Homework 2 2004

CM4650 Polymer Rheology

Due Wednesday February 4, 2004 in class

1. (10 points) Expand completely the expression $\nabla \cdot (\alpha v u)$ where $\alpha$ is a scalar, but it is not constant. Write your answers in Einstein and also in vector (also called Gibbs) notation.

2. (20 points) Through experiments it is determined for a particular flow that the velocity profile is given by

$$v = \begin{pmatrix} \alpha x_2 \\ bx_1 + x_2^2 \\ cx_3 \end{pmatrix}_{123}$$

This is written in the Cartesian coordinate system with unit vectors $\hat{e}_1$, $\hat{e}_2$, and $\hat{e}_3$, and variables $x_1$, $x_2$, and $x_3$; $a$, $b$, and $c$ are constants. Evaluate $\nabla \cdot v$, $\nabla v^2$, and $\nabla v$ for this system.

3. (10 points) What is a tensor? We showed in class that the force $\mathbf{f}$ on an arbitrary surface (area = $dA$, unit normal = $\hat{n}$) at a point in a fluid could be calculated by

$$\mathbf{f} = \hat{n} \cdot \mathbf{\Pi} dA$$

where $\mathbf{\Pi}$ is the total stress tensor at that point. Why is the condition of stress at a point in a fluid described by a tensor rather than some other type of mathematical entity? Please confine your answer to 6 sentences.

4. (10 points) For a Newtonian, incompressible fluid (viscosity = $\mu = 0.01 \text{ Pa s}$) subjected to the velocity profiles given below (velocity in $m/s$ for $x_1$, $x_2$, and $x_3$ given in $m$), calculate the stress tensor $\mathbf{\tau}$.

   a) $\mathbf{v} = \begin{pmatrix} 9x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$

   b) $\mathbf{v} = \begin{pmatrix} -3x_1 \\ -3x_2 \\ 6x_3 \end{pmatrix}_{123}$

5. (10 points) Text 3.8 (what are the boundary conditions for the given flows?)

6. (20 points) Text, Problem 3.14 (Newtonian fluid down an inclined plane)

7. (20 points) Text, Problem 3.18 (Tangential annular flow of a Newtonian Fluid; this is a longer problem)