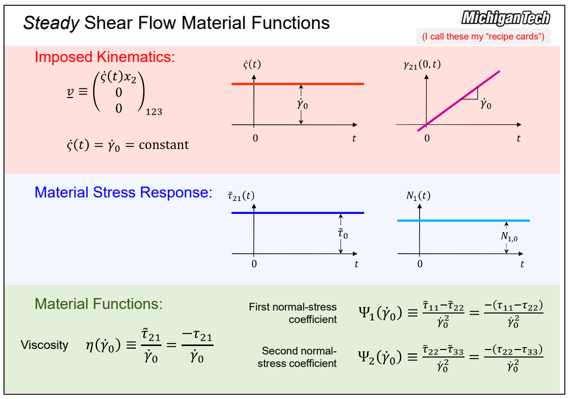
Chapter 5: Material Functions



CM4650
Polymer Rheology
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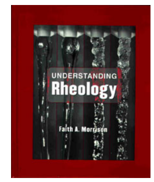

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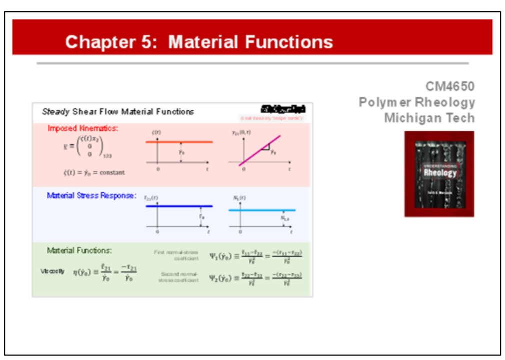

What are
material functions,
and why do we need
them?

2

What are material functions and why do we need them?

NEWTONIAN

Constitutive equation

$$\underline{\underline{\tau}} = -\underline{\underline{\tilde{\tau}}} = -\mu\left(\nabla \underline{v} + (\nabla \underline{v})^T\right)$$

+

Momentum balance

$$\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v}\right) = -\nabla p - \nabla \cdot \underline{\underline{\tau}} + \rho \underline{g}$$

+

Flow scenarios

=

predictions of:

Velocity and pressure fields

$$\underline{v}, p(x, y, z)$$

When we know $\underline{\underline{\tau}}(\underline{v})$, the flow modeling effort is expended on developing the flow scenario and solving the math.

2

What are material functions and why do we need them?

NON-NEWTONIAN

Constitutive equation

$$\underline{\underline{\tau}} = -\underline{\underline{\tilde{\tau}}} = \underline{\underline{\tau}}(\underline{v}) \quad ?$$

+

Momentum balance

$$\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v}\right) = -\nabla p - \nabla \cdot \underline{\underline{\tau}} + \rho \underline{g}$$

+

Flow scenarios

=

~~predictions of:~~

Velocity and pressure fields

~~$\underline{v}, p(x, y, z)$~~

When we don't know $\underline{\underline{\tau}}(\underline{v})$, this approach is a nonstarter.

2

What are material functions and why do we need them?

New approach

Hypothetical Constitutive equation: $\underline{\underline{\tau}} = -\underline{\underline{\tilde{\tau}}} = \underline{\underline{\tau}}(\underline{v})$

\+

Standard Velocity and pressure fields: $\underline{v} \equiv \begin{pmatrix} \dot{\varsigma}(t) x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$ $\quad p = p_0$

=

predictions of:

Material Functions (stress responses)

$$\eta(\dot{\gamma}_0) \equiv \frac{\tilde{\tau}_{21}}{\dot{\gamma}_0} = \frac{-\tau_{21}}{\dot{\gamma}_0}$$

$$\Psi_1(\dot{\gamma}_0) \equiv \frac{\tilde{\tau}_{11} - \tilde{\tau}_{22}}{\dot{\gamma}_0^2} = \frac{-(\tau_{11} - \tau_{22})}{\dot{\gamma}_0^2}$$

$$\Psi_2(\dot{\gamma}_0) \equiv \frac{\tilde{\tau}_{22} - \tilde{\tau}_{33}}{\dot{\gamma}_0^2} = \frac{-(\tau_{22} - \tau_{33})}{\dot{\gamma}_0^2}$$

measurements of

=

material + $\underline{v} \equiv \begin{pmatrix} \dot{\varsigma}(t) x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$ $\quad p = p_0$ (Standard Velocity and pressure fields) + Ability to measure stresses

3

What are material functions and why do we need them?

Modeling

Hypothetical Constitutive equation: $\underline{\underline{\tau}} = -\underline{\underline{\tilde{\tau}}} = \underline{\underline{\tau}}(\underline{v})$

\+

Standard Velocity and pressure fields: $\underline{v} \equiv \begin{pmatrix} \dot{\varsigma}(t) x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$ $\quad p = p_0$

=

predictions of:

Material Functions (stress responses)

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measurements of

=

material + $\underline{v} \equiv \begin{pmatrix} \dot{\varsigma}(t) x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$ $\quad p = p_0$ (Standard Velocity and pressure fields) + Ability to measure stresses

Meet here: **Material Functions**

Measuring

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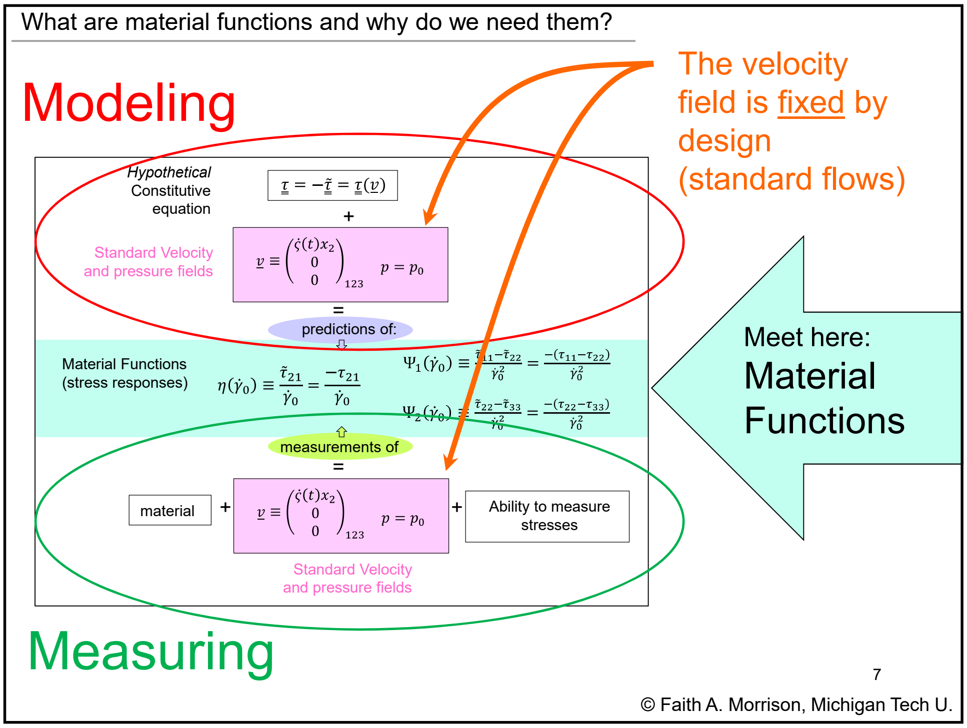
What are material functions and why do we need them?
Modeling
Hypothetical Constitutive equation
$\underline{\underline{\tau}} = -\underline{\underline{\tilde{\tau}}} = \underline{\underline{\tau}}(\underline{v})$
+
Standard Velocity and pressure fields
$\underline{v} \equiv \begin{pmatrix} \dot{\varsigma}(t)x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$ $p = p_0$
=
predictions of:
Material Functions (stress responses)
$\eta(\dot{\gamma}_0) \equiv \frac{\tilde{\tau}_{21}}{\dot{\gamma}_0} = \frac{-\tau_{21}}{\dot{\gamma}_0}$
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measurements of
=
material
+
$\underline{v} \equiv \begin{pmatrix} \dot{\varsigma}(t)x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$ $p = p_0$
+
Ability to measure stresses
Standard Velocity and pressure fields
Measuring
The velocity field is fixed by design (standard flows)
Meet here: Material Functions
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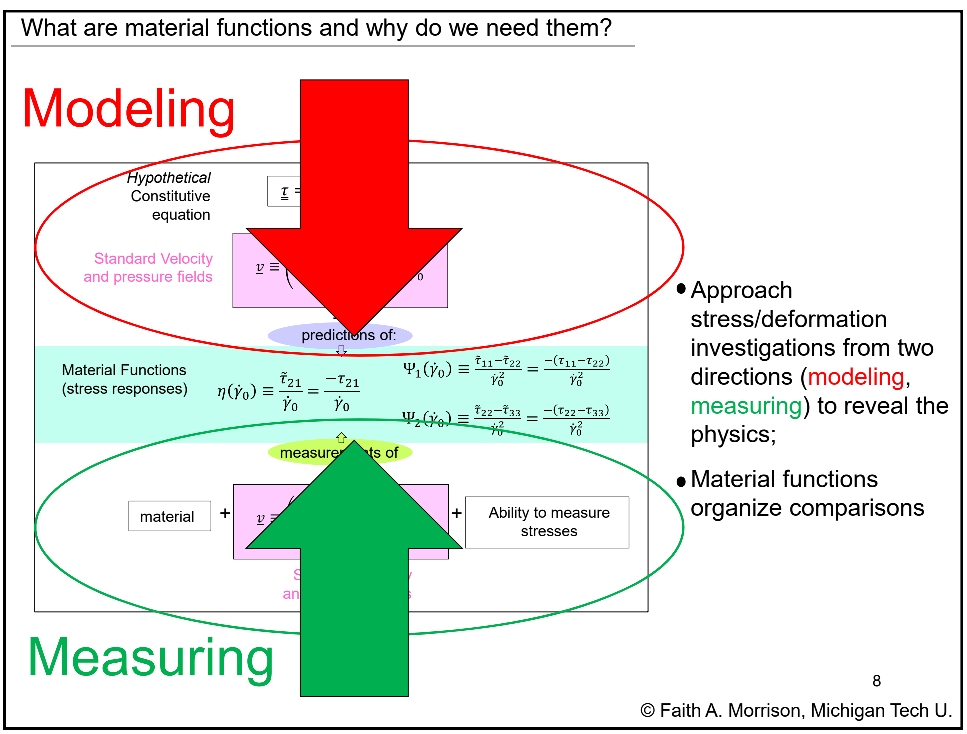
What are material functions and why do we need them?
Modeling
Hypothetical Constitutive equation
Standard Velocity and pressure fields
predictions of:
Material Functions (stress responses)
$\eta(\dot{\gamma}_0) \equiv \frac{\tilde{\tau}_{21}}{\dot{\gamma}_0} = \frac{-\tau_{21}}{\dot{\gamma}_0}$
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material
+
+
Ability to measure stresses
Measuring
• Approach stress/deformation investigations from two directions (modeling, measuring) to reveal the physics;
• Material functions organize comparisons
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What are material functions and why do we need them?

For non-Newtonian fluids:

- We do not know the stress/deformation relationship $\left(\underline{\underline{\tau}}(\underline{v})\right)$
- We approach stress/deformation investigations from two directions (modeling, measuring) to reveal the physics;
- Material functions organize comparisons

Hypothetical Constitutive equation: $\underline{\underline{\tau}} = -\underline{\underline{\tilde{\tau}}} = \underline{\underline{\tau}}(\underline{v})$

\+

Standard Velocity and pressure fields: $\underline{v} \equiv \begin{pmatrix} \dot{\varsigma}(t)x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$ $\quad p = p_0$

=

predictions of:

Material Functions (stress responses):

$$\eta(\dot{\gamma}_0) \equiv \frac{\tilde{\tau}_{21}}{\dot{\gamma}_0} = \frac{-\tau_{21}}{\dot{\gamma}_0}$$

$$\Psi_1(\dot{\gamma}_0) \equiv \frac{\tilde{\tau}_{11}-\tilde{\tau}_{22}}{\dot{\gamma}_0^2} = \frac{-(\tau_{11}-\tau_{22})}{\dot{\gamma}_0^2}$$

$$\Psi_2(\dot{\gamma}_0) \equiv \frac{\tilde{\tau}_{22}-\tilde{\tau}_{33}}{\dot{\gamma}_0^2} = \frac{-(\tau_{22}-\tau_{33})}{\dot{\gamma}_0^2}$$

measurements of

=

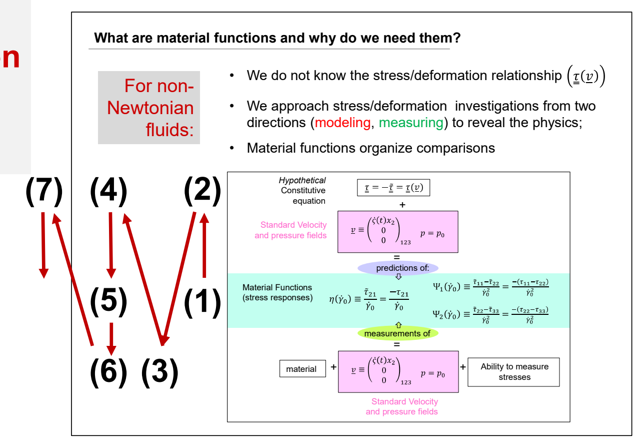
material + $\underline{v} \equiv \begin{pmatrix} \dot{\varsigma}(t)x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$ $\quad p = p_0$ + Ability to measure stresses

Standard Velocity and pressure fields

3

Investigating **Stress/Deformation** Relationships

(**Rheology**)



1. Choose a material function
2. Predict what Newtonian fluids would do
3. See what non-Newtonian fluids **do**
4. Hypothesize a $\underline{\underline{\tau}}(\underline{v})$
5. Predict the material function
6. Compare with what non-Newtonian fluids **do**
7. Reflect, **learn**, revise model, repeat.

