MEMORANDUM

TO: Prof. Davis Hubbard

FROM: Prof. Faith A. Morrison

DATE: 22 April 2014

RE: Investigation of Variability of Bourdon Gauge Sets in the Chemical Engineering Transport Laboratory

Introduction

Our group was asked to investigate the accuracy and reproducibility of the 8 Bourdon gauge pairs associated with the 8 lab stations in room 103 of the Chem. Sci. building. Two Bourdon gauges form a portable device used to measure pressure drops in the flow loops of the Chemical Engineering Transport Laboratory at Michigan Tech.

The objective for this lab was to measure a flow pressure drop with each of the 8 Bourdon gauge pairs and to compare the accuracy and reproducibility of the results. The gauge pairs were used to measure pressure drop across a six-foot section of a ¼" copper tube for flow rates up to 3.5 *gpm*. Results are presented in terms of friction factor versus Reynolds number. We were also asked to identify any problems, inaccuracies, or operational difficulties that we encountered.

Experimental

Pressure measurements were taken at two pressure taps on the ¼" line in the flow loop at Station 4 using each of 8 Bourdon gauge pairs. The water in the apparatus (municipal supply; no treatment applied) was driven by a ½ hp constant speed centrifugal pump through a piping network from one open tank to a second tank. The flow rate was set with a needle valve, and the actual value of flow rate was determined by using a calibrated rotameter (Morrison, 2010a). For the Bourdon gauge evaluations, five flow rates were chosen from 1.5 *gpm* to 3.5 *gpm* measured in 0.5 *gpm* increments. The pressure drop was obtained by connecting the Bourdon gauges to two pressure taps 6.0 ft apart on a nominal ¼" copper tube (type L, $ID = 0.315\ in$, Cooper Development Association, 2016). For each pair, the left Bourdon gauge measured both below and above ambient pressure, while the right Bourdon gauge only measured above ambient pressure. Three replicates of pressure drop at five flow rates were taken and averaged. Data were taken in random order to prevent systematic error; water temperature was recorded.

Results

Fifteen measurements of pressure drop versus flow rate were obtained (Harris, 2009); the raw data may be found in the appendix. The measured flow rates were converted into Reynolds number, Re, using equations (1) and (2).

$$V = \frac{4Q}{\pi D^2} \qquad (1)$$

$$\mathrm{Re} = \frac{\rho V D}{\mu} \qquad (2)$$

where V is the average water velocity, Q is the volumetric flow rate, D is the true inner diameter of the ¼'' tubing, ρ is water density, and μ is water viscosity. Literature values at $25^{o}C$ were used for both water density and water viscosity (Geankoplis, 2003). The measured pressure drops were converted to Fanning friction factor f with equation 3 (Geankoplis, 2003).

$$f = \frac{\left(\frac{1}{4}\right)(P_o - P_L)}{\frac{L}{D}\left(\frac{1}{2}\rho V^2\right)} \qquad (3)$$

where $P_0 - P_L = \Delta p$ is the pressure drop along $L = 6.0\ ft$ of copper tubing. For the eight gauge pairs, friction factor versus Reynolds number is shown in Figure 1. Also shown in Figure 1 is the trend from Nikuradse's data for the friction factor for smooth pipes (Morrison, 2013).

