Final Exam
CBE 425/565 Polymer Rheology
Princeton University
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Please be neat.
This exam is closed book, closed notes. No internet-capable devices are permitted.
See the formula sheet for more definitions/formulas.

Fluid force on a surface S: \( \mathbf{F}_{on} = \iint_S \left[ \mathbf{n} \cdot (-\mathbf{n}) \right]_{\text{surface}} dS \)

Flow rate through surface S: \( Q = \iint_S \left[ \mathbf{n} \cdot \mathbf{v} \right]_{\text{surface}} dS \)

Fluid torque on a surface S: \( \mathbf{T}_{on} = \iint_S \left[ R \times \left( \mathbf{n} \cdot (-\mathbf{n}) \right) \right]_{\text{surface}} dS \)

Spherical coordinates:
\[
\begin{align*}
  x &= r \sin \theta \cos \phi \\
  y &= r \sin \theta \sin \phi \\
  z &= r \cos \theta
\end{align*}
\]
\[
\begin{align*}
  \mathbf{e}_r &= (\sin \theta \cos \phi) \mathbf{e}_x - (\sin \theta \sin \phi) \mathbf{e}_y + \cos \theta \mathbf{e}_z \\
  \mathbf{e}_\theta &= (\cos \theta \cos \phi) \mathbf{e}_x + (\cos \theta \sin \phi) \mathbf{e}_y + (-\sin \theta) \mathbf{e}_z \\
  \mathbf{e}_\phi &= (-\sin \phi) \mathbf{e}_x + \cos \phi \mathbf{e}_y
\end{align*}
\]

Cylindrical coordinates:
\[
\begin{align*}
  x &= r \cos \theta \\
  y &= r \sin \theta \\
  z &= z
\end{align*}
\]
\[
\begin{align*}
  \mathbf{e}_r &= \cos \theta \mathbf{e}_x + \sin \theta \mathbf{e}_y \\
  \mathbf{e}_\theta &= (-\sin \theta) \mathbf{e}_x + \cos \theta \mathbf{e}_y \\
  \mathbf{e}_z &= \mathbf{e}_z
\end{align*}
\]
1. (20 points) What is the 2-component of $\nabla \cdot \left( B^T \right)$? Note that $B$ is a tensor. Work out the solution in Einstein notation and write out all summations in your final answer.

2. (20 points) Calculate the expression for strain $\gamma = \gamma(t', t)$ in the start up of steady shearing experiment. Note that $t'$ ranges from $-\infty$ to $t$ and $t$ is greater than zero.

3. (20 points) Derive the prediction of the Lodge model for the three material functions $\eta, \Psi_1, \Psi_2$ in steady shear flow. For $\eta$ and $\Psi_2$ carry out all integrals. For $\Psi_1$ set up the integral but you do not need to carry out the integration.

4. (20 points) Each part is 5 points.
   a) For the capillary flow data plotted in the figure below, is there evidence of entry/exit losses? Each trend represents a set of runs on different capillaries at a common value of $4Q/\pi R^3$. Explain your answer in one or two sentences.

   b) What is the advantage of cone-and-plate flow over parallel plate flow? Please confine your answer to one or two sentences.

   c) The Weissenberg-Rabinowitsch correction for capillary flow is shown below. Why is this needed? Please confine your answer to one or two sentences.

\[
\dot{\gamma}_R = \frac{4Q}{\pi R^3} \left( \frac{1}{4} + \frac{dln \left( \frac{4Q}{\pi R^3} \right)}{dln \left( \frac{\Delta p R}{2L} \right)} \right)
\]

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d) For shear flow between infinite wide parallel plates separated by a gap of 1.0mm, how far would the upper plate need to move to the right to subject the sample to a strain of 1.8 (180%)?

5. (20 points) The material whose rheological behavior is given in the figure below is made to flow in a capillary rheometer (see figure next page). At steady state, what proportion of the wall shear stress is due to gravity at \( g = 9.80 \, \text{m/s}^2 \)? Please show your calculations to justify your work. You may neglect end effects. The density of the material is \( \rho = 0.892 \, \text{g/cm}^3 \).

\[ \eta_0 = 1.8 \times 10^4 \, \text{poise} \]
\[ \lambda = 1.5 \, \text{s} \]
\[ \alpha = 1.5 \]
\[ n = 0.24 \]
\[ \eta_\infty = 0 \]

Note: 10 poise = 1 Pa s
• Capillary length = 30 mm
• Isothermal
• Incompressible
• $P_1 - P_2$ measured

1.0 mm diameter capillary

$Q$

$P_2$ = exit pressure