Let's get started:

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Investigating **Stress/Deformation** Relationships (**Rheology**)

1. Choose a material function
2. Predict what Newtonian fluids would do
3. See what non-Newtonian fluids **do**
4. Hypothesize a $\underline{\underline{\tau}}(\underline{v})$
5. Predict the material function
6. Compare with what non-Newtonian fluids **do**
7. Reflect, **learn**, revise model, repeat.

1) Choose a material function

kinematics

1. Choice of flow (shear or elongation)

$$\underline{v} = \begin{pmatrix} \dot{\varsigma}(t)x_2 \\ 0 \\ 0 \end{pmatrix}_{123} \qquad \underline{v} = \begin{pmatrix} -\frac{1}{2}\dot{\varepsilon}(t)x_1 \\ -\frac{1}{2}\dot{\varepsilon}(t)x_2 \\ \dot{\varepsilon}(t)x_3 \end{pmatrix}_{123}$$

2. Choice of time dependence of $\dot{\varsigma}(t)$ or $\dot{\varepsilon}(t)$

3. Material functions definitions: will be based on τ_{21}, N_1, N_2 in shear or $\tau_{22}-\tau_{11}$, $\tau_{22}-\tau_{11}$ in elongational flows.

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Steady Shear Flow Material Functions

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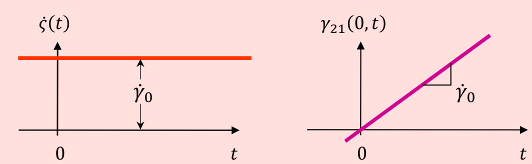
(I call these my "recipe cards")

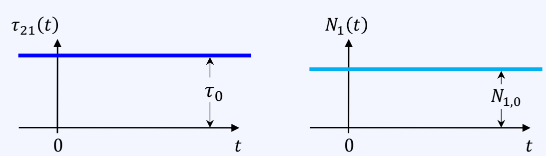
Imposed Kinematics:

$$\underline{v} \equiv \begin{pmatrix} \dot{\varsigma}(t)x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$$

$$\dot{\varsigma}(t) = \dot{\gamma}_0 = \text{constant}$$



Material Stress Response:



Material Functions:

Viscosity $\eta(\dot{\gamma}_0) \equiv \frac{\tilde{\tau}_{21}}{\dot{\gamma}_0} = \frac{-\tau_{21}}{\dot{\gamma}_0}$

First normal-stress coefficient $\Psi_1(\dot{\gamma}_0) \equiv \frac{\tilde{\tau}_{11}-\tilde{\tau}_{22}}{\dot{\gamma}_0^2} = \frac{-(\tau_{11}-\tau_{22})}{\dot{\gamma}_0^2}$

Second normal-stress coefficient $\Psi_2(\dot{\gamma}_0) \equiv \frac{\tilde{\tau}_{22}-\tilde{\tau}_{33}}{\dot{\gamma}_0^2} = \frac{-(\tau_{22}-\tau_{33})}{\dot{\gamma}_0^2}$

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Investigating **Stress/Deformation** Relationships (**Rheology**)

1. Choose a material function
2. Predict what Newtonian fluids would do
3. See what non-Newtonian fluids **do**
4. Hypothesize a $\underline{\underline{\tau}}(\underline{v})$
5. Predict the material function
6. Compare with what non-Newtonian fluids **do**
7. Reflect, **learn**, revise model, repeat.

2) Predict what **Newtonian** fluids would do

How do we predict material functions?

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What are material functions and why do we need them?

New Approach

Constitutive equation $\underline{\underline{\tau}} = -\underline{\underline{\tilde{\tau}}} = \underline{\underline{\tau}}(\underline{v})$

+

Standard Velocity and pressure fields $\underline{v} \equiv \begin{pmatrix} \dot{\varsigma}(t) x_2 \\ 0 \\ 0 \end{pmatrix}_{123} \quad p = p_0$

=

predictions of:

Material Functions (stress responses)

$$\eta(\dot{\gamma}_0) \equiv \frac{\tilde{\tau}_{21}}{\dot{\gamma}_0} = \frac{-\tau_{21}}{\dot{\gamma}_0}$$

$$\Psi_1(\dot{\gamma}_0) \equiv \frac{\tilde{\tau}_{11} - \tilde{\tau}_{22}}{\dot{\gamma}_0^2} = \frac{-(\tau_{11} - \tau_{22})}{\dot{\gamma}_0^2}$$

$$\Psi_2(\dot{\gamma}_0) \equiv \frac{\tilde{\tau}_{22} - \tilde{\tau}_{33}}{\dot{\gamma}_0^2} = \frac{-(\tau_{22} - \tau_{33})}{\dot{\gamma}_0^2}$$

How do we predict material functions?

We must know the Constitutive Equation.

3

Investigating **Stress/Deformation** Relationships (**Rheology**)

1. Choose a material function
2. Predict what Newtonian fluids would do
3. See what non-Newtonian fluids **do**
4. Hypothesize a $\underline{\underline{\tau}}(\underline{v})$
5. Predict the material function
6. Compare with what non-Newtonian fluids **do**
7. Reflect, **learn**, revise model, repeat.

2) Predict what **Newtonian** fluids would do

What does the **Newtonian** Fluid model predict in steady shearing?

$$\underline{\underline{\tau}} = -\mu\dot{\underline{\underline{\gamma}}} = -\mu[\nabla \underline{v} + (\nabla \underline{v})^T]$$

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Investigating **Stress/Deformation** Relationships (**Rheology**)

What does the **Newtonian** Fluid model predict in steady shearing?

$$\underline{\underline{\tau}} = -\mu\dot{\underline{\underline{\gamma}}} = -\mu[\nabla \underline{v} + (\nabla \underline{v})^T]$$

You try.

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Investigating **Stress/Deformation** Relationships (**Rheology**)

1. Choose a material function
2. Predict what Newtonian fluids would do
3. See what non-Newtonian fluids **do**
4. Hypothesize a $\underline{\underline{\tau}}(\underline{v})$
5. Predict the material function
6. Compare with what non-Newtonian fluids **do**
7. Reflect, **learn**, revise model, repeat.

3) See what non-Newtonian fluids **do**

*What do we **measure** for these material functions?*

(for polymer solutions, for example)

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3) See what non-Newtonian fluids do

Steady shear viscosity and first normal stress coefficient

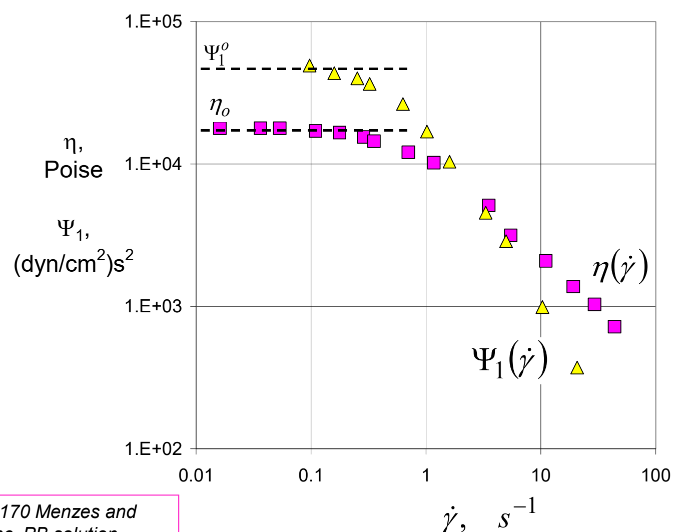

Figure 6.1, p. 170 Menzes and Graessley conc. PB solution

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3) See what non-Newtonian fluids do

Steady shear viscosity and first normal stress coefficient

- 813 kg/mol
- 517 kg/mol
- 350 kg/mol
- 200 kg/mol

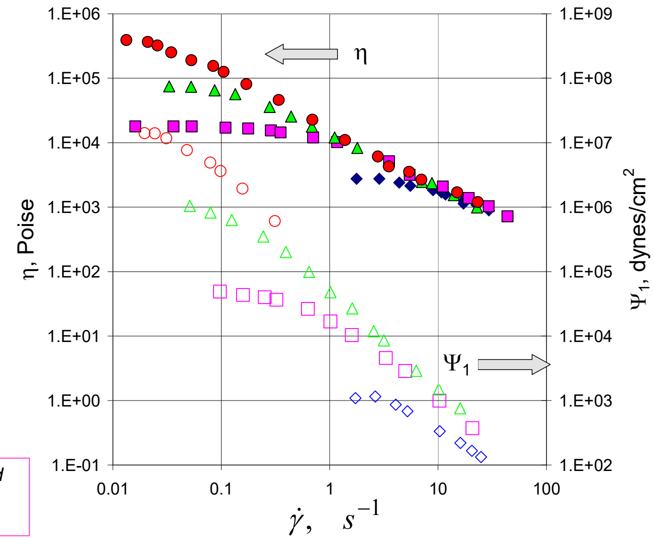

Figure 6.2, p. 171 Menzes and Graessley conc. PB solution; c=0.0676 g/cm³

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3) See what non-Newtonian fluids do

Steady shear viscosity for linear and branched PDMS

- + linear 131 kg/mole
- ▲ branched 156 kg/mole
- □ linear 418 kg/mol
- ◆ branched 428 kg/mol

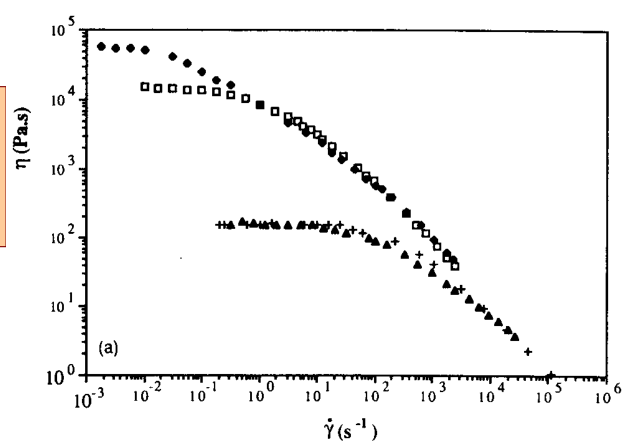

Figure 6.3, p. 172 Piau et al., linear and branched PDMS

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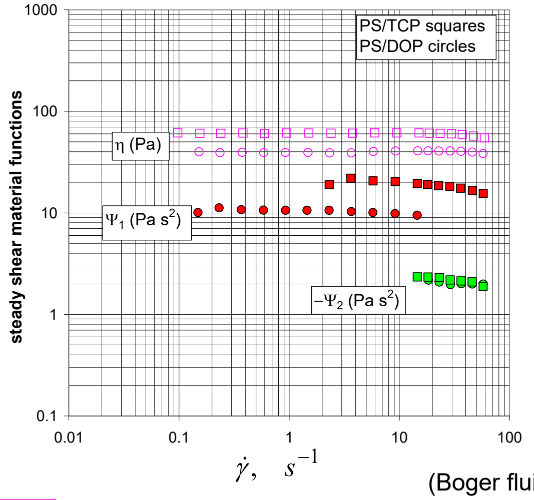
3) See what non-Newtonian fluids do

Steady shear viscosity and first and ***second*** normal stress coefficient



(Boger fluid)

Figure 6.6, p. 174 Magda et al.; Polystyrene solns

Investigating **Stress/Deformation** Relationships (**Rheology**)

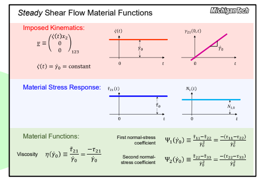
1. Choose a material function
2. Predict what Newtonian fluids would do
3. See what non-Newtonian fluids **do**
4. Hypothesize a $\underline{\underline{\tau}}(\underline{v})$
5. Predict the material function
6. Compare with what non-Newtonian fluids **do**
7. Reflect, **learn**, revise model, repeat.

4) Hypothesize a $\underline{\underline{\tau}}(\underline{v})$

(how do we do **that**, exactly?)

4) Hypothesize a $\underline{\underline{\tau}}(\underline{v})$

What have the investigations of the ***steady shear*** material functions taught us so far?



- Newtonian constitutive equation is inadequate

  1. Predicts constant shear viscosity (does not predict rate dependence)
  2. Predicts no shear normal stresses (a nonlinear effect; these stresses are generated for many fluids)

- Behavior depends on the material (chemical structure, molecular weight, concentration)

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4) Hypothesize a $\underline{\underline{\tau}}(\underline{v})$

Can we fix the Newtonian Constitutive Equation?

$$\underline{\underline{\tau}} = -\mu\left[\nabla\underline{v} + (\nabla\underline{v})^T\right]$$

Let's replace μ with a function of shear rate because we want to predict a rate dependence

$$\underline{\underline{\tau}} = -(\text{function of } \dot{\gamma}_0)\underline{\underline{\dot{\gamma}}}$$

New hypothesis for $\underline{\underline{\tau}}(\underline{v})$

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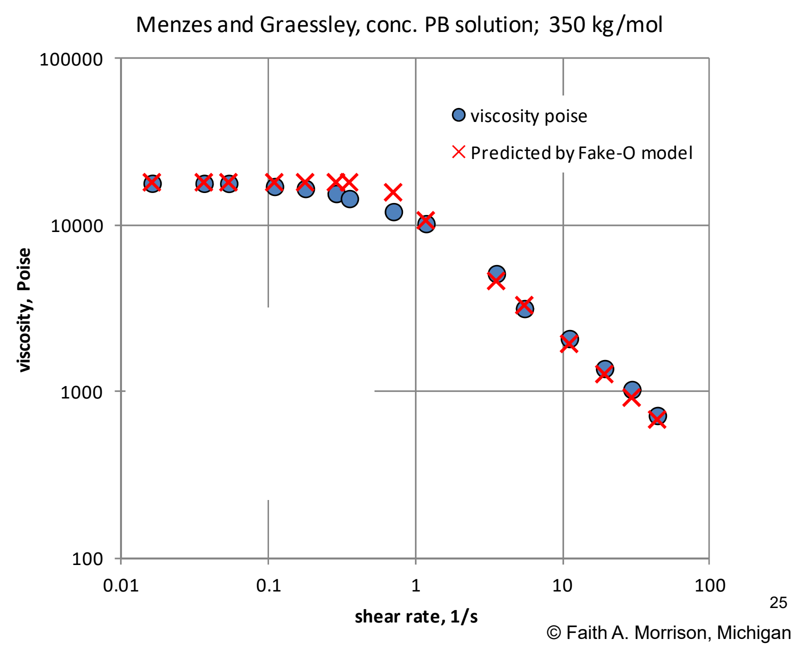
4) Hypothesize a $\underline{\underline{\tau}}(\underline{v})$



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What are material functions and why do we need them?