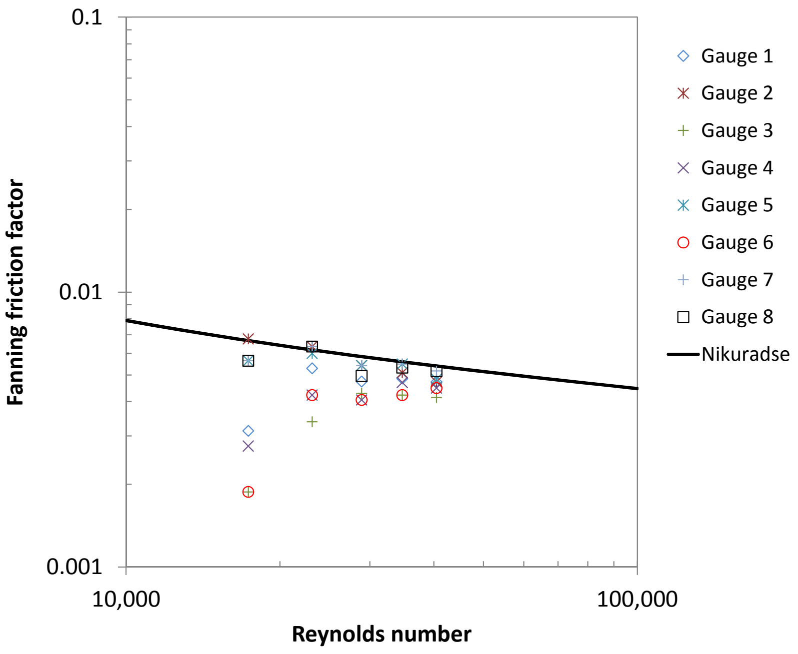


Figure 1: Friction factor versus Reynolds number for each of eight Bourdon gauge pairs. Also shown is a line indicating the trend in Nikuradse's smooth-pipe data obtained from a correlation (Morrison, 2013).

Discussion

Stable flows were readily established and no operational difficulties were encountered. The flow rates chosen for our experiments ranged from 1.5 to 3.5 gpm; in the ¼ inch tube these flow rates correspond to Reynolds numbers between 10^4 and 10^5, and the flows were turbulent. In smooth pipes, literature values of the friction factor expected for this range of Reynolds number are from 0.007 to 0.005 (Morrison, 2013).

The experimental data fall at or below the literature trend and show a degree of spread that is quite significant at the lowest Reynolds numbers tested (from 3% above the literature value to 72% below) The spread diminishes at higher Reynolds numbers to between 2 and 4% below the literature value. In most cases, the eight Bourdon gauge pairs did not agree in their values for $\Delta p(Q)$, as discussed in full below.

An error analysis was performed, and we determined that almost all of the error in measured friction factor could be attributed to reading and calibration error of the Bourdon gauges. Following standard protocols, we estimated the standard error e_s in each of the quantities in equation 3 and found that the resulting error in Fanning friction factor was insensitive to all but the value for the error in Δp, $e_s(\Delta p)$ (see error propagation worksheets in the appendix). Consideration of the sources of reading error in Δp and taking into account both minimum and maximum error estimates, we found that a reasonable estimate for the reading error is $e_s(\Delta p) \approx 0.58\, psi$. The errors in Δp and V (the value of $e_s(V)$ was estimated as $\pm 1\ in/s$) were propagated into error in f for each measurement. Because V^2 appears in the denominator of equation 3, the effect of $e_s(\Delta p)$ on the error in f is amplified at low flow rates. Error bars representing 95% confidence intervals of $\pm 2e_s$ (twice the propagated standard error) on f are included in Figure 2; Figure 3 includes the same results plotted for each Bourdon gauge pair individually. In Figures 2 and 3 the lower ranges of error bars are omitted when the act of subtracting $2e_s$ would result in a negative f. Error was found to be a strong function of flow rate Q.

For all Bourdon gauge pairs the error in f is quite large at low Reynolds numbers, and we judge the Bourdon gauge sets to be unusable below 2.5 gpm in the ¼ in tube, which corresponds to $\Delta p \approx 4\, psi$. For values of f that correspond to flow rates above 2.5 gpm, the measured values of f match, or nearly match, the literature values within the uncertainty determined by the error propagation calculation (Figure 3). Bourdon gauge pairs 1, 3, 4, and 6 had poor correspondance with the literature values (friction factor was between 10 and 73% below the literature values). Bourdon gauge pairs 2, 5, 7, and 8 showed better correspondance (between 1 and 21% below the literature values). In all cases measured friction factors were systematically below the literature values.

