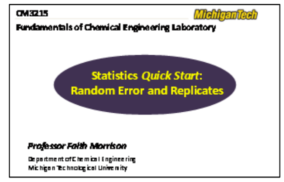
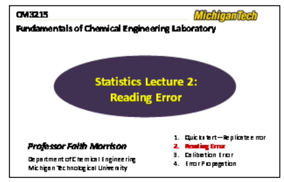
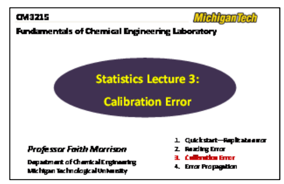
Where are we in our discussion of error analysis?

Let's revisit:







1

Obtaining a Good Estimate of a *Measured* Quantity

Summary:

Replicate error:

- Measure the quantity several times – replicates
- The average value is a good estimate of the quantity we are measuring if only random errors are present
- The 95% confidence interval comes from $\pm(**)e_s$
- $(**) = 2$ if the number of replicates is 7 or higher
- $(**)$ comes from the Student's t distribution if $N < 7$
- Report one sig fig on error (unless that digit is 1 or 2)

Reading error:

- Determine signal needed to change reading
- Determine half smallest division or decimal place
- Determine average of fluctuations
- Max of those $/\sqrt{3}$ =reading error
- use $\pm 2e_s$ for 95% confidence interval

Calibration error:

- Determine manufacturer maximum error allowable
- Assume least significant digit varies by ± 1
- Calibrate in-house
- Use largest uncertainty as determined above
- Replication cannot reduce calibration error

Measured quantities, e.g.: mass, temperature, DC current, time interval, etc.

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Obtaining a Good Estimate of a Quantity

- Replicate error
- Reading error
- Calibration error

*But what do we do when we obtain a quantity from a **calculation?***

$$\rho = \frac{M_F - M_E}{V_{pyc}}$$

$$\mu = \rho \alpha \Delta t$$

$$Q = \dot{m} C_p (T_{out} - T_{in})$$

$$etc.$$

Answer:
Propagate the **error** through the calculation

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Fundamentals of Chemical Engineering Laboratory

Statistics Lecture 4: Error Propagation

Professor Faith Morrison
Department of Chemical Engineering
Michigan Technological University

1. Quick start—Replicate error
2. Reading Error
3. Calibration Error
4. **Error Propagation**
5. Least Squares Curve Fitting

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Fundamentals of Chemical Engineering Laboratory

Statistics Lecture 4:

Error Propagation

Professor Faith Morrison

Department of Chemical Engineering
Michigan Technological University

References:
- *Dealing with Data,* Arthur J. Lyon (Pergamon Press, NY 1970)

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Example 1:

What is the uncertainty (95% confidence interval) in $\rho_{bluefluid}$ as determined in the lab?

Image source: www.coleparmer.com

Image source: //en.wikipedia.org/wiki/Relative_density

Example 1:

What is the uncertainty (95% confidence interval) in $\rho_{bluefluid}$ as determined in the lab?

$$\rho_{bluefluid} = \frac{M_{full} - M_{empty}}{V_{pycnometer}}$$

- The value of density obtained is a function of three measurements
- Each measurement has its own uncertainty

Image source: www.coleparmer.com

Image source: //en.wikipedia.org/wiki/Relative_density

Example 1:

$e_s \equiv$ Standard Error

What is the uncertainty (95% confidence interval) in $\rho_{bluefluid}$ as determined in the lab?

Three error sources on each measured quantity:

$e_s = \frac{s}{\sqrt{n}}$	Standard error of replicates
$e_s = \frac{e_R}{\sqrt{3}}$	Standard error due to Reading Error
$e_s =$ (as determined)	Standard error due to Calibration Error

For each variable, determine the three e_s, then pick the largest

(or average if they are close and you want to be less conservative)

Handy worksheet for replicate error

Replicate Error Worksheet

CM3215 Fundamentals of Chemical Engineering Lab
Prof. Faith Morrison

Michigan Tech
Michigan Technological University
Department of Chemical Engineering

This worksheet guides the user through the calculation of the standard error and 95% confidence interval on a quantity that has been measured n times (replicated). The replicate-error-related standard error e_s may subsequently be used in propagation-of-error calculations of derived quantities.

Replicated Variable, Y:						Units:		
Measured values $Y_1, Y_2, \dots, Y_n$		Sample Mean, $\bar{Y}$	Sample Variance, s^2	Sample Standard Deviation, s	Standard Error, $e_s = \frac{s}{\sqrt{n}}$	95% Confidence Interval based on n replicates (*Student's t* distribution)		(include units)
Y_1						$n = 1$	n/a	$\pm$
Y_2						$n = 2$	$\pm 12.7 e_s$	
Y_3						$n = 3$	$\pm 4.30 e_s$	
Y_4						$n = 4$	$\pm 3.18 e_s$	
Y_5						$n = 5$	$\pm 2.78 e_s$	
Y_6						$n = 6$	$\pm 2.57 e_s$	
Y_7						$n \geq 7$	$\pm 2 e_s$	
						∞	$\pm 1.96 e_s$	

$$\bar{Y} \equiv \frac{1}{n}\sum_{i=1}^{n} Y_i \qquad s^2 \equiv \frac{1}{(n-1)^2}\sum_{i=1}^{n}(Y_i - \bar{Y})^2$$

