The steady shear viscosity function $\eta$ can be fit to experimental data to an arbitrarily high precision.

Does this mean that Generalized Newtonian Fluid models are okay to use in all situations?

**Not necessarily.** A constitutive model needs to be able to predict *all stresses* in *all flows*, not just shear stresses in steady shearing. We need to check predictions.

For example, does the GNF predict the shear normal stresses?

$$ \tau = \begin{pmatrix} \tau_{11} & \tau_{12} & 0 \\ \tau_{21} & \tau_{22} & 0 \\ 0 & 0 & \tau_{33} \end{pmatrix}_{123} $$

**Generalized Newtonian Fluid (GNF) constitutive equation**

$$ \tau = -\eta(\dot{\gamma}) \begin{pmatrix} 2 \frac{\partial v_1}{\partial x_1} + \frac{\partial v_2}{\partial x_2} + \frac{\partial v_3}{\partial x_3} \\ \frac{\partial v_1}{\partial x_2} + 2 \frac{\partial v_2}{\partial x_1} + \frac{\partial v_3}{\partial x_3} \\ \frac{\partial v_1}{\partial x_3} + \frac{\partial v_2}{\partial x_2} + 2 \frac{\partial v_3}{\partial x_1} \end{pmatrix}_{123} $$

**In Shear Flow:**

$$ v = \begin{pmatrix} v_1 \\ 0 \\ 0 \end{pmatrix}_{123}, \quad \dot{v} = \frac{\partial v_1}{\partial x_1}, \quad \tau = \begin{pmatrix} \tau_{11} & \tau_{12} & \tau_{13} \\ \tau_{21} & \tau_{22} & \tau_{23} \\ \tau_{31} & \tau_{32} & \tau_{33} \end{pmatrix}_{123} = \begin{pmatrix} 0 & -\eta(\dot{\gamma}) \frac{\partial v_1}{\partial x_2} & 0 \\ -\eta(\dot{\gamma}) \frac{\partial v_1}{\partial x_3} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{123} $$

**No matter what we pick for the function $\eta(\dot{\gamma})$, we cannot predict shear normal stresses with a Generalized Newtonian Fluid.**
What does the GNF predict for start-up shear stresses?

**imposed shear rate**

\[ \dot{\gamma}_{21} = \frac{v_1(t)}{H} \]

Shear stress response

What the data show:

- \( \tau_{21}(t) \)

What the GNF models predict:

- \( \tau_{21}(t) \)

No matter what we pick for the function \( \eta(\dot{\gamma}) \), we cannot predict the time-dependence of shear start-up correctly with a GNF.

Start-up shear stresses

What the data show:

- \( \tau_{21}(t) \), increasing \( \dot{\gamma} \)

What the GNF models predict:

- \( \tau_{21}(t) \), increasing \( \dot{\gamma} \)

Correctly captures rate dependence

misses start-up effects

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**What does the GNF predict in steady elongational flow?**

- **imposed deformation**
  - (steady state)

**elongational stress response**

- **What the data show:**
  \[ \lim_{\epsilon \to 0} \eta = 3\eta_0 \]
  Trouton’s Rule
  *(there is limited elongational viscosity data available)*

- **What the GNF models predict:**
  \[ \eta = 3\eta \]
  For all deformation rates
  If a material shear-thins, GNF predicts it will tension-thin.

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**Summary:** *Generalized Newtonian Fluid Constitutive Equations*

**PRO:**
- A first constitutive equation
- Can match steady shearing data very well
- Simple to calculate with
- Found to predict pressure-drop/flow rate relationships well

**CON:**
- Fails to predict shear normal stresses
- Fails to predict start-up or cessation effects (time-dependence, memory) – only a function of instantaneous velocity gradient
- Derived ad hoc from shear observations; unclear of validity in non-shear flows