We evaluated the agreement among the eight Bourdon gauge pairs by treating the measurements on different Bourdon gauge pairs at each flow rate as replicate measurements of $\Delta p(Q)$ (Figure 4). Aggregating the Δp results across the Bourdon gauges allows us to calculate a “replicate” standard deviation s. As shown in Figure 4, we find that all the measured data fall within $\pm 2s$ of the mean across pairs, allowing us to conclude that the differences among the gauges are due to random errors in

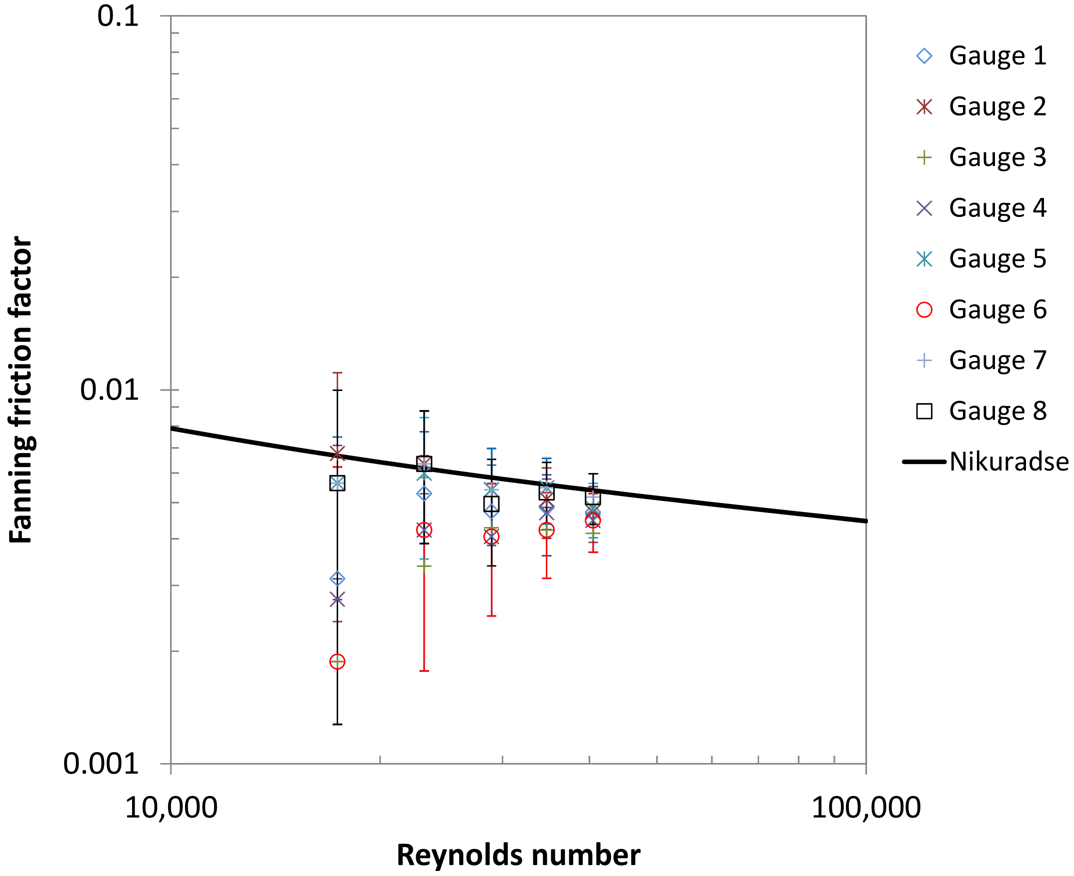


Figure 2: Friction factor versus Reynolds number for each Bourdon gauge pair with 95% confidence intervals of $\pm 2e_s$ as determined through propagation of reading error estimates. Figure 3 includes each data set plotted individually.

performance or calibration. For the measurements reported here, the standard deviations were found to be $0.44 \leq s \leq 0.56\,psi$ (varies with Q). The near agreement between $s \approx 0.56$ and the uncertainty estimated from error propagation $e_s(\Delta p) = 0.58\,psi$ supports that reasonable estimates were made in the error propagation calculations.

The data lie systematically below the literature values; a possible reason for this is the design of the laboratory flow loop. In Nikuradse's experiments, care was taken to have a very long pipe to allow the flow to develop to its true, long-pipe structure. By contrast, the laboratory apparatus contains a 90^{o} bend within 10 inches of the upstream pressure tap. This bend may prevent the flow from becoming well established, and this entry-effect may reduce the pressure drop that develops.