Constitutive equation $\underline{\underline{\tau}} = -\underline{\underline{\tilde{\tau}}} = \underline{\underline{\tau}}(\underline{v})$ ← **We are guessing this**

+

Standard Velocity and pressure fields

$$\underline{v} \equiv \begin{pmatrix} \dot{\varsigma}(t)x_2 \\ 0 \\ 0 \end{pmatrix}_{123} \qquad p = p_0$$

=

predictions of:

Material Functions (stress responses)

$$\eta(\dot{\gamma}_0) \equiv \frac{\tilde{\tau}_{21}}{\dot{\gamma}_0} = \frac{-\tau_{21}}{\dot{\gamma}_0}$$

$$\Psi_1(\dot{\gamma}_0) \equiv \frac{\tilde{\tau}_{11}-\tilde{\tau}_{22}}{\dot{\gamma}_0^2} = \frac{-(\tau_{11}-\tau_{22})}{\dot{\gamma}_0^2}$$

$$\Psi_2(\dot{\gamma}_0) \equiv \frac{\tilde{\tau}_{22}-\tilde{\tau}_{33}}{\dot{\gamma}_0^2} = \frac{-(\tau_{22}-\tau_{33})}{\dot{\gamma}_0^2}$$

3

Investigating **Stress/Deformation** Relationships (**Rheology**)

1. Choose a material function
2. Predict what Newtonian fluids would do
3. See what non-Newtonian fluids **do**
4. Hypothesize a $\underline{\underline{\tau}}(\underline{v})$
5. Predict the material function
6. Compare with what non-Newtonian fluids **do**
7. Reflect, **learn**, revise model, repeat.

5) Predict the material function (with new $\underline{\underline{\tau}}(\underline{v})$)

What does this model predict for steady shear viscosity?

$$\underline{\underline{\tau}} = -M(\dot{\gamma}_0)[\nabla\underline{v} + (\nabla\underline{v})^T]$$

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5) Predict the material function

What does this model predict for steady shear viscosity?

$$\underline{\underline{\tau}} = -M(\dot{\gamma}_0)[\nabla\underline{v} + (\nabla\underline{v})^T]$$

You try.

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5) Predict the material function

What does this model predict for steady shear viscosity?

$$\underline{\underline{\tau}} = -M(\dot{\gamma}_0)[\nabla \underline{v} + (\nabla \underline{v})^T]$$

Answer:

$$\eta = M(\dot{\gamma}_0)$$

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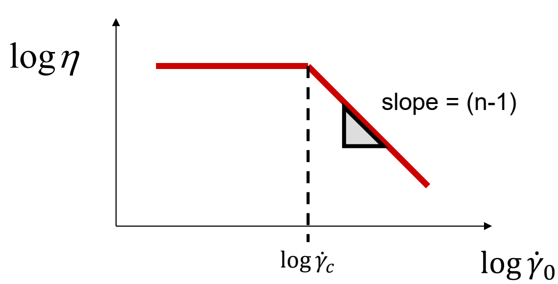
5) Predict the material function

If we choose:

$$M(\dot{\gamma}_0) = \begin{cases} M_0 & \dot{\gamma}_0 > \dot{\gamma}_c \\ m\dot{\gamma}_0^{n-1} & \dot{\gamma}_0 \geq \dot{\gamma}_c \end{cases}$$

Fake-O Model®

(we are forcing the observed rate dependence)



Problem solved!
The model and the experiments for $\eta(\dot{\gamma})$ match.

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5) Predict the material function

But what about the normal stresses?

$$\underline{\underline{\tau}} = -M(\dot{\gamma}_0)[\nabla \underline{v} + (\nabla \underline{v})^T]$$

$$M(\dot{\gamma}_0) = \begin{cases} M_0 & \dot{\gamma}_0 > \dot{\gamma}_c \\ m\dot{\gamma}_0^{n-1} & \dot{\gamma}_0 \geq \dot{\gamma}_c \end{cases}$$

$$\nabla v = \begin{pmatrix} 0 & 0 & 0 \\ \dot{\gamma}_0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{123} \qquad \underline{\underline{\dot{\gamma}}} = \begin{pmatrix} 0 & \dot{\gamma}_0 & 0 \\ \dot{\gamma}_0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{123}$$

$$\underline{\underline{\tau}} = \begin{pmatrix} 0 & -M(\dot{\gamma}_0)\dot{\gamma}_0 & 0 \\ -M(\dot{\gamma}_0)\dot{\gamma}_0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{123}$$

$$\Rightarrow \Psi_1 = \Psi_2 = 0$$

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Investigating **Stress/Deformation** Relationships (**Rheology**)

1. Choose a material function
2. Predict what Newtonian fluids would do
3. See what non-Newtonian fluids **do**
4. Hypothesize a $\underline{\underline{\tau}}(\underline{v})$
5. Predict the material function
6. Compare with what non-Newtonian fluids **do**
7. Reflect, **learn**, revise model, repeat.

6) Compare with what non-Newtonian fluids **do**

?

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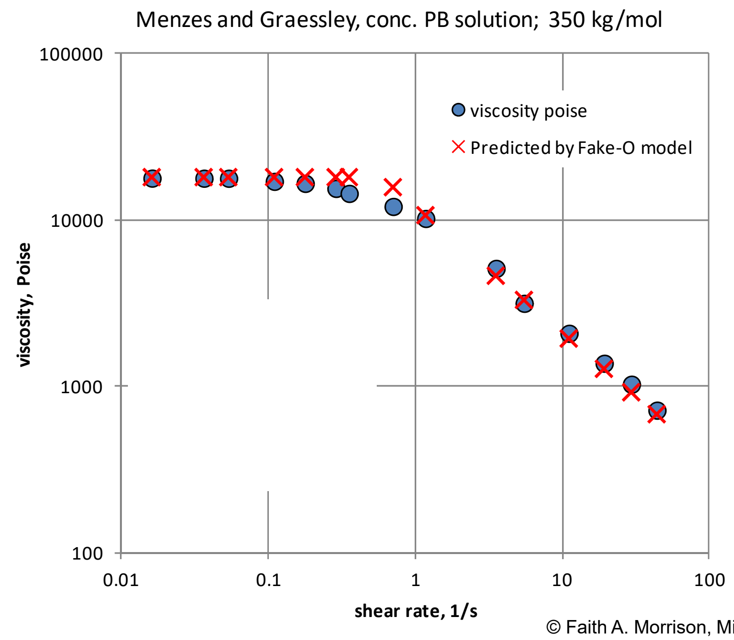
6) Compare with what non-Newtonian fluids do



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6) Compare with what non-Newtonian fluids do

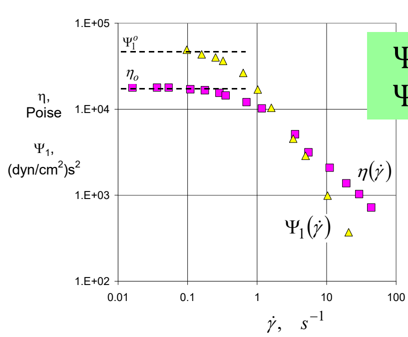
Fake-O Model Predictions:

Steady shear:

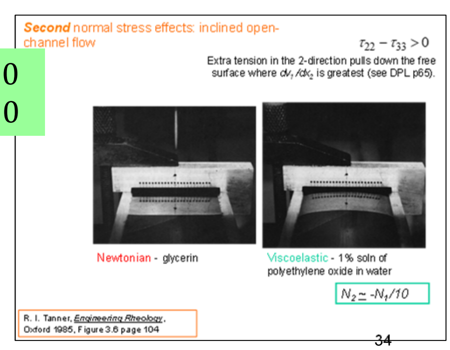
$$\eta = \begin{cases} M_0 & \dot{\gamma}_0 \le \dot{\gamma}_c \\ m\dot{\gamma}_0^{n-1} & \dot{\gamma}_0 > \dot{\gamma}_c \end{cases}$$

$$\Psi_1 = 0$$
$$\Psi_2 = 0$$

Polymer Behavior:



$$\Psi_1 \neq 0$$
$$\Psi_2 \neq 0$$



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6) Compare with what non-Newtonian fluids do

Fake-O Model Predictions:

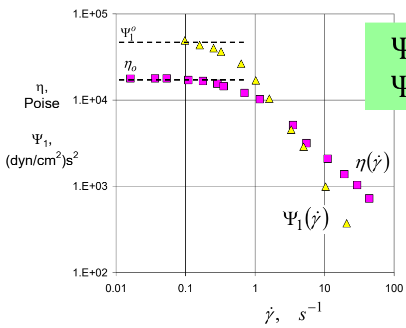
Steady shear:

✓ $\eta = \begin{cases} M_0 & \dot{\gamma}_0 \le \dot{\gamma}_c \\ m\dot{\gamma}_0^{n-1} & \dot{\gamma}_0 > \dot{\gamma}_c \end{cases}$

✗ $\Psi_1 = 0$

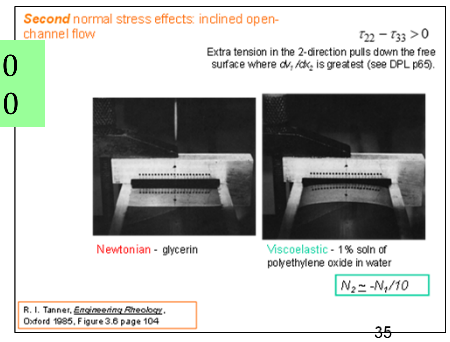
✗ $\Psi_2 = 0$

Polymer Behavior:



$\Psi_1 \neq 0$

$\Psi_2 \neq 0$



6) Compare with what non-Newtonian fluids do

Why did we get 0 for the normal stresses?

$$\underline{\underline{\tau}} = -M(\dot{\gamma}_0)\left[\nabla \underline{v} + (\nabla \underline{v})^T\right]$$

$$M(\dot{\gamma}_0) = \begin{cases} M_0 & \dot{\gamma}_0 > \dot{\gamma}_c \\ m\dot{\gamma}_0^{n-1} & \dot{\gamma}_0 \ge \dot{\gamma}_c \end{cases}$$

$$\underline{\underline{\tau}} = -M(\dot{\gamma}_0)\begin{pmatrix} 0 & \dot{\gamma}_0 & 0 \\ \dot{\gamma}_0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{123}$$

It appears that $\underline{\underline{\tau}}$ should not be simply proportional to $\underline{\underline{\dot{\gamma}}}$

$$\Rightarrow \Psi_1 = \Psi_2 = 0$$

Need to try something else . . .

$$\underline{\underline{\tau}} = -\mu\dot{\underline{\underline{\gamma}}} + \underline{\underline{I}} f(\underline{v})$$

$$\underline{\underline{\tau}} = f(\underline{v})\nabla\underline{v} \cdot (\nabla\underline{v})^T$$

$$\underline{\underline{\tau}} = A[\nabla\underline{v} \cdot (\nabla\underline{v})^T] + B\nabla\underline{v} + C(\nabla\underline{v})^T$$

...

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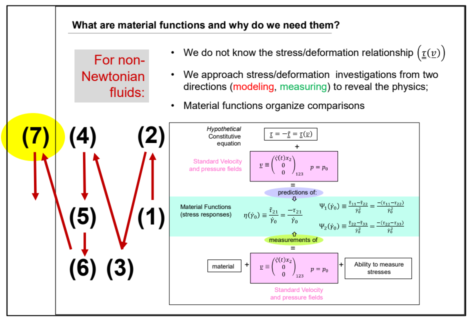
Investigating **Stress/Deformation** Relationships (**Rheology**)

1. Choose a material function
2. Predict what Newtonian fluids would do
3. See what non-Newtonian fluids **do**
4. Hypothesize a $\underline{\underline{\tau}}(\underline{v})$
5. Predict the material function
6. Compare with what non-Newtonian fluids **do**
7. Reflect, **learn**, revise model, repeat.

7) Reflect, learn, revise model, repeat

- ✓ Reflect
- ✓ Learn
- ✓ Revise model
- ✓ Propose new $\underline{\underline{\tau}}(\underline{v})$
- ✓ Repeat cycle….



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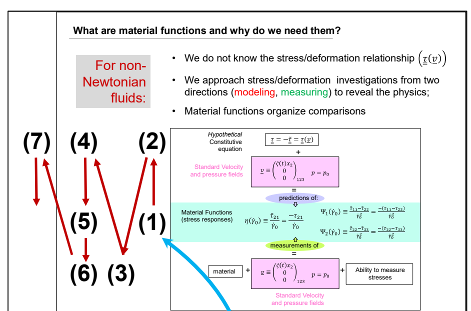
7) Reflect, learn, revise model, repeat

- ✓ Reflect
- ✓ Learn
- ✓ Revise model
- ✓ Propose new $\underline{\underline{\tau}}(\underline{v})$
- ✓ Repeat cycle….

What shall we guess next?

To sort out how to fix the Newtonian equation, we need more observations (to give us ideas).



Let's try another material function that's not a steady flow (but stick to shear).

1) Choose a material function

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Investigating **Stress/Deformation** Relationships (**Rheology**)

1. Choose a material function
2. Predict what Newtonian fluids would do
3. See what non-Newtonian fluids **do**
4. Hypothesize a $\underline{\underline{\tau}}(\underline{v})$
5. Predict the material function
6. Compare with what non-Newtonian fluids **do**
7. Reflect, **learn**, revise model, repeat.