11-Sep-14

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www.chem.mtu.edu/~fmorriso/cm3215/ReplicateErrorWorksheet.pdf

Handy worksheet for reading error

Reading Error Worksheet

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Department of Chemical Engineering

This worksheet guides the user through the calculation of the standard error and 95% confidence scale or off a digital readout (yielding value X and subject to reading error). The reading-error-related standard error e_s may subsequently be used in propagation of error calculations of derived quantities.

		Reading error			
	Measured Quantity: (give symbol)				
	Representative value:	(include units)		Quantity or Not Applicable	
	issue	contribution to error			
Reading error, e_R:	Resolution	How much signal does it take to cause the reading to change?	1		
	Limitation on marked scale or digital readout	Half smallest division or decimal place	2		
	Fluctuations with time of observation	(max-min)/2	3		
		Maximum of 1, 2 & 3:	e_R=		(units)
	Standard error based on reading error:	$e_s = e_R/\sqrt{3}$	e_s=		
		95% Confidence Interval on the reading: +/-1.96e_s			

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www.chem.mtu.edu/~fmorriso/cm3215/ReadingErrorWorksheet.pdf

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Calibration Error Worksheet

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Prof. Faith Morrison

The error e_s is defined as the "best-case" standard error for a quantity as determined for a brand-new unit by a manufacturer or for a particular device by someone with authority to certify the value. For example, the technical specifications of a device may indicate that it is accurate to a value $\pm 2e_s$. Alternatively, a value of a constant (the viscometer constant α, for example) may be provided by the manufacturer with no specific uncertainty. In this case, the method of "least significant digit" is appropriate for evaluating the uncertainty. Finally, a user may take steps to calibrate a meter on site; this determination of error (likely to be greater than the "best case" error) has the advantage of reflecting issues associated with the particular unit in question.

Quantity:	Symbol:	Representative value: (include units)	
		Estimate of e_s: (or Not Applicable)	
Method 1: Manufacturer maximum error allowable	$2\,e_s \approx$		
Method 2: Least significant digit on provided value	Least significant digit varies by at least ± 1		
Method 3: User calibration	$2e_s \approx$		
	Maximum of Methods 1 - 3	$e_s=$	95% C.I.: quantity$\pm 2e_s$
		$2e_s =$	(units)

Handy worksheet for calibration error

www.chem.mtu.edu/~fmorriso/cm3215/CalibrationErrorWorksheet.pdf

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Example 1: What is the uncertainty (95% confidence interval) in $\rho_{bluefluid}$ as determined in the lab?

First:

What are the uncertainties e_{x_i} for M_{full}, M_{empty}, and V_{pyc}?

You try.

Image source: www.coleparmer.com

Image source: //en.wikipedia.org/wiki/Relative_density

Example 1: What is the uncertainty (95% confidence interval) in $\rho_{bluefluid}$ as determined in the lab?

First:

What are the uncertainties e_{x_i} for M_{full}, M_{empty}, and V_{pyc}?

Standard errors:

M_{full}:	$= 30.800\ g$
M_{empty}:	$= 13.410\ g$
$V_{pycnometer}$	$= 10.00\ ml$

Example 1: What is the uncertainty (95% confidence interval) in $\rho_{bluefluid}$ as determined in the lab?

First:

What are the uncertainties e_{x_i} for M_{full}, M_{empty}, and V_{pyc}?

Standard errors:

M_{full}:	$= 30.800\ g$	(reading)
M_{empty}:	$= 13.410\ g$	
$V_{pycnometer}$	$= 10.00\ ml$	(calibration)

Now, how to combine?
Propagation of Errors

Error Propagation

We seek to combine the errors associated with the various quantities in a calculation

$$\rho_{bluefluid} = \frac{M_{full} - M_{empty}}{V_{pycnometer}}$$

We use an analysis based on the calculation of variance.
We use the Taylor series expansion of a nonlinear function.

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Error Propagation

We use an analysis based on the Taylor series expansion of a nonlinear function.

Taylor series:

(higher order terms)

$$f(x_1, x_2, x_3) = f^0 + \frac{\partial f}{\partial x_1} x_1 + \frac{\partial f}{\partial x_2} x_2 + \frac{\partial f}{\partial x_3} x_3 + h.o.t.$$

A calculation of the function $f(x_1, x_2, x_3)$ from uncertain values of x_1, x_2, x_3 is a random variable of mean $\bar{f}$ and variance σ_f^2:

$$\sigma_f^2 = \left(\frac{\partial f}{\partial x_1}\right)^2 \sigma_{x_1}^2 + \left(\frac{\partial f}{\partial x_2}\right)^2 \sigma_{x_2}^2 + \left(\frac{\partial f}{\partial x_3}\right)^2 \sigma_{x_3}^2 + \text{Covariance terms, if } x_i \text{ are correlated}$$

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Error Propagation

We use an analysis based on the Taylor series expansion of a nonlinear function.