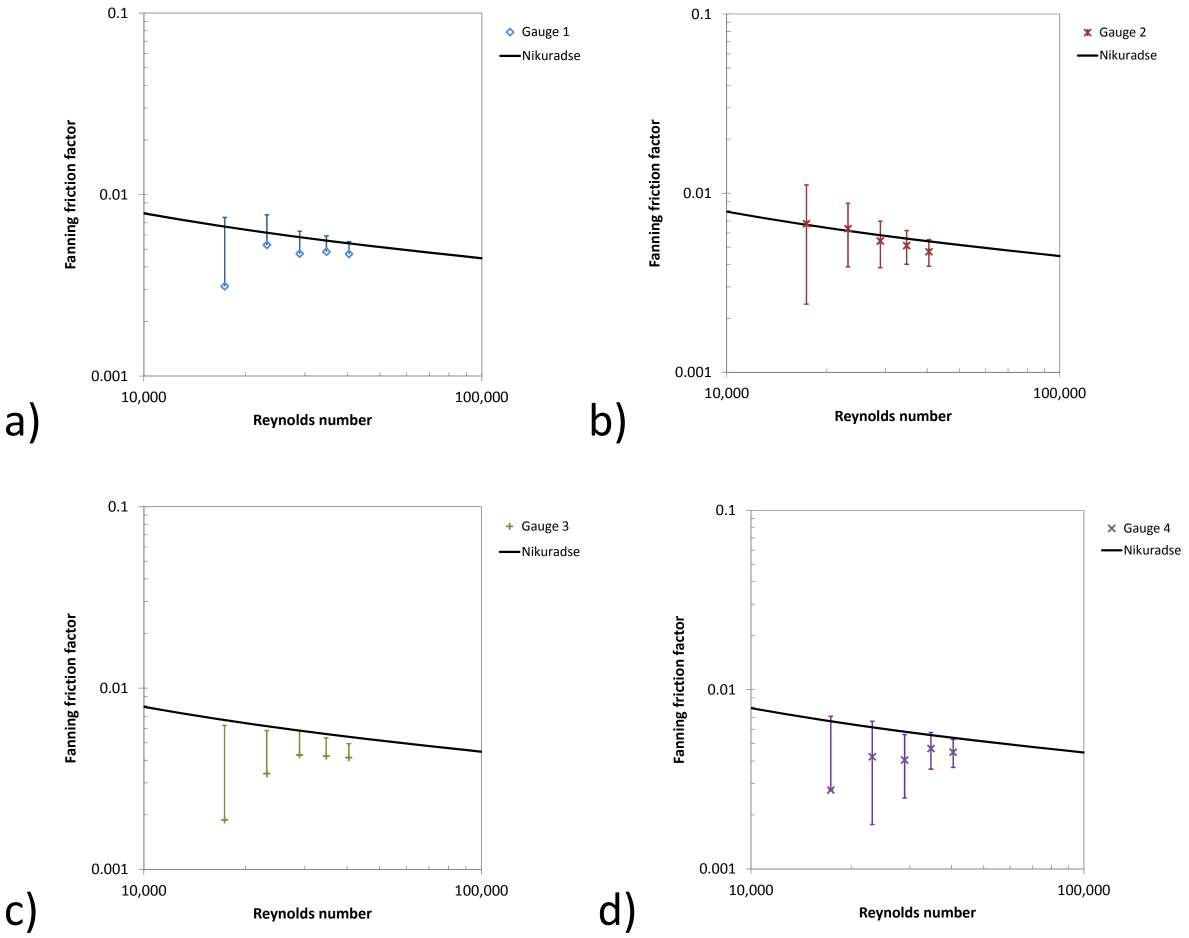


Figure 3 (Part 1): Fanning friction factor versus Reynolds number for the eight Bourdon gauge pairs. Each pair of gauges is shown singly, with the appropriate error bars: Gauges 1-8 are shown as Figures 3a-3h. The lower range of error bars is omitted when the act of subtracting $2e_s$ would result in a negative f.

