Predictions of the (single-mode) Maxwell Model

\[ \tau + \frac{\eta}{G} \frac{\partial \tau}{\partial t} = -\eta \frac{\partial \nu}{\partial t} \]
\[ \tau(t) = -\int_{t_0}^{t} \left( \frac{\eta}{\lambda} \right) e^{-\frac{(t-t')}{\lambda}} \gamma(t') \, dt' \]

- Steady shear
  - \( \eta = \eta_0 \)
  - \( \Psi_1 = \Psi_2 = 0 \)
  - Fails to predict shear normal stresses.
  - Fails to predict shear-thinning.

- Steady elongation
  - \( \bar{\eta} = 3\eta_0 \)
  - Trouton’s rule

Steady shear viscosity and first normal stress coefficient

**BOGER FLUIDS**

There are some systems with a constant viscosity but still start-up effects.

Figure 6.5, p. 173 Binnington and Boger; PIB soln
Steady shear viscosity and first and **second** normal stress coefficient

**BOGER FLUIDS**

**Figure 6.6, p. 174 Magda et al.; PS solns**

© Faith A. Morrison, Michigan Tech U.

### Step Shear Strain Material Functions

**Kinematics:**

\[ \dot{\gamma} = \begin{cases} \dot{\gamma}(t) \gamma_2 & \text{if } \dot{\gamma}(t) \neq 0 \\ 0 & \text{if } \dot{\gamma}(t) = 0 \end{cases} \]

\[ \dot{\gamma}(t) = \lim_{\varepsilon \to 0} \frac{\gamma_0 t}{\varepsilon}, \quad 0 \leq t < \varepsilon \]

\[ \dot{\gamma} = \text{constant} = \gamma_0 \]

**Material Functions:**

\[ G(t, \gamma_0) = \frac{-\tau_{21}(t, \gamma_0)}{\gamma_0} \]

First normal-stress relaxation modulus

Relaxation modulus

\[ G_{\psi_1} = \frac{-\left(\tau_{11} - \tau_{22}\right)}{\gamma_0^2} \]

Second normal-stress relaxation modulus

\[ G_{\psi_2} = \frac{-\left(\tau_{22} - \tau_{33}\right)}{\gamma_0^2} \]

© Faith A. Morrison, Michigan Tech U.
Predictions of the (single-mode) Maxwell Model

\[
\frac{\tau}{G} - \eta_0 \frac{\partial \tau}{\partial t} = -\eta_0 \gamma \\
\tau(t) = -\int_{t_0}^{t} \left( \frac{\eta_0}{\lambda} \right) e^{-\frac{(t-t')}{\lambda}} \gamma(t') \, dt'
\]

- **Shear start-up**: \( \eta^+(t) = \eta_0 \left( 1 - e^{-t/\lambda} \right) \)  
  \( \Psi_1^+(t) = \Psi_2^+(t) = 0 \)  
  **Does** predict a gradual build-up of stresses on start-up.

- **Step shear strain**: 
  \( G(t) = \frac{\eta_0}{\lambda} e^{-t/\lambda} \)  
  \( G_{\Psi_1} = G_{\Psi_2} = 0 \)  
  **Does** predict a reasonable relaxation function in step strain (but no normal stresses again).

---

**Step-Shear-Strain Material Function** \( G(t) \) for Maxwell Model

© Faith A. Morrison, Michigan Tech U.
Comparison to experimental data

Figure 8.4, p. 274 data from Einaga et al., PS 20% soln in chlorinated diphenyl

© Faith A. Morrison, Michigan Tech U.