1) Choose a material function

kinematics

1. Choice of flow (shear or elongation)

$$\underline{v} = \begin{pmatrix} \dot{\varsigma}(t)x_2 \\ 0 \\ 0 \end{pmatrix}_{123} \qquad \underline{v} = \begin{pmatrix} -\frac{1}{2}\dot{\varepsilon}(t)x_1 \\ -\frac{1}{2}\dot{\varepsilon}(t)x_2 \\ \dot{\varepsilon}(t)x_3 \end{pmatrix}_{123}$$

2. Choice of time dependence of $\dot{\varsigma}(t)$ or $\dot{\varepsilon}(t)$

3. Material functions definitions: will be based on τ_{21}, N_1, N_2 in shear or $\tau_{22}-\tau_{11}$, $\tau_{22}-\tau_{11}$ in elongational flows.

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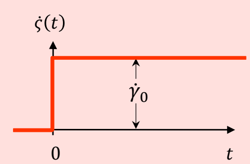
Start-up of Steady Shear Flow Material Functions

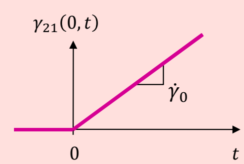
Michigan Tech

Imposed Kinematics:

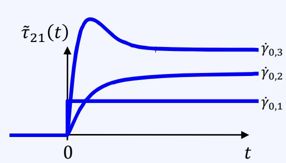
$$\underline{v} \equiv \begin{pmatrix} \dot{\varsigma}(t)x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$$

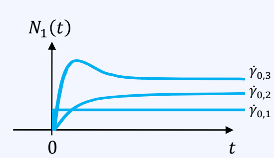
$$\dot{\varsigma}(t) = \begin{cases} 0 & t < 0 \\ \dot{\gamma}_0 & t \geq 0 \end{cases}$$





Material Stress Response:





Material Functions:

Shear stress growth function $\eta^+(t,\dot{\gamma}_0) \equiv \frac{\tilde{\tau}_{21}(t)}{\dot{\gamma}_0} = \frac{-\tau_{21}(t)}{\dot{\gamma}_0}$

First normal-stress growth coefficient $\Psi_1^+(t,\dot{\gamma}_0) \equiv \frac{\tilde{\tau}_{11}-\tilde{\tau}_{22}}{\dot{\gamma}_0^2}$

Second normal-stress growth coefficient $\Psi_2^+(t,\dot{\gamma}_0) \equiv \frac{\tilde{\tau}_{22}-\tilde{\tau}_{33}}{\dot{\gamma}_0^2}$

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Investigating **Stress/Deformation** Relationships (**Rheology**)

1. Choose a material function
2. Predict what Newtonian fluids would do
3. See what non-Newtonian fluids **do**
4. Hypothesize a $\underline{\underline{\tau}}(\underline{v})$
5. Predict the material function
6. Compare with what non-Newtonian fluids **do**
7. Reflect, **learn**, revise model, repeat.

2) Predict what Newtonian fluids would do

What does the **Newtonian** Fluid model predict in start-up of steady shearing?

$$\underline{\underline{\tau}} = -\mu\left[\nabla \underline{v} + (\nabla \underline{v})^T\right]$$

Again, since we know $\underline{v}$, we can just substitute it into the constitutive equation and calculate the stresses.

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2) Predict what Newtonian fluids would do (round 2)

What does the **Newtonian** Fluid constitutive equation predict in start-up of steady shearing?

$$\underline{\underline{\tau}} = -\mu\left[\nabla \underline{v} + (\nabla \underline{v})^T\right]$$

You try.

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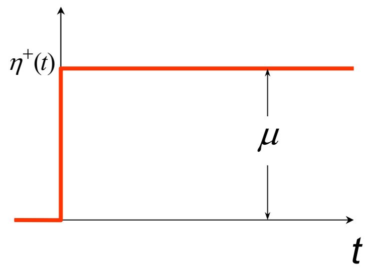
2) Predict what Newtonian fluids would do (round 2)

Material functions predicted for *start-up of steady shearing* of a Newtonian fluid

$$\eta^+(t) = \begin{cases} 0 & t < 0 \\ \mu & t \geq 0 \end{cases}$$



$$\Psi_1^+ \equiv \frac{-(\tau_{11} - \tau_{33})}{\dot{\gamma}_0^2} = 0$$

$$\Psi_2^+ \equiv \frac{-(\tau_{22} - \tau_{33})}{\dot{\gamma}_0^2} = 0$$

Do these predictions match observations?

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Investigating **Stress/Deformation** Relationships (**Rheology**)

1. Choose a material function
2. Predict what Newtonian fluids would do
3. See what non-Newtonian fluids **do**
4. Hypothesize a $\underline{\underline{\tau}}(\underline{v})$
5. Predict the material function
6. Compare with what non-Newtonian fluids **do**
7. Reflect, **learn**, revise model, repeat.

3) See what non-Newtonian fluids **do**

*What do we **measure** for these material functions?*

(for polymer solutions, for example)

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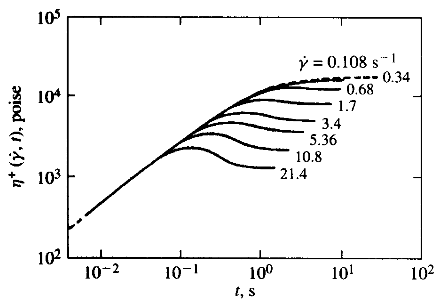
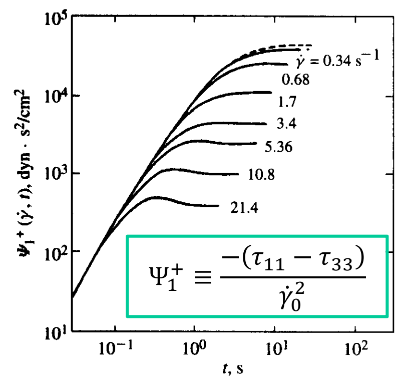
3) See what non-Newtonian fluids do (round 2)

Startup of Steady Shearing

$$\underline{v} \equiv \begin{pmatrix} \dot{\varsigma}(t) x_2 \\ 0 \\ 0 \end{pmatrix}_{123} \qquad \dot{\varsigma}(t) = \begin{cases} 0 & t < 0 \\ \dot{\gamma}_0 & t \geq 0 \end{cases}$$



$$\eta^+(t) = \begin{cases} 0 & t < 0 \\ \mu & t \geq 0 \end{cases}$$



$$\Psi_1^+ \equiv \frac{-(\tau_{11} - \tau_{33})}{\dot{\gamma}_0^2}$$

Figures 6.49, 6.50, p. 208 Menezes and Graessley, Polybutadiene soln

Investigating **Stress/Deformation** Relationships (**Rheology**)

1. Choose a material function
2. Predict what Newtonian fluids would do
3. See what non-Newtonian fluids **do**
4. Hypothesize a $\underline{\underline{\tau}}(\underline{v})$
5. Predict the material function
6. Compare with what non-Newtonian fluids **do**
7. Reflect, **learn**, revise model, repeat.

3) See what non-Newtonian fluids **do**

What about other non-steady flows?

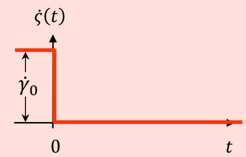
Cessation of Steady Shear Flow Material Functions

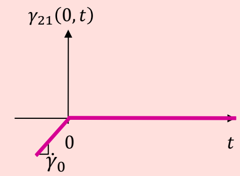
Michigan Tech

Imposed Kinematics:

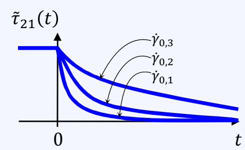
$$\underline{v} \equiv \begin{pmatrix} \dot{\varsigma}(t) x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$$

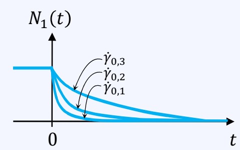
$$\dot{\varsigma}(t) = \begin{cases} \dot{\gamma}_0 & t < 0 \\ 0 & t \geq 0 \end{cases}$$





Material Stress Response:





Material Functions:

Shear stress decay function $\eta^-(t,\dot{\gamma}_0) \equiv \dfrac{\tilde{\tau}_{21}(t)}{\dot{\gamma}_0} = \dfrac{-\tau_{21}(t)}{\dot{\gamma}_0}$

First normal-stress decay coefficient $\Psi_1^-(t,\dot{\gamma}_0) \equiv \dfrac{\tilde{\tau}_{11}-\tilde{\tau}_{22}}{\dot{\gamma}_0^2}$

Second normal-stress decay coefficient $\Psi_2^-(t,\dot{\gamma}_0) \equiv \dfrac{\tilde{\tau}_{22}-\tilde{\tau}_{33}}{\dot{\gamma}_0^2}$

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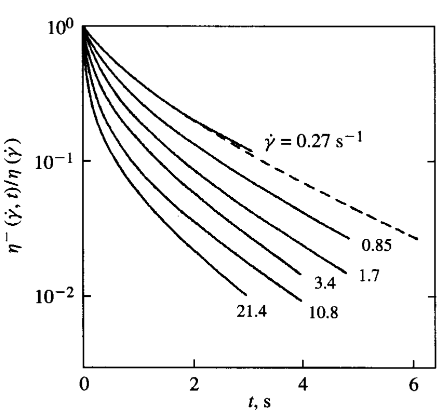
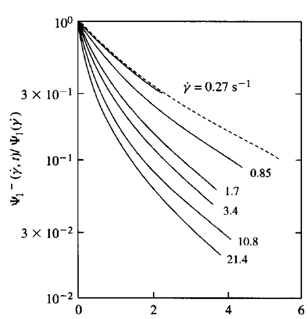
3) See what non-Newtonian fluids do (round 3)

Cessation of Steady Shearing

$$\underline{v} \equiv \begin{pmatrix} \dot{\varsigma}(t) x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$$

$$\dot{\varsigma}(t) = \begin{cases} \dot{\gamma}_0 & t < 0 \\ 0 & t \geq 0 \end{cases}$$





Figures 6.51, 6.52, p. 209
Menezes and Graessley, PB soln

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Investigating **Stress/Deformation** Relationships (**Rheology**)

1. Choose a material function
2. Predict what Newtonian fluids would do
3. See what non-Newtonian fluids **do**
4. Hypothesize a $\underline{\underline{\tau}}(\underline{v})$
5. Predict the material function
6. Compare with what non-Newtonian fluids **do**
7. Reflect, **learn**, revise model, repeat.

5) Predict the material function

What does the Fake-O model® predict for start-up and cessation of shear?

$$\underline{\underline{\tau}} = -M(\dot\gamma_0)\left[\nabla \underline{v} + (\nabla \underline{v})^T\right]$$

$$M(\dot\gamma_0) = \begin{cases} M_0 & \dot\gamma_0 > \dot\gamma_c \\ m\dot\gamma_0^{n-1} & \dot\gamma_0 \geq \dot\gamma_c \end{cases}$$

5) Predict the material functions (rounds 2 & 3)

What does the Fake-O model® predict for start-up and cessation of shear?

$$\underline{\underline{\tau}} = -M(\dot\gamma_0)\left[\nabla \underline{v} + (\nabla \underline{v})^T\right]$$

$$M(\dot\gamma_0) = \begin{cases} M_0 & \dot\gamma_0 > \dot\gamma_c \\ m\dot\gamma_0^{n-1} & \dot\gamma_0 \geq \dot\gamma_c \end{cases}$$

You try.

2018: Homework 3

Investigating **Stress/Deformation** Relationships (**Rheology**)

1. Choose a material function
2. Predict what Newtonian fluids would do
3. See what non-Newtonian fluids **do**
4. Hypothesize a $\underline{\underline{\tau}}(\underline{v})$
5. Predict the material function
6. Compare with what non-Newtonian fluids **do**
7. Reflect, **learn**, revise model, repeat.

6) Compare with what non-Newtonian fluids **do**

?