Taylor series:

(higher order terms)

$$f(x_1,x_2,x_3) = f^0 + \frac{\partial f}{\partial x_1}x_1 + \frac{\partial f}{\partial x_2}x_2 + \frac{\partial f}{\partial x_3}x_3 + h.o.t.$$

A calculation of the function $f(x_1,x_2,x_3)$ from uncertain values of x_1,x_2,x_3 is a random variable of mean $\bar{f}$ and variance σ_f^2:

neglect

$$\sigma_f^2 = \left(\frac{\partial f}{\partial x_1}\right)^2\sigma_{x_1}^2 + \left(\frac{\partial f}{\partial x_2}\right)^2\sigma_{x_2}^2 + \left(\frac{\partial f}{\partial x_3}\right)^2\sigma_{x_3}^2 + \text{Covariance terms, if } x_i \text{ are correlated}$$

Note: covariance terms are not always zero or small; but they often are. For now, this is fine.

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Error Propagation

(To avoid confusion with other variances, we use e_{x_i} nomenclature for errors)

$$e_{s_f}^2 = \left(\frac{\partial f}{\partial x_1}\right)^2 e_{x_1}^2 + \left(\frac{\partial f}{\partial x_2}\right)^2 e_{x_2}^2 + \left(\frac{\partial f}{\partial x_3}\right)^2 e_{x_3}^2$$

$$\rho_{bluefluid} = f\left(M_{full}, M_{empty}, V_{pycnometer}\right)$$

$e_{s_{M_{full}}}$

$e_{s_{M_{empty}}}$

$e_{s_{V_{pycnometer}}}$

We estimate these standard errors with our 3 worksheets

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Error Propagation

$$e_{s_f}^2 = \left(\frac{\partial f}{\partial x_1}\right)^2 e_{x_1}^2 + \left(\frac{\partial f}{\partial x_2}\right)^2 e_{x_2}^2 + \left(\frac{\partial f}{\partial x_3}\right)^2 e_{x_3}^2$$

These come from the formula for $\rho_{bluefluid}$

$$\rho_{bluefluid} = f\left(M_{full}, M_{empty}, V_{pycnometer}\right)$$

$$\rho_{BF} = \frac{M_F - M_E}{V_{pyc}}$$

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Error Propagation

$$e_{s_f}^2 = \left(\frac{\partial f}{\partial x_1}\right)^2 e_{x_1}^2 + \left(\frac{\partial f}{\partial x_2}\right)^2 e_{x_2}^2 + \left(\frac{\partial f}{\partial x_3}\right)^2 e_{x_3}^2$$

$$f = \rho_{BF} = \frac{M_F - M_E}{V_{pyc}}$$

$$\frac{\partial \rho_{BF}}{\partial M_F} =$$

$$\frac{\partial \rho_{BF}}{\partial M_E} =$$

$$\frac{\partial \rho_{BF}}{\partial V_{pyc}} =$$

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Error Propagation

$$e_{s_f}^2 = \left(\frac{\partial f}{\partial x_1}\right)^2 e_{x_1}^2 + \left(\frac{\partial f}{\partial x_2}\right)^2 e_{x_2}^2 + \left(\frac{\partial f}{\partial x_3}\right)^2 e_{x_3}^2$$

We seek this, the standard error of the calculated property, $f = \rho_{bluefluid}$

$$\rho_{bluefluid} = f\left(M_{full}, M_{empty}, V_{pycnometer}\right)$$

$$\rho_{BF} = \frac{M_F - M_E}{V_{pyc}}$$

Think of the squared partial derivatives as the weighting functions for the individual *squared* standard errors

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Error Propagation Worksheet

CM3215 Fundamentals of Chemical Engineering Lab
Prof. Faith Morrison

This worksheet guides the user through the determination of the standard error e_{s_f} of a quantity $f(x_1, x_2, x_3, x_4, x_5)$ that is calculated from measured quantities x_1, x_2, x_3, x_4 and x_5. The x_i are subject to random errors. The standard error e_{x_i} (replicate, reading, calibration; use the largest) for each variable x_i is determined first, and these uncertainties are propagated to determine e_{s_f} using the relationship given below.

