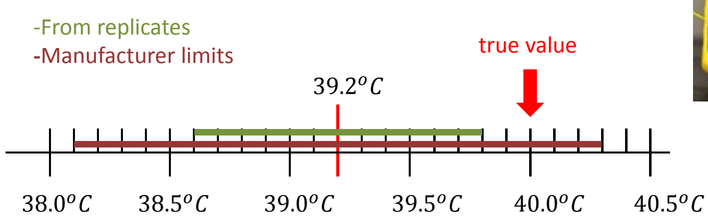
Factory Calibration



Observations:

- The narrow spread of the replicates testifies to the ***precision*** of the measurement
- The fact that the true value of T is not included in the spread of the replicates, speaks poorly of the ***accuracy*** of the measurement
- The *error limits supplied by the manufacturer* do encompass the true value, however
- **Replicates can only tell us about precision, not accuracy.**

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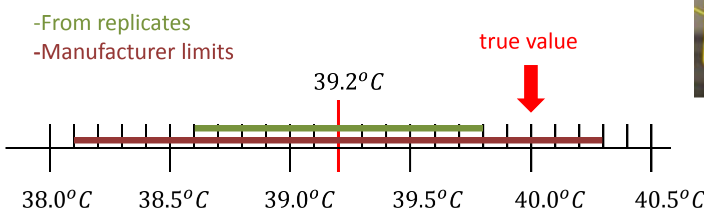
Factory Calibration



If a *single* temperature indicator were used over and over, the spread of the replicates would be even more narrow.

Always consider the manufacturer's error limits along with replicates to estimate a sensor's accuracy.

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Obtaining a Good Estimate of a Quantity

Summary:

Replicate error:

- Measure the quantity several times – replicates
- The average value is a good estimate of the quantity we are measuring if only random errors are present
- The 95% confidence interval comes from $\pm(**)e_s$
- $(**) = 2$ if the number of replicates is 7 or higher
- $(**)$ comes from the Student's t distribution if $N < 7$
- Report one sig fig on error (unless that digit is 1 or 2)

Reading error:

- Determine signal needed to change reading
- Determine half smallest division or decimal place
- Determine average of fluctuations
- Max of those $/\sqrt{3}$ =reading error
- use $\pm 2e_s$ for 95% confidence interval

Calibration error:

- Determine manufacturer maximum error allowable
- Assume least significant digit varies by ± 1
- Calibrate in-house
- Use largest uncertainty as determined above
- Replication cannot reduce calibration error

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Obtaining a Good Estimate of a Quantity

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- use $\pm 2e_s$ for 95% confidence interval

Calibration error:

- Determine manufacturer maximum error allowable
- Assume least significant digit varies by ± 1
- Calibrate in-house
- Use largest uncertainty as determined above
- **Replication cannot reduce calibration error**

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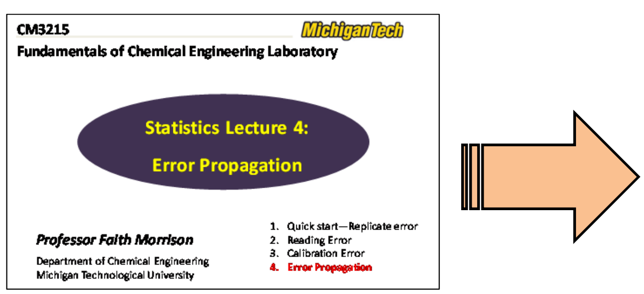
Next:
Error
Propagation



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