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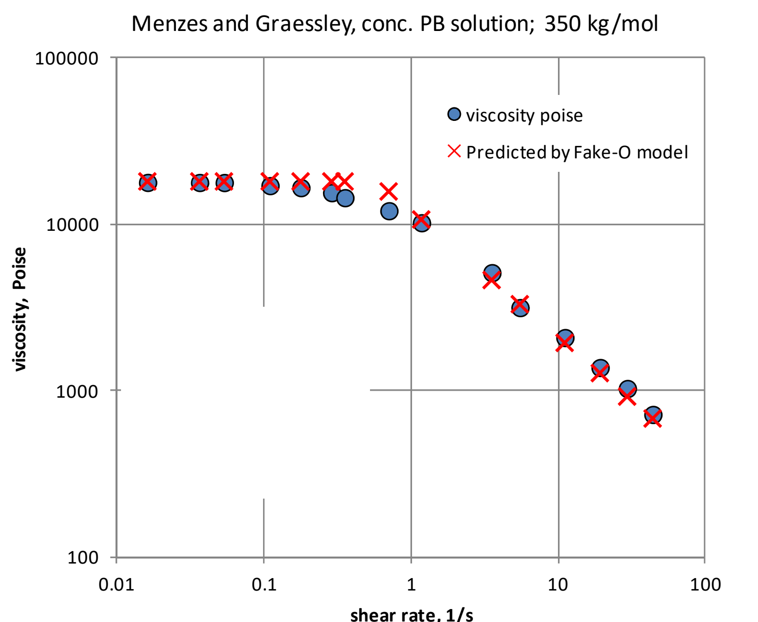
6) Compare with what non-Newtonian fluids do (rounds 2&3)



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6) Compare with what non-Newtonian fluids do (rounds 2&3)

Fake-O Model Shear Material Function Predictions:

Start-up of steady shear:

$$\eta^+(t) = \begin{cases} 0 & t < 0 \\ M(\dot{\gamma}_0) & t \geq 0 \end{cases}$$

where $M(\dot{\gamma}_0) = \begin{cases} M_0 & \dot{\gamma}_0 \leq \dot{\gamma}_c \\ m\dot{\gamma}_0^{n-1} & \dot{\gamma}_0 > \dot{\gamma}_c \end{cases}$

$$\Psi_1^+(t) = 0$$

$$\Psi_2^+(t) = 0$$

Cessation of steady shear:

$$\eta^-(t) = \begin{cases} M(\dot{\gamma}_0) & t < 0 \\ 0 & t \geq 0 \end{cases}$$

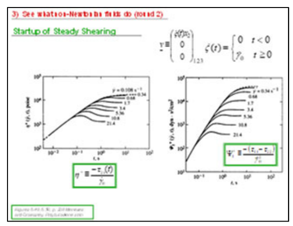
where $M(\dot{\gamma}_0) = \begin{cases} M_0 & \dot{\gamma}_0 \leq \dot{\gamma}_c \\ m\dot{\gamma}_0^{n-1} & \dot{\gamma}_0 > \dot{\gamma}_c \end{cases}$

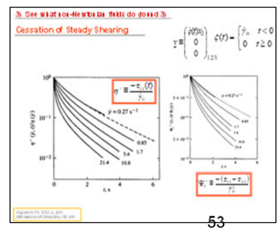
$$\Psi_1^-(t) = 0$$

$$\Psi_2^-(t) = 0$$

Polymer Behavior:

- captures rate dependence, but, $\Psi_1^+ \neq 0$, $\Psi_2^+ \neq 0$
- Also, response is not instantaneous





53

6) Compare with what non-Newtonian fluids do (rounds 2&3)

$$\underline{\underline{\tau}} = -M(\dot{\gamma}_0)[\nabla \underline{v} + (\nabla \underline{v})^T]$$

Observations

$$M(\dot{\gamma}_0) = \begin{cases} M_0 & \dot{\gamma}_0 > \dot{\gamma}_c \\ m\dot{\gamma}_0^{n-1} & \dot{\gamma}_0 \geq \dot{\gamma}_c \end{cases}$$

•The Fake-O model® predicts an instantaneous stress response, and this is not what is observed for polymers

•The predicted unsteady material functions depend on the shear rate, which is observed for polymers

$$\eta^+ = \eta^+(t, \dot{\gamma}_0)$$

Progress here

•No shear normal stresses are predicted

54

6) Compare with what non-Newtonian fluids do (rounds 2&3)

$$\underline{\underline{\tau}} = -M(\dot{\gamma}_0)\left[\nabla \underline{v} + (\nabla \underline{v})^T\right]$$

Observations

$$M(\dot{\gamma}_0) = \begin{cases} M_0 & \dot{\gamma}_0 > \dot{\gamma}_c \\ m\dot{\gamma}_0^{n-1} & \dot{\gamma}_0 \geq \dot{\gamma}_c \end{cases}$$

- The Fake-O model® predicts an instantaneous stress response, and this is not what is observed for polymers ← **Lacks memory**
- The predicted unsteady material functions depend on the shear rate, which is observed for polymers

$$\eta^+ = \eta^+(t, \dot{\gamma}_0)$$ ← **Progress here**

- No shear normal stresses are predicted ← **Related to nonlinearities**

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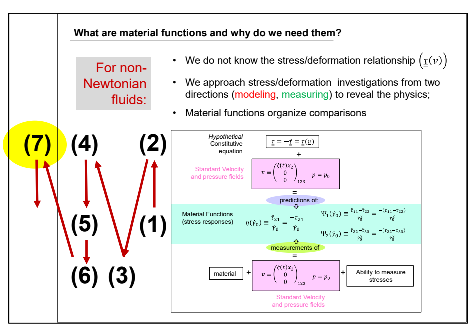
Investigating **Stress/Deformation** Relationships (**Rheology**)

1. Choose a material function
2. Predict what Newtonian fluids would do
3. See what non-Newtonian fluids **do**
4. Hypothesize a $\underline{\underline{\tau}}(\underline{v})$
5. Predict the material function
6. Compare with what non-Newtonian fluids **do**
7. Reflect, **learn**, revise model, repeat.

7) Reflect, learn, revise model, repeat

- ✓ Reflect
- ✓ Learn
- ✓ Revise model
- ✓ Propose new $\underline{\underline{\tau}}(\underline{v})$
- ✓ Repeat cycle….



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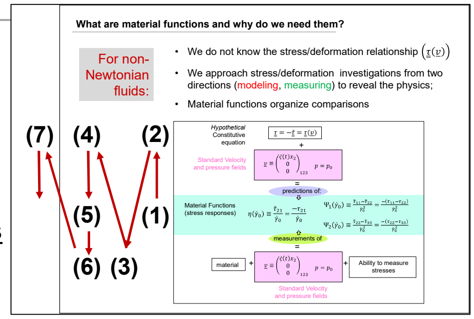
7) Reflect, learn, revise model, repeat



To proceed to better-designed constitutive equations, we need to know more about material behavior, i.e. we need more material functions to predict, and we need measurements of these material functions.

•More non-steady material functions (material functions that tell us about memory)

•Material functions that tell us about nonlinearity (strain)

•**Material functions for a different flow**

Back to step 1

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Investigating **Stress/Deformation** Relationships (**Rheology**)

1. Choose a material function
2. Predict what Newtonian fluids would do
3. See what non-Newtonian fluids **do**
4. Hypothesize a $\underline{\underline{\tau}}(\underline{v})$
5. Predict the material function
6. Compare with what non-Newtonian fluids **do**
7. Reflect, **learn**, revise model, repeat.

1) Choose a material function

kinematics

1. Choice of flow (shear or elongation)

$$\underline{v} = \begin{pmatrix} \dot{\varsigma}(t) x_2 \\ 0 \\ 0 \end{pmatrix}_{123} \qquad \underline{v} = \begin{pmatrix} -\frac{1}{2}\dot{\varepsilon}(t) x_1 \\ -\frac{1}{2}\dot{\varepsilon}(t) x_2 \\ \dot{\varepsilon}(t) x_3 \end{pmatrix}_{123}$$

2. Choice of time dependence of $\dot{\varsigma}(t)$ or $\dot{\varepsilon}(t)$

3. Material functions definitions: will be based on τ_{21}, N_1, N_2 in shear or $\tau_{22}-\tau_{11}$, $\tau_{22}-\tau_{11}$ in elongational flows.

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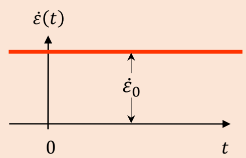
Steady Elongational Flow Material Functions

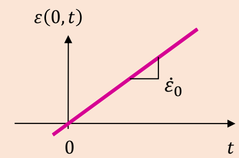
Michigan Tech

Imposed Kinematics:

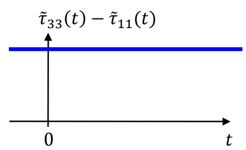
$$\underline{v} \equiv \begin{pmatrix} -\frac{1}{2}\dot{\varepsilon}(t)x_1 \\ -\frac{1}{2}\dot{\varepsilon}(t)x_2 \\ \dot{\varepsilon}(t)x_3 \end{pmatrix}_{123}$$

$$\dot{\varepsilon}(t) = \dot{\varepsilon}_0 = \text{constant}$$





Material Stress Response:



Material Functions:

Elongational Viscosity $\quad \eta_e(\dot{\varepsilon}_0) \equiv \dfrac{\tilde{\tau}_{33} - \tilde{\tau}_{11}}{\dot{\varepsilon}_0} = \dfrac{-(\tau_{33} - \tau_{11})}{\dot{\varepsilon}_0}$

Alternatively, $\bar{\eta}(\dot{\varepsilon}_0)$

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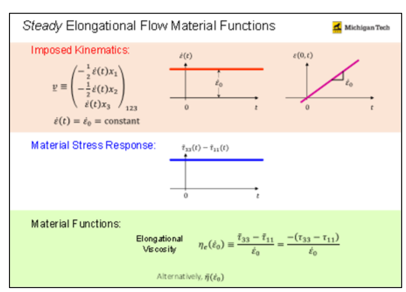
2) Predict what Newtonian fluids would do.

1. Choose a material function
2. Predict what Newtonian fluids would do
3. See what non-Newtonian fluids **do**
4. Hypothesize a $\underline{\underline{\tau}}(\underline{v})$
5. Predict the material function
6. Compare with what non-Newtonian fluids **do**
7. Reflect, **learn**, revise model, repeat.

2) Predict what Newtonian fluids would do.

?

$$\underline{\underline{\tau}} = -\mu \dot{\underline{\underline{\gamma}}}$$



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3) See what non-Newtonian fluids do

1. Choose a material function
2. Predict what Newtonian fluids would do
3. See what non-Newtonian fluids **do**
4. Hypothesize a $\underline{\underline{\tau}}(\underline{v})$
5. Predict the material function
6. Compare with what non-Newtonian fluids **do**
7. Reflect, **learn**, revise model, repeat.

3) See what non-Newtonian fluids **do**

?

You try.

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Investigating **Stress/Deformation** Relationships (**Rheology**)

1. Choose a material function
2. Predict what Newtonian fluids would do
3. See what non-Newtonian fluids **do**
4. Hypothesize a $\underline{\underline{\tau}}(\underline{v})$
5. Predict the material function
6. Compare with what non-Newtonian fluids **do**
7. Reflect, **learn**, revise model, repeat.

6) Compare with what non-Newtonian fluids **do**

?

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3) See what non-Newtonian fluids do

Steady State Elongation Viscosity

Both tension thinning and thickening are observed.

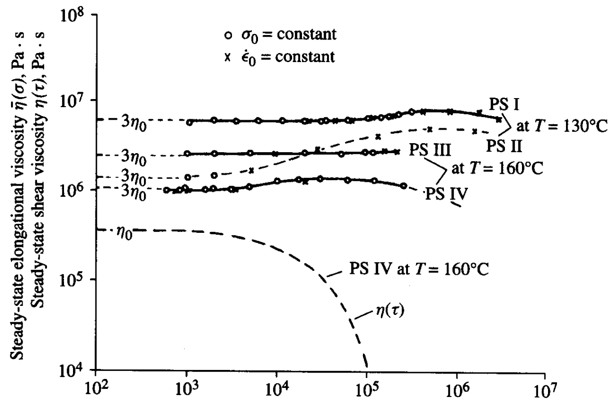


Figure 6.60, p. 215
Munstedt.; PS melt

Trouton ratio: $Tr \equiv \frac{\bar{\eta}}{\eta_0}$

Investigating **Stress/Deformation** Relationships (**Rheology**)

1. Choose a material function
2. Predict what Newtonian fluids would do
3. See what non-Newtonian fluids **do**
4. Hypothesize a $\underline{\underline{\tau}}(\underline{v})$
5. Predict the material function
6. Compare with what non-Newtonian fluids **do**
7. Reflect, **learn**, revise model, repeat.

5) Predict the material function (with new $\underline{\underline{\tau}}(\underline{v})$)

What does the Fake-O®-model predict for steady elongational viscosity?

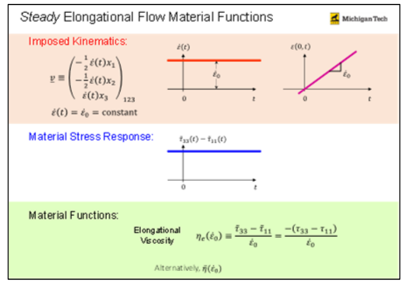
$$\underline{\underline{\tau}} = -M(\dot{\gamma}_0)\left[\nabla \underline{v} + (\nabla \underline{v})^T\right]$$

$$M(\dot{\gamma}_0) = \begin{cases} M_0 & \dot{\gamma}_0 > \dot{\gamma}_c \\ m\dot{\gamma}_0^{n-1} & \dot{\gamma}_0 \geq \dot{\gamma}_c \end{cases}$$

5) Predict the material function

$$\underline{\underline{\tau}} = -M(\dot{\gamma}_0)[\nabla \underline{v} + (\nabla \underline{v})^T]$$

$$M(\dot{\gamma}_0) = \begin{cases} M_0 & \dot{\gamma}_0 > \dot{\gamma}_c \\ m\dot{\gamma}_0^{n-1} & \dot{\gamma}_0 \geq \dot{\gamma}_c \end{cases}$$



$$\eta_e = ?$$

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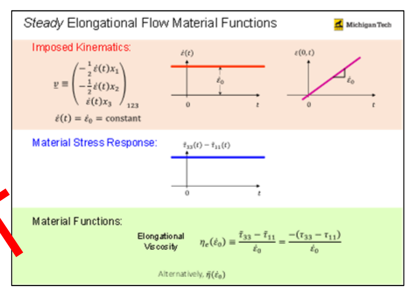
5) Predict the material function

$$\underline{\underline{\tau}} = -M(\dot{\gamma}_0)[\nabla \underline{v} + (\nabla \underline{v})^T]$$

$$M(\dot{\gamma}_0) = \begin{cases} M_0 & \dot{\gamma}_0 > \dot{\gamma}_c \\ m\dot{\gamma}_0^{n-1} & \dot{\gamma}_0 \geq \dot{\gamma}_c \end{cases}$$



DOESN'T WORK

$$\eta_e = ?$$

(What is $\dot{\gamma}_0$ doing in an elongational calculation?)

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Investigating **Stress/Deformation** Relationships (**Rheology**)

1. Choose a material function
2. Predict what Newtonian fluids would do
3. See what non-Newtonian fluids **do**
4. Hypothesize a $\underline{\underline{\tau}}(\underline{v})$
5. Predict the material function
6. Compare with what non-Newtonian fluids **do**
7. Reflect, **learn**, revise model, repeat.

What if we make the following replacement?

$$\dot{\gamma}_0 \rightarrow \frac{\partial v_1}{\partial x_2}$$

This at least can be written for any flow and it is equal to the shear rate in shear flow.