$f(x_1, x_2, x_3, x_4, x_5)$:	Formula for f:	Representative value of f: (include units)	95% C.I. of f: $(f \pm 2e_{s_f})$ (include units)

Measured quantities, x_i			$\frac{\partial f}{\partial x_i}$	$e_{x_i} = \frac{s_i}{\sqrt{N}}$ or $\frac{e_{R_i}}{\sqrt{3}}$ or e_{s_i}	$\left(\frac{\partial f}{\partial x_i}\right)^2 e_{x_i}^2$
x_i	Symbol	Representative value			
x_1					
x_2					
x_3					
x_4					
x_5					

$$e_{s_f}^2 = \left(\frac{\partial f}{\partial x_1}\right)^2 e_{x_1}^2 + \left(\frac{\partial f}{\partial x_2}\right)^2 e_{x_2}^2 + \left(\frac{\partial f}{\partial x_3}\right)^2 e_{x_3}^2 + \left(\frac{\partial f}{\partial x_4}\right)^2 e_{x_4}^2 + \left(\frac{\partial f}{\partial x_5}\right)^2 e_{x_5}^2$$

$e_{s_f}^2 =$

$e_{s_f} =$ units

Standard error of calculated quantity, f

Handy worksheet for error propagation

www.chem.mtu.edu/~fmorriso/cm3215/ErrorPropagationWorksheet.pdf

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Example 1:

What is the uncertainty (95% confidence interval) in $\rho_{bluefluid}$ as determined in the lab?

Data:

$$M_F = 30.800\ g$$
$$M_E = 13.410\ g$$
$$V_{pyc} = 10.00\ ml$$

Formula:

$$\rho_{BF} = \frac{M_F - M_E}{V_{pyc}}$$

Image source: www.coleparmer.com

Image source: //en.wikipedia.org/wiki/Relative_density

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Example 1: What is the uncertainty (95% confidence interval) in $\rho_{bluefluid}$ as determined in the lab?

$f(x_1, x_2, x_3, x_4, x_5)$:	Formula for f: $\rho_{BF} = \frac{M_F - M_E}{V_{pyc}}$	Representative value of f: (include units)	95% C.I. of f: $(f \pm 2e_{sf})$ (include units)

Measured quantities, x_i			$\frac{\partial f}{\partial x_i}$	$e_{x_i} = \frac{s_i}{\sqrt{N}}$ or $\frac{e_{R_i}}{\sqrt{3}}$ or e_{s_i}	$\left(\frac{\partial f}{\partial x_i}\right)^2 e_{x_i}^2$
x_i	Symbol	Representative value			
x_1					
x_2					
x_3					
x_4					
x_5					

$$e_{s_f}^2 = \left(\frac{\partial f}{\partial x_1}\right)^2 e_{x_1}^2 + \left(\frac{\partial f}{\partial x_2}\right)^2 e_{x_2}^2 + \left(\frac{\partial f}{\partial x_3}\right)^2 e_{x_3}^2 + \left(\frac{\partial f}{\partial x_4}\right)^2 e_{x_4}^2 + \left(\frac{\partial f}{\partial x_5}\right)^2 e_{x_5}^2$$

$e_{s_f}^2 =$	
$e_{s_f} =$	units

Standard error of calculated quantity, f

Example 1: What is the uncertainty (95% confidence interval) in $\rho_{bluefluid}$ as determined in the lab?

$f(x_1,x_2,x_3,x_4,x_5)$:	Formula for f: $\rho_{BF} = \frac{M_F - M_E}{V_{pyc}}$	Representative value of f: (include units) $1.739\ g/ml$	95% C.I. of f: $(f \pm 2e_{sf})$ (include units)

Measured quantities, x_i					
x_i	Symbol	Representative value	$\frac{\partial f}{\partial x_i}$	$e_{x_i} = \frac{s_i}{\sqrt{N}}$ or $\frac{e_{R_i}}{\sqrt{3}}$ or e_{s_i}	$\left(\frac{\partial f}{\partial x_i}\right)^2 e_{x_i}^2$
x_1	M_F	$30.800\ g$			
x_2	M_E	$13.410\ g$			
x_3	V_{pyc}	$10.00\ ml$			
x_4					
x_5					

Note: The units used here __must__ work in the formula without any additional unit conversion.

$$e_{s_f}^2 = \left(\frac{\partial f}{\partial x_1}\right)^2 e_{x_1}^2 + \left(\frac{\partial f}{\partial x_2}\right)^2 e_{x_2}^2 + \left(\frac{\partial f}{\partial x_3}\right)^2 e_{x_3}^2 + \left(\frac{\partial f}{\partial x_4}\right)^2 e_{x_4}^2 + \left(\frac{\partial f}{\partial x_5}\right)^2 e_{x_5}^2$$

$e_{s_f}^2 =$

$e_{s_f} =$ units

Standard error of calculated quantity, f

Example 1: What is the uncertainty (95% confidence interval) in $\rho_{bluefluid}$ as determined in the lab?