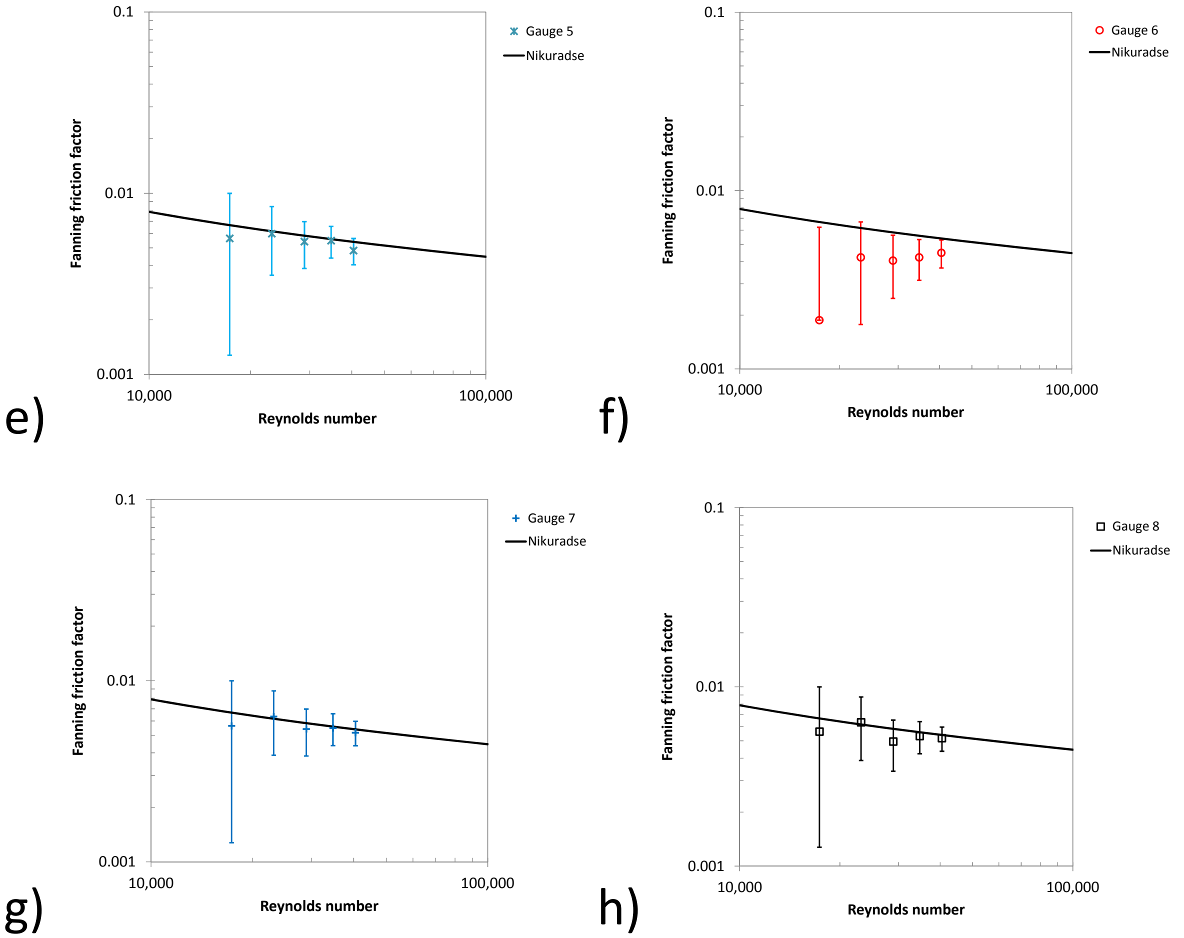


Figure 3 (Part 2): Fanning friction factor versus Reynolds number for the eight Bourdon gauge pairs. Each pair of gauges is shown singly, with the appropriate error bars: Gauges 1-8 are shown as Figures 3a-3h. The lower range of error bars is omitted when the act of subtracting $2e_s$ would result in a negative f.