4) Hypothesize a constitutive equation

$$\underline{\underline{\tau}} = -M(\dot{\gamma}_0)\left[\nabla \underline{v} + (\nabla \underline{v})^T\right]$$

$$M(\dot{\gamma}_0) = \begin{cases} M_0 & \dot{\gamma}_0 > \dot{\gamma}_c \\ m\dot{\gamma}_0^{n-1} & \dot{\gamma}_0 \geq \dot{\gamma}_c \end{cases}$$

Observations

- The model contains parameters that are specific to shear flow – makes it impossible to adapt for elongational or mixed flows
- Also, the model should only contain quantities that are independent of coordinate system (i.e. **invariant**)

We will try to salvage the model by replacing the flow-specific kinetic parameter with something that is **frame-invariant** and not **flow-specific**.

4) Hypothesize a constitutive equation

We will take out the shear rate $\dot{\gamma}_0$ and replace with the magnitude of the rate-of-deformation tensor $\left|\underline{\underline{\dot{\gamma}}}\right|$ (which is related to the second invariant of that tensor).

$$\underline{\underline{\tau}} = -M\left(\left|\underline{\underline{\dot{\gamma}}}\right|\right)\left[\nabla\underline{v} + (\nabla\underline{v})^T\right]$$

$$M\left(\left|\underline{\underline{\dot{\gamma}}}\right|\right) = \begin{cases} M_0 & \left|\underline{\underline{\dot{\gamma}}}\right| > \dot{\gamma}_c \\ m\left|\underline{\underline{\dot{\gamma}}}\right|^{n-1} & \left|\underline{\underline{\dot{\gamma}}}\right| \geq \dot{\gamma}_c \end{cases}$$

(Hold that thought; finish the chapter)

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1) Choose a material function – elongational flow

1. Choose a material function
2. Predict what Newtonian fluids would do
3. See what non-Newtonian fluids **do**
4. Hypothesize a $\underline{\underline{\tau}}(\underline{v})$
5. Predict the material function
6. Compare with what non-Newtonian fluids **do**
7. Reflect, **learn**, revise model, repeat.

The other elongational experiments are analogous to shear experiments *(see text)*

- Elongational stress growth
- Elongational stress cessation *(nearly impossible)*
- Elongational creep
- Step elongational strain
- Small-amplitude Oscillatory Elongation (SAOE) *(Redundant with SAOS)*

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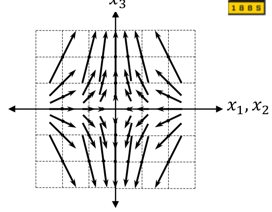
Part II-A. Continuum versus molecular modeling

Michigan Tech

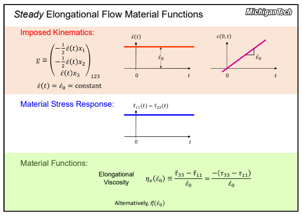
Elongation Material Functions

$$\underline{v} \equiv \begin{pmatrix} -\frac{1}{2}\dot{\varepsilon}(t)x_1 \\ -\frac{1}{2}\dot{\varepsilon}(t)x_2 \\ \dot{\varepsilon}(t)x_3 \end{pmatrix}_{123}$$

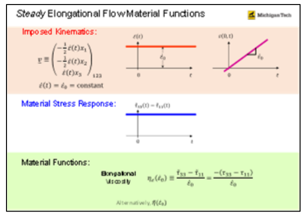


"Recipe cards"

a) Steady



b) Start-up



c) Cessation

(currently unobservable)

d) Step strain

(exists, but less often discussed)

e) SAOE

(exists, but easily converted to SAOS so is redundant)

f) Creep

(exists)

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Steady Elongational Flow Material Functions

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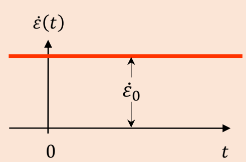
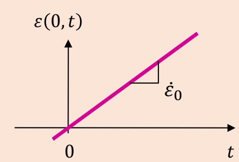
Imposed Kinematics:

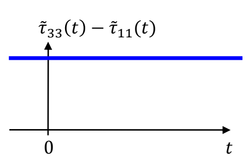
$$\underline{v} \equiv \begin{pmatrix} -\frac{1}{2}\dot{\varepsilon}(t)x_1 \\ -\frac{1}{2}\dot{\varepsilon}(t)x_2 \\ \dot{\varepsilon}(t)x_3 \end{pmatrix}_{123}$$

$$\dot{\varepsilon}(t) = \dot{\varepsilon}_0 = \text{constant}$$





Material Stress Response:



Material Functions:

Elongational Viscosity

$$\eta_e(\dot{\varepsilon}_0) \equiv \frac{\tilde{\tau}_{33} - \tilde{\tau}_{11}}{\dot{\varepsilon}_0} = \frac{-(\tau_{33} - \tau_{11})}{\dot{\varepsilon}_0}$$

Alternatively, $\bar{\eta}(\dot{\varepsilon}_0)$

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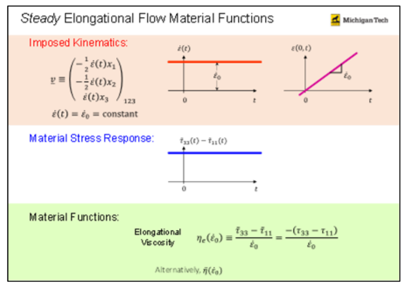
3) See what non-Newtonian fluids do

1. Choose a material function
2. Predict what Newtonian fluids would do
3. See what non-Newtonian fluids **do**
4. Hypothesize a $\underline{\underline{\tau}}(\underline{v})$
5. Predict the material function
6. Compare with what non-Newtonian fluids **do**
7. Reflect, **learn**, revise model, repeat.

3) See what non-Newtonian fluids **do**

?



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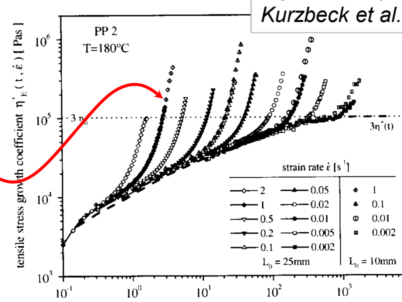
3) See what non-Newtonian fluids do

Start-up of Steady Elongation

Figure 6.64, p. 218
Kurzbeck et al.; PP

Strain-hardening



Fit to an advanced constitutive equation (12 mode pom-pom model)

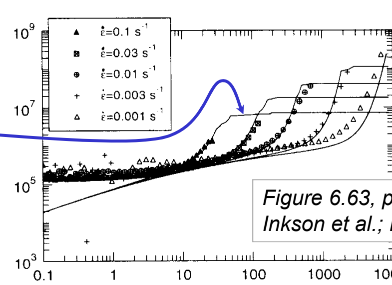


Figure 6.63, p. 217
Inkson et al.; LDPE

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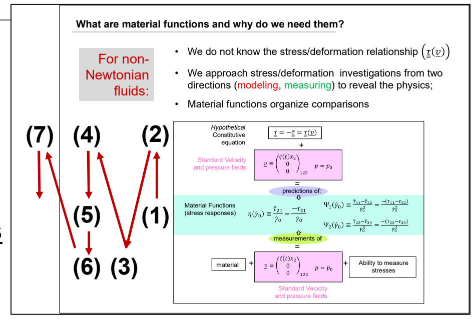
7) Reflect, learn, revise model, repeat

What now?

To proceed to better-designed constitutive equations, we need to know more about material behavior, i.e. we need more material functions to predict, and we need measurements of these material functions.



•More non-steady material functions (material functions that tell us about memory)

•**Material functions that tell us about nonlinearity (strain)**

•Material functions for a different flow

Back to step 1

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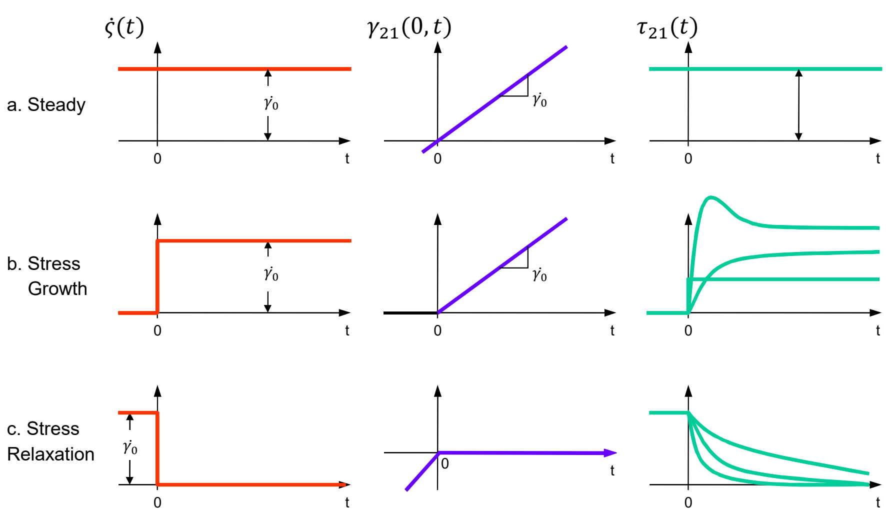
1) Choose a material function – Rate based

Summary of shear rate kinematics *(part 1)*



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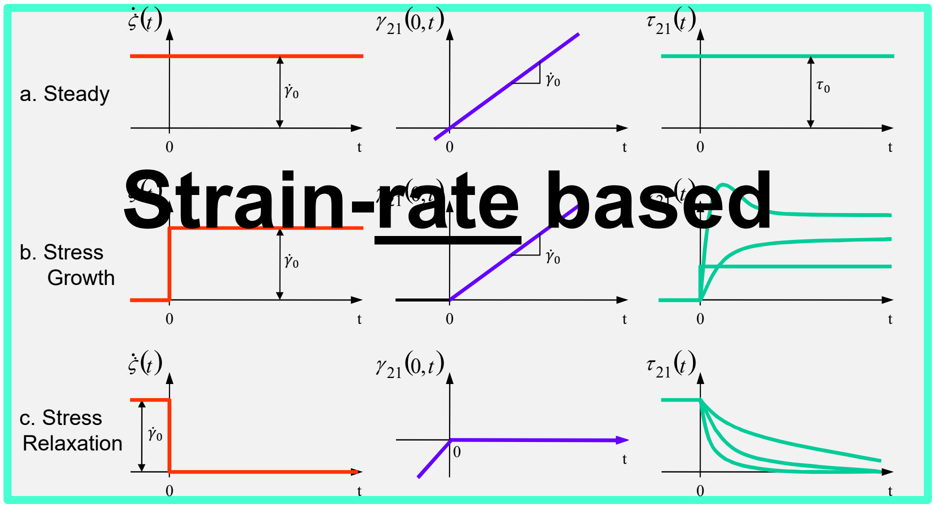
1) Choose a material function – Rate based

Summary of shear rate kinematics *(part 1, rate-based)*



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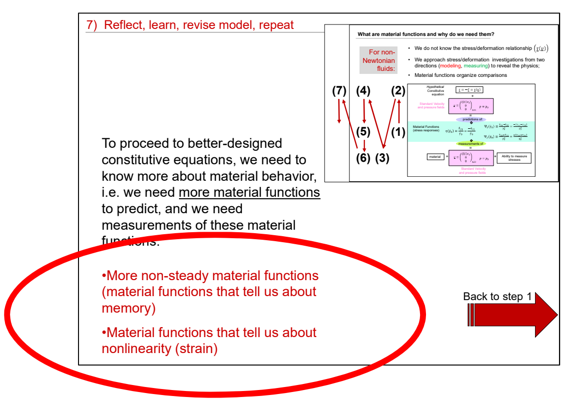
1) Choose a material function



The next three families of material functions incorporate the concept of **strain**.

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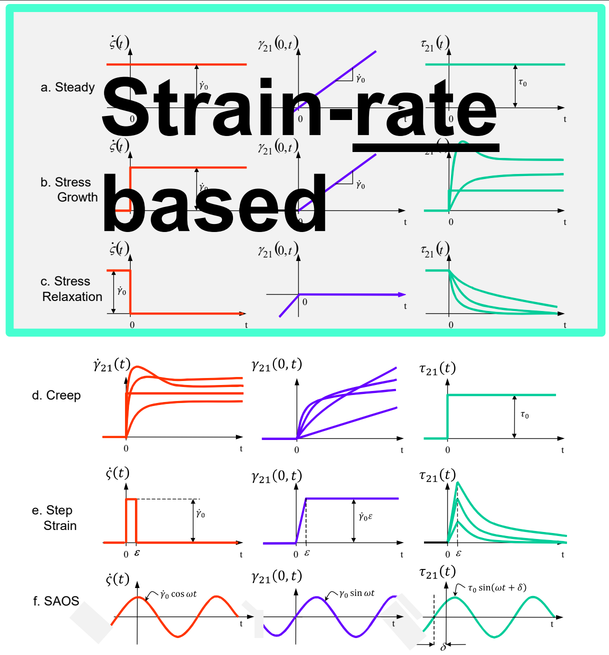
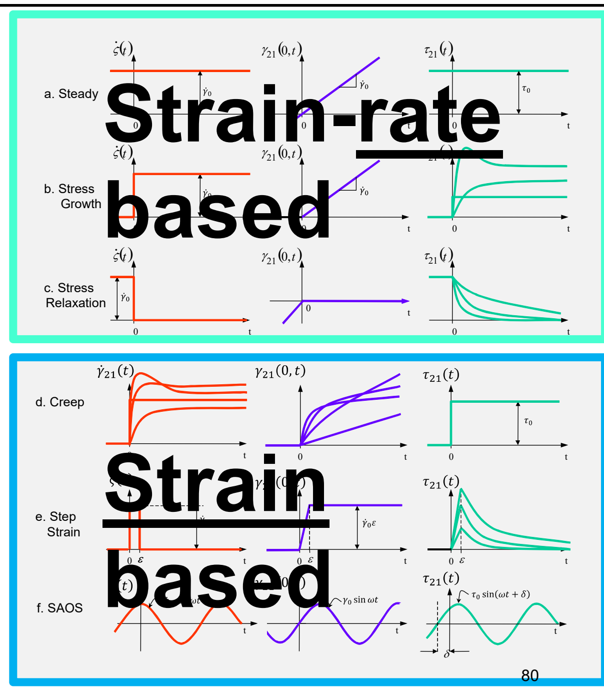
The first three recipe cards were strain-rate based.