$f(x_1,x_2,x_3,x_4,x_5)$:	Formula for f: $\rho_{BF} = \frac{M_F - M_E}{V_{pyc}}$	Representative value of f: (include units) $1.739\ g/ml$	95% C.I. of f: $(f \pm 2e_{sf})$ (include units)

Measured quantities, x_i					
x_i	Symbol	Representative value	$\frac{\partial f}{\partial x_i}$	$e_{x_i} = \frac{s_i}{\sqrt{N}}$ or $\frac{e_{R_i}}{\sqrt{3}}$ or e_{s_i}	$\left(\frac{\partial f}{\partial x_i}\right)^2 e_{x_i}^2$
x_1	M_F	$30.800\ g$			
x_2	M_E	$13.410\ g$			
x_3	V_{pyc}	$10.00\ ml$			
x_4					
x_5					

You try.

$$e_{s_f}^2 = \left(\frac{\partial f}{\partial x_1}\right)^2 e_{x_1}^2 + \left(\frac{\partial f}{\partial x_2}\right)^2 e_{x_2}^2 + \left(\frac{\partial f}{\partial x_3}\right)^2 e_{x_3}^2 + \left(\frac{\partial f}{\partial x_4}\right)^2 e_{x_4}^2 + \left(\frac{\partial f}{\partial x_5}\right)^2 e_{x_5}^2$$

$e_{s_f}^2 =$

$e_{s_f} =$ units

Standard error of calculated quantity, f

Example 1: What is the uncertainty (95% confidence interval) in $\rho_{bluefluid}$ as determined in the lab?

$f(x_1, x_2, x_3, x_4, x_5)$:	Formula for f: $\rho_{BF} = \frac{M_F - M_E}{V_{pyc}}$	Representative value of f: (include units) $1.739\ g/ml$	95% C.I. of f: $(f \pm 2e_{sf})$ (include units) $1.739 \pm 0.007\ g/ml$

Measured quantities, x_i			$\frac{\partial f}{\partial x_i}$	$e_{x_i} = \frac{s_i}{\sqrt{N}}$ or $\frac{e_{R_i}}{\sqrt{3}}$ or e_{s_i}	$\left(\frac{\partial f}{\partial x_i}\right)^2 e_{x_i}^2$
x_i	Symbol	Representative value			
x_1	M_F	$30.800\ g$			
x_2	M_E	$13.410\ g$			
x_3	V_{pyc}	$10.00\ ml$			
x_4					
x_5					

$$e_{s_f}^2 = \left(\frac{\partial f}{\partial x_1}\right)^2 e_{x_1}^2 + \left(\frac{\partial f}{\partial x_2}\right)^2 e_{x_2}^2 + \left(\right.$$

Example 1: What is the uncertainty (95% confidence interval) in $\rho_{bluefluid}$ as determined in the lab?

Excel is an excellent tool for error propagation

Error propagation Worksheet									
$f(x_1, x_2, x_3)$			f	ρ_{BF}	1.739	g/ml	$2e_s$		g/ml
	x_i	value		df/dx_i	$(df/dx_i)^2$	e_{xi}	e_{xi}^2	$(df/dx_i)^2 e_{xi}^2$	
x_1	M_F	30.800	g						g^2/ml^2
x_2	M_E	13.410	g						g^2/ml^2
x_3	V_{pyc}	10.000	ml						g^2/ml^2
							e_s^2		g^2/ml^2
							e_s		g/ml

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Example 1:

What is the uncertainty (95% confidence interval) in $\rho_{bluefluid}$ as determined in the lab?

Excel is an excellent tool for error propagation

Error propagation Worksheet									
							e_s^2	1.21E-05	g^2/ml^2
							e_s	0.0035	g/ml

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Example 1:

What is the uncertainty (95% confidence interval) in $\rho_{bluefluid}$ as determined in the lab?

Answer from error propagation:

$$1.739 \pm 0.007 \; g/ml$$

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Summary: Error Analysis with Real Numbers

- To understand the accuracy of our numbers, we need to determine a ***confidence interval***.

$\bar{x} \pm 2e_s$ with 95.0% confidence

For replicate data with $n < 7$, replace "2" with $t_{0.025,n-1}$

- The **Standard error e_s** for a measured quantity is the largest of:
  - e_s determined by *replicates* $e_s = s/\sqrt{n}$ or
  - e_s by estimate of *reading error* $e_s = e_R/\sqrt{3}$ or
  - e_s by estimate of *calibration error* $e_s = \text{max error}/2$

- Standard error e_f for derived quantities (arrived at from equations), is obtained at through **error propagation,** which is a combination of *variances*.

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Example 2: Replicates revisited

In Example 1, we calculated a value of ρ_{BF} along with its uncertainty from a single determination of density using error propagation. In lab, we have replicates of density measurements. How does the result from the single value compare to the result determined from replicates?

i	ρ_{BFi}
	g/cm
1	1.7162
2	1.7162
3	1.69942
4	1.7110
5	1.7152
6	1.70616
7	1.73097
8	1.73746
9	1.727

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Example 2: Replicates revisited

In Example 1, we calculated a value of ρ_{BF} along with its uncertainty from a single determination of density using error propagation. In lab, we have replicates of density measurements. How does the result from the single value compare to the result determined from replicates?


Replicate Worksheet

i	ρ_{BFi}				
	g/cm		**n=**	9	
			mean ρ=	1.718	g^2/ml^2
1	1.7162		**s^2=**	0.00015	g^2/ml^2
2	1.7162		**s=**	0.0121	g/cm
3	1.69942		**s/sqrt(n)=**	0.0040	g/cm
4	1.7110		**$2e_s$=**	0.008	g/cm
5	1.7152		**te_s=**	0.009	g/cm
6	1.70616				
7	1.73097				
8	1.73746				
9	1.727				

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Example 2: Replicates revisited

In Example 1, we calculated a value of ρ_{BF} along with its uncertainty from a single determination of density using error propagation. In lab, we have replicates of density measurements. How does the result from the single value compare to the result determined from replicates?