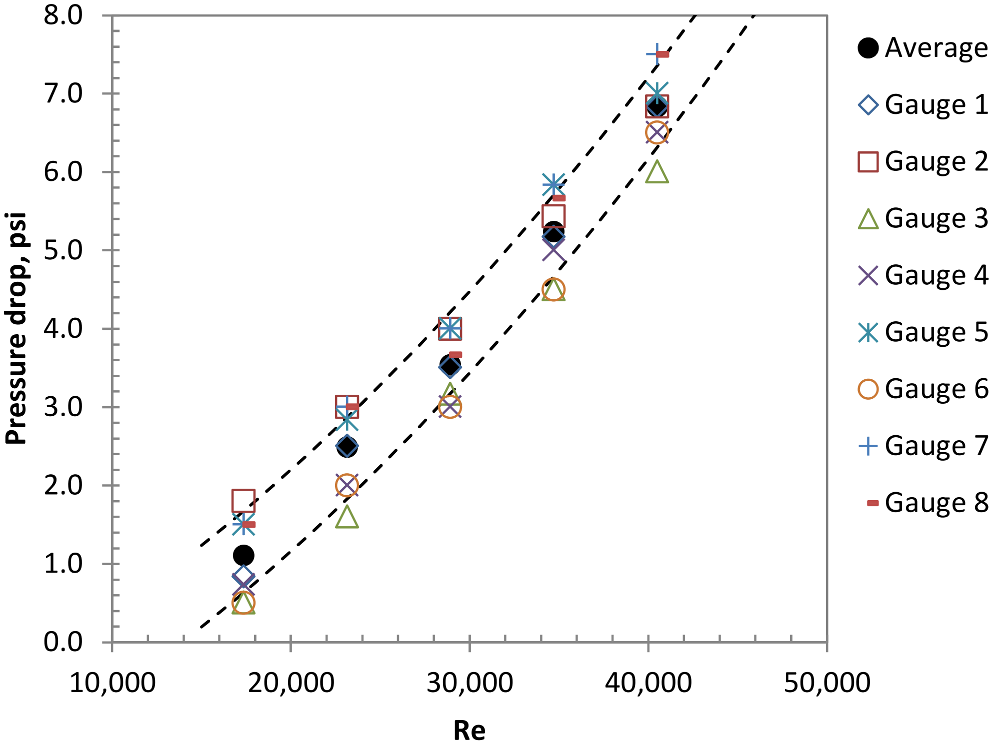


Figure 4: Pressure drops (an average of triplicates) as a function of Reynolds number from eight Bourdon gauge pairs. Also shown is the average across all gauge pairs (filled circles) and the 95% confidence interval $\pm 2s$ (dashed lines) constructed from the sample standard deviation calculated among all the gauge pairs, s.

Conclusions

Eight Bourdon gauge pairs were tested on a flow loop and the measured pressure drops varied with a standard deviation that ranged from $0.44 \leq s \leq 0.56\ psi$. Error propagation showed that the error in Δp, $e_s(\Delta p)$, dominates the error in Fanning friction factor and agrees with the standard deviation among the Bourdon gauge pairs. Due to the established uncertainty of the Bourdon gauge pairs, it is not recommended to use the Bourdon gauge pairs for flow rates below $2.5\ gpm$ on the ¼ in tube. Bourdon gauge pairs 2, 5, 7, and 8 appear to be more accurate than pairs 1, 3, 4, and 6. Friction factors calculated from the pressure drop data are systematically below the literature trend for $f(\mathrm{Re})$ but agree with the literature value within the established uncertainties. Improved agreement with the literature may result if the length of the straight pipe is increased.

References

Copper Development Association, Inc., 2006. *The Copper Tube Handbook*, "Dimensions and Physical Characteristics of Copper Tube: Type L," (pg. 21), 260 Madison Avenue, New York, available at www.chem.mtu.edu/~fmorriso/cm3215/copper_tube_handbook.pdf, accessed 22 January 2010.

Christie J. Geankoplis, 2003. *Transport Processes and Separation Process Principles*, 4th edition (Upper Saddle River, NJ: Pearson Education, Inc.).