79

The first three recipe cards were strain-rate based.

The second three recipe cards are strain based.



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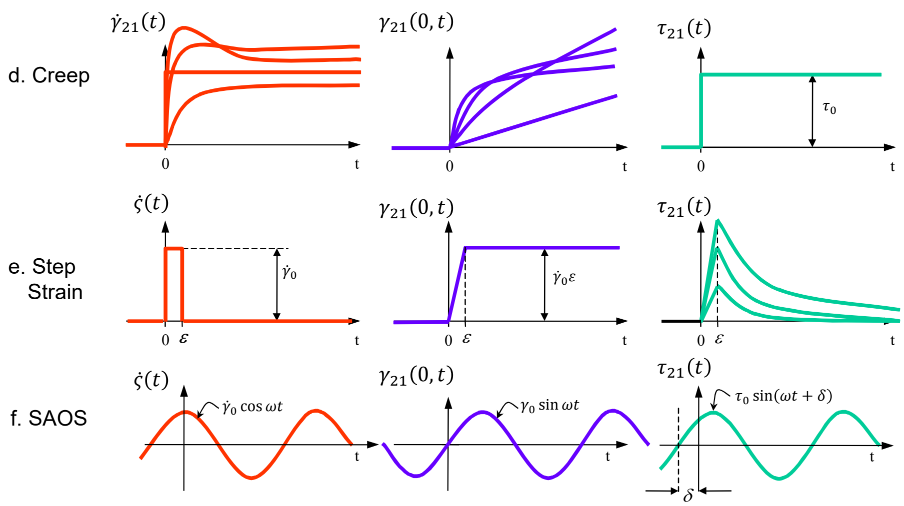
1) Choose a material function – Strain based

Summary of shear rate kinematics *(part 2; strain-based)*



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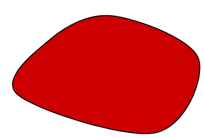
Investigating **Stress/Deformation** Relationships (**Rheology**)

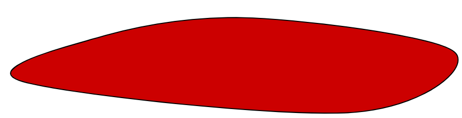
1) Choose a material function – Strain based

What is ***strain***?



Initial or reference state

Strain is a measure of *deformation* (change in shape)



Final state

(shape change relative to some other state)

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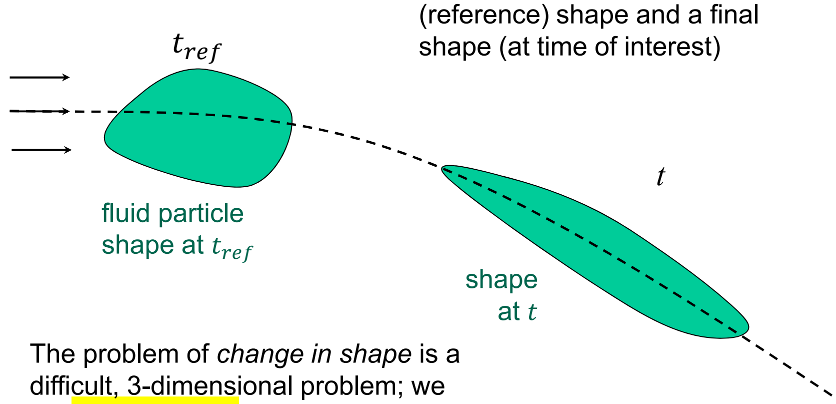
What is **strain**? Answer: change in shape

Deformation (=strain)

We need a way to quantify "change in shape" due to flow.

There must be an initial (reference) shape and a final shape (at time of interest)



The problem of *change in shape* is a difficult, 3-dimensional problem; we can **start simple** with unidirectional flow (shear).

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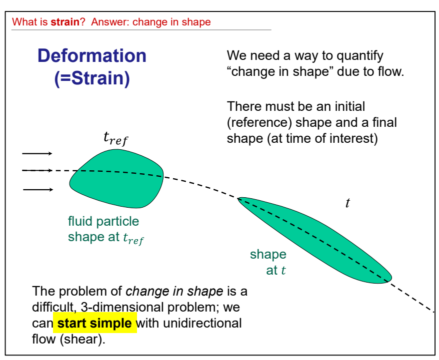
What is **strain**? Answer: change in shape


What is **strain**? Answer: change in shape

Deformation (=Strain)

We need a way to quantify "change in shape" due to flow.

There must be an initial (reference) shape and a final shape (at time of interest)

t_{ref}

fluid particle shape at t_{ref}

t

shape at t

The problem of *change in shape* is a difficult, 3-dimensional problem; we can **start simple** with unidirectional flow (shear).

NOTE:

Strain can be conceptually complicated

Let's begin with the answer

84

What is shear strain? Summary

Strain is our measure of deformation (change of shape)

For shear flow (steady or unsteady, $\dot{\gamma}_{21}(t) = \dot{\varsigma}$):

$$\gamma_{21}(t_1,t_2) = \int_{t_1}^{t_2} \dot{\varsigma}(t')dt'$$	**Strain** is the integral of strain rate	*Strain accumulates as the flow progresses*
$$\frac{d\gamma_{21}}{dt} = \dot{\gamma}_{21}(t)$$ Deformation rate	The **time derivative of strain** is the strain rate	*The strain rate is the rate of instantaneous shape change*

Shear strain tensor:

$$\frac{d\underline{\underline{\dot{\gamma}}}}{dt} = \frac{d(\underline{\underline{\gamma}})}{dt}$$

(see next page)

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Strain Tensor—small strains

Strain is our measure of deformation (change of shape)

Infinitesimal strain tensor:

$$\frac{d\underline{\underline{\dot{\gamma}}}}{dt} = \frac{d(\underline{\underline{\gamma}})}{dt}$$

More on this later.

$$\underline{\underline{\gamma}}(t_1,t_2) = \begin{pmatrix} \int_{t_1}^{t_2} \dot{\gamma}_{11}(t')dt' & \int_{t_1}^{t_2} \dot{\gamma}_{12}(t')dt' & \int_{t_1}^{t_2} \dot{\gamma}_{13}(t')dt' \\ \int_{t_1}^{t_2} \dot{\gamma}_{21}(t')dt' & \int_{t_1}^{t_2} \dot{\gamma}_{22}(t')dt' & \int_{t_1}^{t_2} \dot{\gamma}_{23}(t')dt' \\ \int_{t_1}^{t_2} \dot{\gamma}_{31}(t')dt' & \int_{t_1}^{t_2} \dot{\gamma}_{32}(t')dt' & \int_{t_1}^{t_2} \dot{\gamma}_{33}(t')dt' \end{pmatrix}_{123}$$

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What is elongational strain? Summary

Strain is our measure of deformation (change of shape)

For uniaxial elongational flow (steady or unsteady, $\dot{\varepsilon}(t)$):

$$\varepsilon(t_{ref}, t) = \int_{t_{ref}}^{t} \dot{\varepsilon}(t')dt'$$

Strain is the integral of strain rate

Strain accumulates as the flow progresses

$$\frac{d\varepsilon}{dt} = \dot{\varepsilon}(t)$$

Deformation rate

The **time derivative of strain** is the strain rate

The strain rate is the rate of instantaneous shape change

(finite-strain tensor is complicated) $\frac{d\underline{\underline{\dot{\gamma}}}}{dt} = \frac{d\underline{\underline{?}}}{dt}$ *(see Ch9)*

Now we can continue with material functions based on strain.

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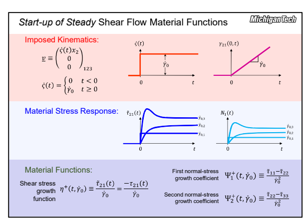
Practice with strain

❑ What is the strain in start-up of steady shear?

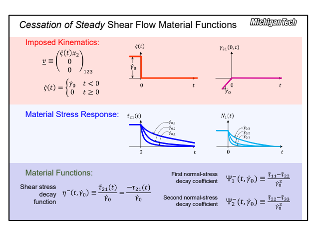
(let $t_{ref} = -\infty$)



Answer: $\dot{\gamma}_0 t$

❑ What is the strain in cessation of steady shear?

(let $t_{ref} = 0$)



Answer: 0

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Start-up of Steady Shear Flow Material Functions

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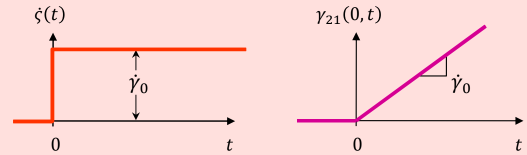
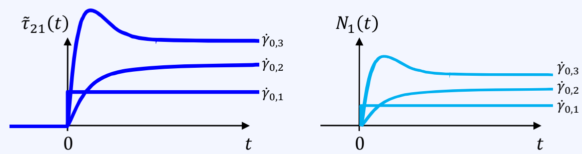
Imposed Kinematics:

$$\underline{v} \equiv \begin{pmatrix} \dot\varsigma(t) x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$$

$$\dot\varsigma(t) = \begin{cases} 0 & t \le 0 \\ \dot\gamma_0 & t < 0 \end{cases}$$



Material Stress Response:



Material Functions:

Shear stress growth function
$$\eta^+(t,\dot\gamma_0) \equiv \frac{\tilde\tau_{21}(t)}{\dot\gamma_0} = \frac{-\tau_{21}(t)}{\dot\gamma_0}$$

First normal-stress growth coefficient
$$\Psi_1^+(t,\dot\gamma_0) \equiv \frac{\tilde\tau_{11}-\tilde\tau_{22}}{\dot\gamma_0^2}$$

Second normal-stress growth coefficient
$$\Psi_2^+(t,\dot\gamma_0) \equiv \frac{\tilde\tau_{22}-\tilde\tau_{33}}{\dot\gamma_0^2}$$

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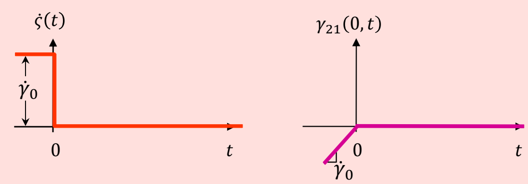
Cessation of Steady Shear Flow Material Functions

Michigan Tech

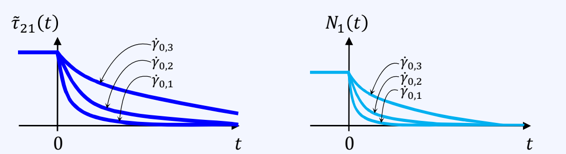
Imposed Kinematics:

$$\underline{v} \equiv \begin{pmatrix} \dot\varsigma(t) x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$$

$$\dot\varsigma(t) = \begin{cases} \dot\gamma_0 & t < 0 \\ 0 & t \ge 0 \end{cases}$$



Material Stress Response:



Material Functions:

Shear stress decay function
$$\eta^-(t,\dot\gamma_0) \equiv \frac{\tilde\tau_{21}(t)}{\dot\gamma_0} = \frac{-\tau_{21}(t)}{\dot\gamma_0}$$

First normal-stress decay coefficient
$$\Psi_1^-(t,\dot\gamma_0) \equiv \frac{\tilde\tau_{11}-\tilde\tau_{22}}{\dot\gamma_0^2}$$

Second normal-stress decay coefficient
$$\Psi_2^-(t,\dot\gamma_0) \equiv \frac{\tilde\tau_{22}-\tilde\tau_{33}}{\dot\gamma_0^2}$$

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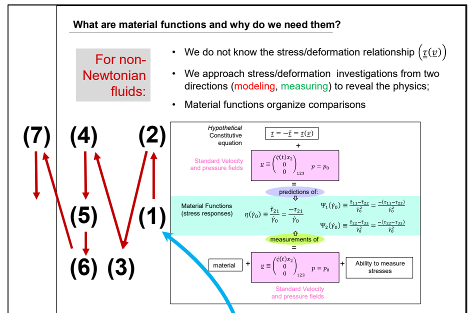
Investigating **Stress/Deformation** Relationships (**Rheology**)

- ✓ Reflect
- ✓ Learn
- ✓ Revise model
- ✓ Propose new $\underline{\underline{\tau}}(\underline{v})$
- ✓ Repeat cycle….

What shall we do next?

To sort out how to model non-Newtonian behavior, we need more observations (to give us ideas).



We want to try another material function.

1) Choose a material function

91

Investigating **Stress/Deformation** Relationships (**Rheology**)

1. Choose a material function
2. Predict what Newtonian fluids would do
3. See what non-Newtonian fluids **do**
4. Hypothesize a $\underline{\underline{\tau}}(\underline{v})$
5. Predict the material function
6. Compare with what non-Newtonian fluids **do**
7. Reflect, **learn**, revise model, repeat.

1) Choose a material function

kinematics

1. Choice of flow (shear or elongation)

$$\underline{v} = \begin{pmatrix} \dot{\varsigma}(t)x_2 \\ 0 \\ 0 \end{pmatrix}_{123} \qquad \underline{v} = \begin{pmatrix} -\frac{1}{2}\dot{\varepsilon}(t)x_1 \\ -\frac{1}{2}\dot{\varepsilon}(t)x_2 \\ \dot{\varepsilon}(t)x_3 \end{pmatrix}_{123}$$

2. Choice of time dependence of $\dot{\varsigma}(t)$ or $\dot{\varepsilon}(t)$

3. Material functions definitions: will be based on τ_{21}, N_1, N_2 in shear or $\tau_{22}-\tau_{11}$, $\tau_{22}-\tau_{11}$ in elongational flows.