Solution:

Single measurement:

$$1.739 \pm 0.007\ g/ml$$

9 replicates:

$$1.718 \pm 0.009\ g/ml$$

Do they agree?

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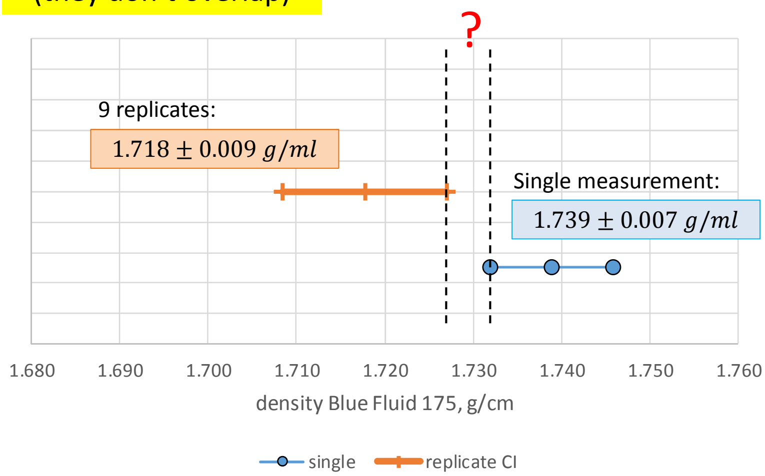
Example 2: Replicates revisited

(they don't overlap)



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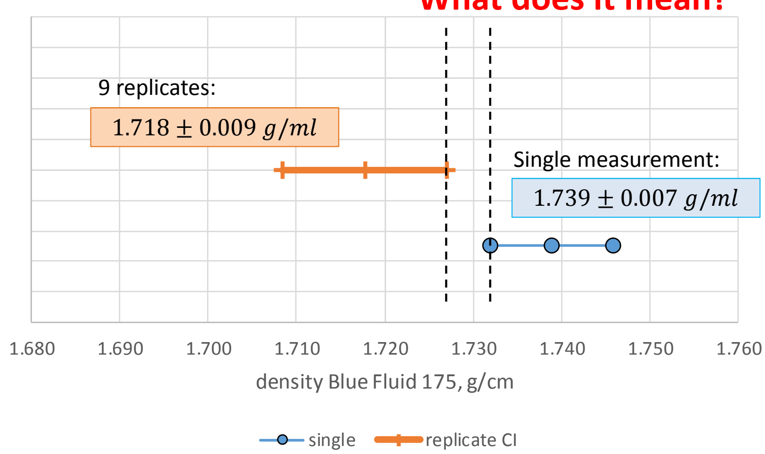
Example 2: Replicates revisited

(they don't overlap)

Why not?
What does it mean?



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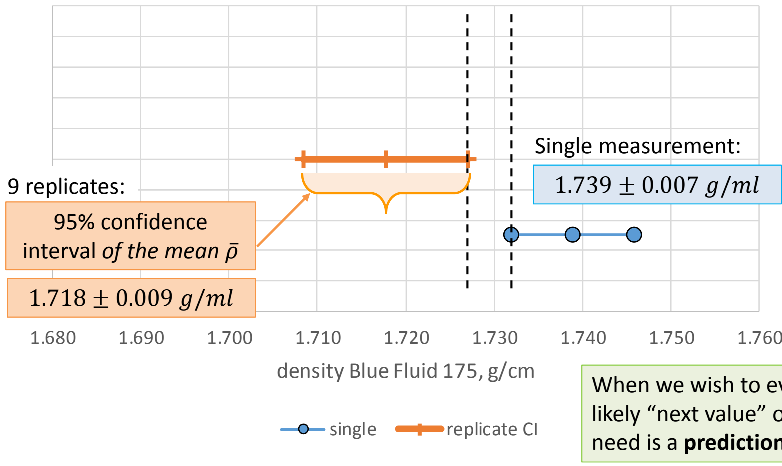
Why don't the two error intervals overlap?