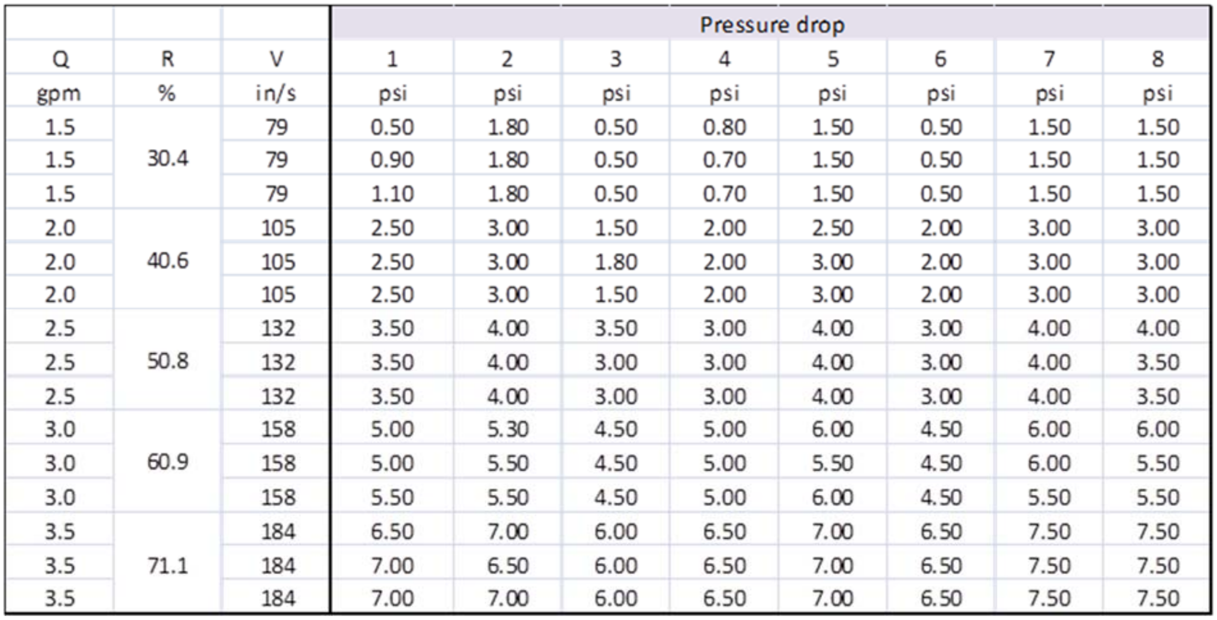
Ben Harris, 2009. "Lab 4 Makeup: Bourdon Gauge Reliability with Respect to Reynolds Number and Friction Factor," CM3215 Laboratory, Department of Chemical Engineering, Michigan Technological University, Houghton, MI, 9 October 2009.

Faith A. Morrison, 2014. "Data Archive for CM3215 Laboratory Data," Department of Chemical Engineering, Michigan Technological University, Houghton, MI, available at www.chem.mtu.edu/~fmorriso/cm3215/archive.html, accessed 22 January 2010(a).

Faith A. Morrison, 2013. "Data Correlation for Friction Factor in Smooth Pipes," (pg. 532-3) from *An Introduction to Fluid Mechanics* (Cambridge University Press, New York).

Appendix: Raw data and Error Analysis Worksheets

Table A1: Raw data for Investigation of Variability of Eight Bourdon Gauge Sets in the Chemical Engineering Junior Laboratory; Faith A. Morrison, Michigan Tech, 22 January 2010 (Harris, 2009).



			Pressure drop							
Q	R	V	1	2	3	4	5	6	7	8
gpm	%	in/s	psi	psi	psi	psi	psi	psi	psi	psi
1.5	30.4	79	0.50	1.80	0.50	0.80	1.50	0.50	1.50	1.50
1.5		79	0.90	1.80	0.50	0.70	1.50	0.50	1.50	1.50
1.5		79	1.10	1.80	0.50	0.70	1.50	0.50	1.50	1.50
2.0	40.6	105	2.50	3.00	1.50	2.00	2.50	2.00	3.00	3.00
2.0		105	2.50	3.00	1.80	2.00	3.00	2.00	3.00	3.00
2.0		105	2.50	3.00	1.50	2.00	3.00	2.00	3.00	3.00
2.5	50.8	132	3.50	4.00	3.50	3.00	4.00	3.00	4.00	4.00
2.5		132	3.50	4.00	3.00	3.00	4.00	3.00	4.00	3.50
2.5		132	3.50	4.00	3.00	3.00	4.00	3.00	4.00	3.50
3.0	60.9	158	5.00	5.30	4.50	5.00	6.00	4.50	6.00	6.00
3.0		158	5.00	5.50	4.50	5.00	5.50	4.50	6.00	5.50
3.0		158	5.50	5.50	4.50	5.00	6.00	4.50	5.50	5.50
3.5	71.1	184	6.50	7.00	6.00	6.50	7.00	6.50	7.50	7.50
3.5		184	7.00	6.50	6.00	6.50	7.00	6.50	7.50	7.50
3.5		184	7.00	7.00	6.00	6.50	7.00	6.50	7.50	7.50