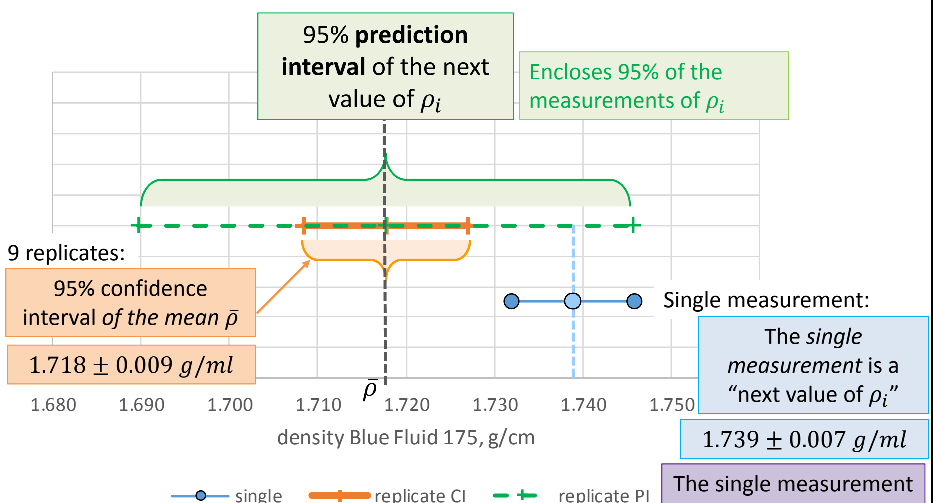
When averaging replicates of ρ, we are estimating the **mean value, $\bar{\rho}$**.
The 95% CI we calculate $(\pm ts/\sqrt{n})$ is the confidence interval **of the mean**.



When we wish to evaluate the likely "next value" of ρ_i, what we need is a **prediction interval**.

Prediction interval of next value of x: $\pm t_{0.025,n-1}s$

$\approx \pm 2s$



The single measurement *does* lie within the **95%PI** of the next value

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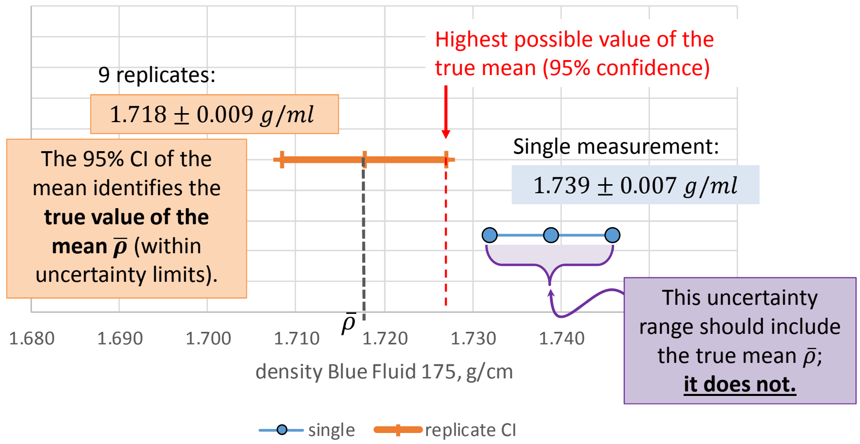
Example 3: Evaluate Error Propagation

But, is our Error Propagation answer **right?**



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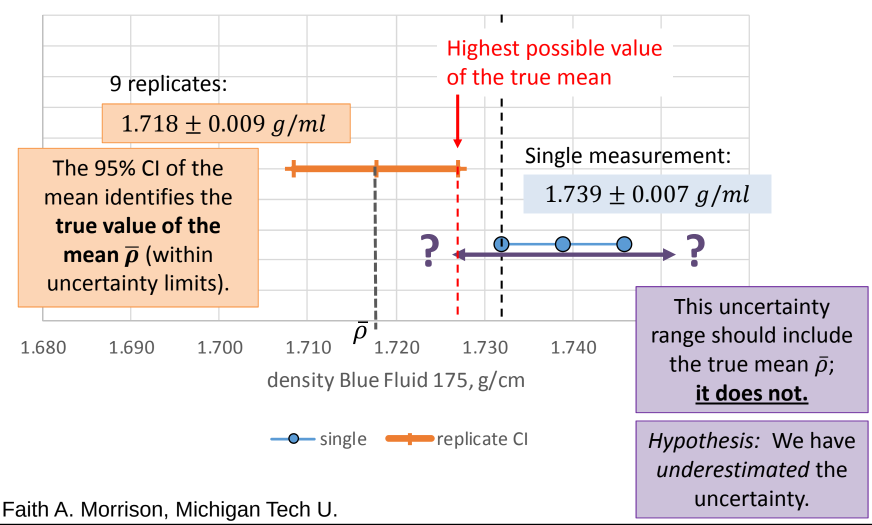


Example 2: Replicates revisited


Could it be we underestimated the **reading error on mass**?

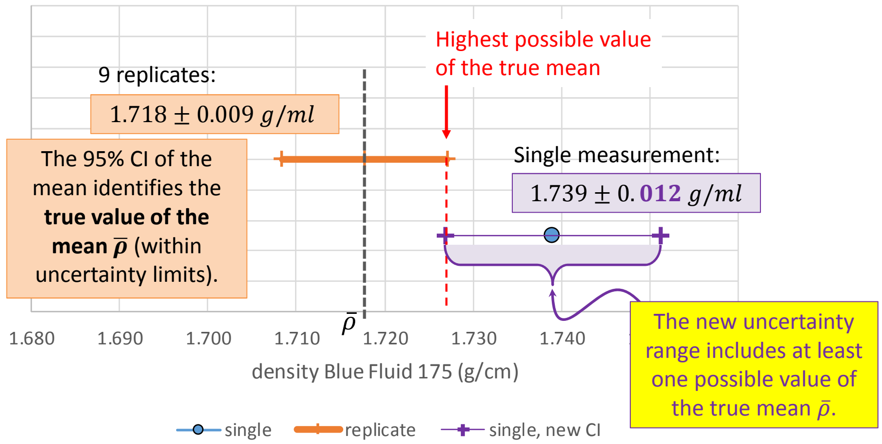
					new reading error on mass:			0.01	g
Error propagation Worksheet									
$f(x_1,x_2,x_3)$			f	ρ_{BF}	**1.739**	**g/ml**	**$2e_s$**	**0.007**	**g/ml**
	x_i	**value**		**df/dx_i**	**$(df/dx_i)^2$**	**e_{xi}**	**e_{xi}^2**	**$(df/dx_i)^2 e_{xi}^2$**	
x_1	M_F	30.800	g	0.10	0.010	5.8E-03	3.3E-05	3.33E-07	g^2/ml^2
x_2	M_E	13.410	g	-0.10	0.010	5.8E-03	3.3E-05	3.33E-07	g^2/ml^2
x_3	V_{pyc}	10.000	ml	-0.174	0.0302	0.02	4.0E-04	1.210E-05	g^2/ml^2
							e_s^2	1.28E-05	g^2/ml^2
							e_s	0.0036	g/ml

Example 2: Replicates revisited


Could it be we underestimated the **calibration error on volume**?

					reading error on mass:			1.00E-04	g
					new reading error on vol:			0.035	ml
Error propagation Worksheet									
$f(x_1,x_2,x_3)$			f	ρ_{BF}	**1.739**	**g/ml**	**$2e_s$**	**0.012**	**g/ml**
	x_i	**value**		**df/dx_i**	**$(df/dx_i)^2$**	**e_{xi}**	**e_{xi}^2**	**$(df/dx_i)^2 e_{xi}^2$**	
x_1	M_F	30.800	g	0.10	0.010	5.8E-05	3.3E-09	3.33E-11	g^2/ml^2
x_2	M_E	13.410	g	-0.10	0.010	5.8E-05	3.3E-09	3.33E-11	g^2/ml^2
x_3	V_{pyc}	10.000	ml	-0.174	0.0302	0.035	1.2E-03	3.705E-05	g^2/ml^2
							e_s^2	3.70E-05	g^2/ml^2
							e_s	0.0061	g/ml

Example 2: Replicates revisited



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Summary: Error Analysis with Real Numbers

- To understand the accuracy of our numbers, we need to determine a ***confidence interval.***
  $\bar{x} \pm 2e_s$ with 95.0% confidence — For replicate data with $n < 7$, replace "2" with $t_{0.025,n-1}$
- The **Standard error e_s** for a measured quantity is the largest of:
  - e_s determined by *replicates* $e_s = s/\sqrt{n}$ or
  - e_s by estimate of *reading error* $e_s = e_R/\sqrt{3}$ or
  - e_s by estimate of *calibration error* $e_s = \text{max error}/2$
- Standard error e_f for derived quantities (arrived at from equations), is obtained through **error propagation,** which is a combination of *variances*.
- Replication always improves the **estimation of the mean.** — The answer from replicates is more reliable than single values.
- The **prediction interval of the next value of x** should encompass 95% of all measured values. — 95% PI: $\bar{x} \pm 2s$ or $\bar{x} \pm t_{0.025,n-1}s$ if $n < 7$
- The weighting values $\left(\frac{\partial f}{\partial x_i}\right)^2 e_{x_i}^2$ indicate the **impact** of individual errors on the final value.
- **Estimates** for e_s (particularly those obtained through e_R) may need to be re-evaluated, if unreasonably narrow confidence intervals are identified.

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Fundamentals of Chemical Engineering Laboratory

Error Analysis for Laboratory Data

1. Quick start—Replicate error
2. Reading Error
3. Calibration Error
4. Error Propagation

Professor Faith Morrison

Department of Chemical Engineering
Michigan Technological University

Final takeaway:

1. You must know the uncertainty in your numbers
2. The 3 worksheets help you assess: replicate, reading, and calibration error
3. Final worksheet helps you carry out error propagation
4. These are the tools you need to determine the uncertainty in your numbers.

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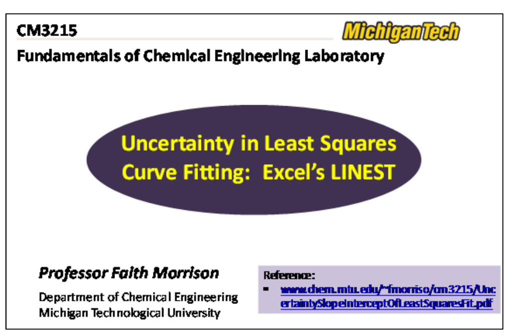
Next: Least Squares (an application of error propagation)



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