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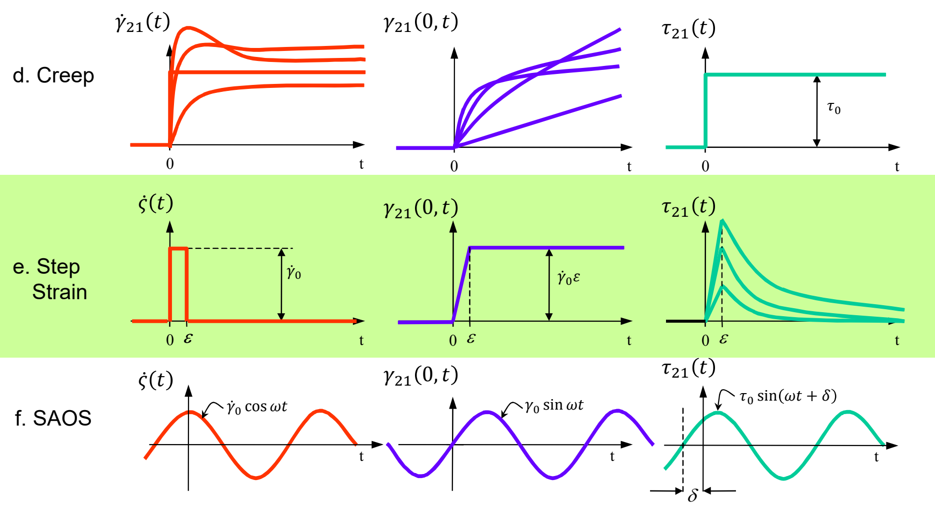
1) Choose a material function – Strain based

Summary of shear rate kinematics *(part 2; strain-based)*



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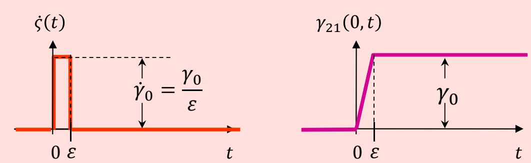
Step Strain Shear Flow Material Functions

Michigan Tech

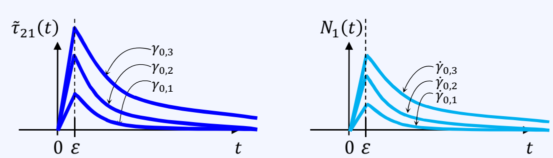
Imposed Kinematics:

$$\underline{v} \equiv \begin{pmatrix} \dot{\varsigma}(t) x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$$

$$\dot{\varsigma}(t) = \lim_{\varepsilon \to 0} \begin{cases} 0 & t \le 0 \\ \gamma_0/\varepsilon & 0 < t < \varepsilon \\ 0 & t \ge \varepsilon \end{cases}$$



Material Stress Response:



Material Functions:

Relaxation modulus $G(t,\gamma_0) \equiv \frac{\tilde{\tau}_{21}(t,\gamma_0)}{\gamma_0} = \frac{-\tau_{21}(t,\gamma_0)}{\gamma_0}$

First normal-stress relaxation modulus $G_{\Psi_1}(t,\gamma_0) \equiv \frac{\tilde{\tau}_{11} - \tilde{\tau}_{22}}{\gamma_0^2}$

Second normal-stress relaxation modulus $G_{\Psi_2}(t,\gamma_0) \equiv \frac{\tilde{\tau}_{22} - \tilde{\tau}_{33}}{\gamma_0^2}$

94

1) Choose a material function – Strain based

What is the **strain** in the step-strain flow?

?

You try.

95

1) Choose a material function – Strain based

What is the **strain** in the step-strain flow?

$$\gamma_{21}(-\infty,t) = \int_{-\infty}^{t} \dot{\varsigma}(t')dt'$$

$$= \int_{-\infty}^{t} \lim_{\varepsilon\to 0}\left(\begin{cases} 0 & t'<0 \\ \gamma_0/\varepsilon & 0\le t'<\varepsilon \\ 0 & t\ge\varepsilon \end{cases}\right)dt'$$

$$= \lim_{\varepsilon\to 0}\int_{0}^{\varepsilon}\frac{\gamma_0}{\varepsilon}dt'$$

$$= \gamma_0$$

The strain imposed is a constant

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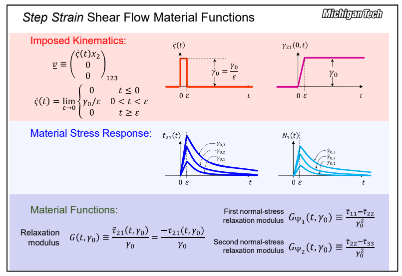
2) Predict what Newtonian fluids would do.

1. Choose a material function
2. Predict what Newtonian fluids would do
3. See what non-Newtonian fluids **do**
4. Hypothesize a $\underline{\underline{\tau}}(\underline{v})$
5. Predict the material function
6. Compare with what non-Newtonian fluids **do**
7. Reflect, **learn**, revise model, repeat.

2) Predict what Newtonian fluids would do.

?



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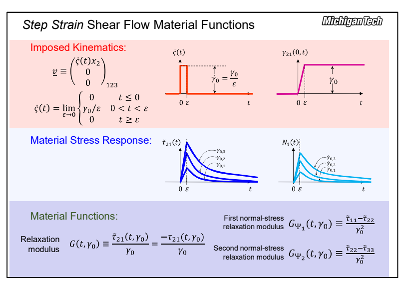
2) Predict what Newtonian fluids would do.

1. Choose a material function
2. Predict what Newtonian fluids would do
3. See what non-Newtonian fluids **do**
4. Hypothesize a $\underline{\underline{\tau}}(\underline{v})$
5. Predict the material function
6. Compare with what non-Newtonian fluids **do**
7. Reflect, **learn**, revise model, repeat.

2) Predict what Newtonian fluids would do.

You try.



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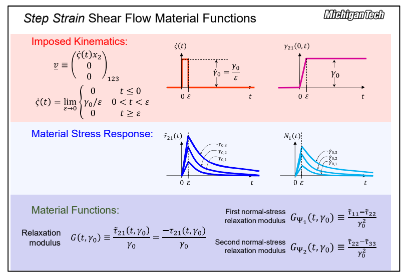
2) Predict what Newtonian fluids would do.

1. Choose a material function
2. Predict what Newtonian fluids would do
3. See what non-Newtonian fluids **do**
4. Hypothesize a $\underline{\underline{\tau}}(\underline{v})$
5. Predict the material function
6. Compare with what non-Newtonian fluids **do**
7. Reflect, **learn**, revise model, repeat.

2) Predict what Newtonian fluids would do.

Answer: mostly zero

(Dirac delta function)



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3) See what non-Newtonian fluids do

1. Choose a material function
2. Predict what Newtonian fluids would do
3. See what non-Newtonian fluids **do**
4. Hypothesize a $\underline{\underline{\tau}}(\underline{v})$
5. Predict the material function
6. Compare with what non-Newtonian fluids **do**
7. Reflect, **learn**, revise model, repeat.

3) See what non-Newtonian fluids **do**

?

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3) See what non-Newtonian fluids do

Step shear strain - *strain dependence*

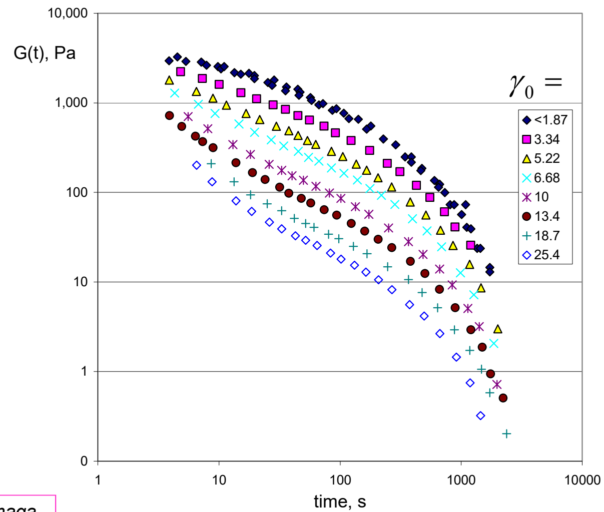

Figure 6.57, p. 212 Einaga et al.; PS soln

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3) See what non-Newtonian fluids do

Linear viscoelastic limit

$$\lim_{\gamma_0 \to 0} G(t, \gamma_0) = G(t)$$

At small strains the relaxation modulus is independent of strain.

The polystyrene solutions on the previous slide show time-strain independence, i.e. the curves have the same shape at different strains.

Damping function, $h(\gamma_0)$:

$$h(\gamma_0) \equiv \frac{G(t, \gamma_0)}{G(t)}$$

The damping function summarizes the non-linear effects as a function of strain amplitude.

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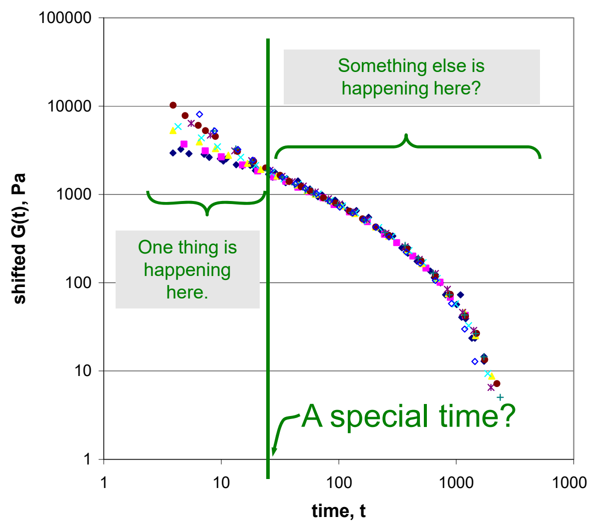
3) See what non-Newtonian fluids do

Step shear strain - **Damping Function**



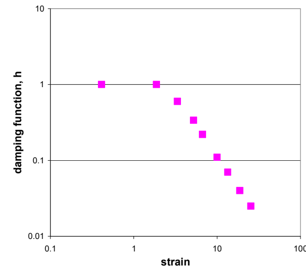


Figure 6.58, p. 213
Einaga et al.; PS soln

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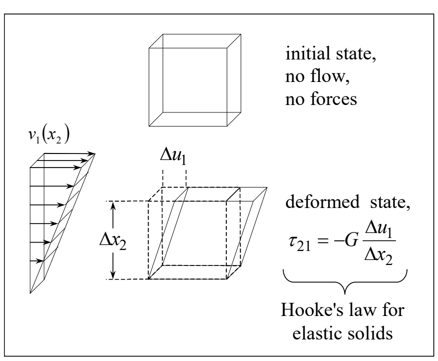
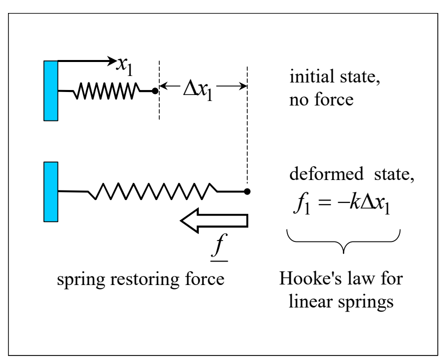
3) See what non-Newtonian fluids do

What types of materials generate stress in proportion to the strain imposed? Answer: elastic solids

Hooke's Law for elastic solids: $\tau_{21}(t) = -G\gamma_{21}(0,t)$
(shear flow)





Similar to the linear spring law

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2) Predict what Newtonian fluids would do.

1. Choose a material function
2. Predict what Newtonian fluids would do
3. See what non-Newtonian fluids **do**
4. Hypothesize a $\underline{\underline{\tau}}(\underline{v})$
5. Predict the material function
6. Compare with what non-Newtonian fluids **do**
7. Reflect, **learn**, revise model, repeat.

Hookean solids

2) Predict what ~~Newtonian fluids~~ would do.

Hookean Solid Constitutive Equation

$$\underline{\underline{\tau}} = -G\underline{\underline{\gamma}}(0,t)$$

In shear: $\tau_{21}(t) = -G\gamma_{21}(0,t)$

?

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2) Predict what Newtonian fluids would do.

1. Choose a material function
2. Predict what Newtonian fluids would do
3. See what non-Newtonian fluids **do**
4. Hypothesize a $\underline{\underline{\tau}}(\underline{v})$
5. Predict the material function
6. Compare with what non-Newtonian fluids **do**
7. Reflect, **learn**, revise model, repeat.

Hookean solids

2) Predict what ~~Newtonian fluids~~ would do.

Hookean Solid Constitutive Equation

$$\underline{\underline{\tau}} = -G\underline{\underline{\gamma}}(0,t)$$

Answer: much more interesting.

Solution:
- *Recipe card gives kinematics*
- *Calculate* $\underline{\underline{\tau}}(t)$
- Calculate material functions

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Investigating **Stress/Deformation** Relationships (**Rheology**)

1. Choose a material function
2. Predict what Newtonian fluids would do
3. See what non-Newtonian fluids **do**
4. Hypothesize a $\underline{\underline{\tau}}(\underline{v})$
5. Predict the material function
6. Compare with what non-Newtonian fluids **do**
7. Reflect, **learn**, revise model, repeat.

1) Choose a material function

kinematics

1. Choice of flow (shear or elongation)

$$\underline{v} = \begin{pmatrix} \dot{\varsigma}(t)x_2 \\ 0 \\ 0 \end{pmatrix}_{123} \qquad \underline{v} = \begin{pmatrix} -\frac{1}{2}\dot{\varepsilon}(t)x_1 \\ -\frac{1}{2}\dot{\varepsilon}(t)x_2 \\ \dot{\varepsilon}(t)x_3 \end{pmatrix}_{123}$$

2. Choice of time dependence of $\dot{\varsigma}(t)$ or $\dot{\varepsilon}(t)$

3. Material functions definitions: will be based on τ_{21}, N_1, N_2 in shear or $\tau_{22} - \tau_{11}$, $\tau_{22} - \tau_{11}$ in elongational flows.

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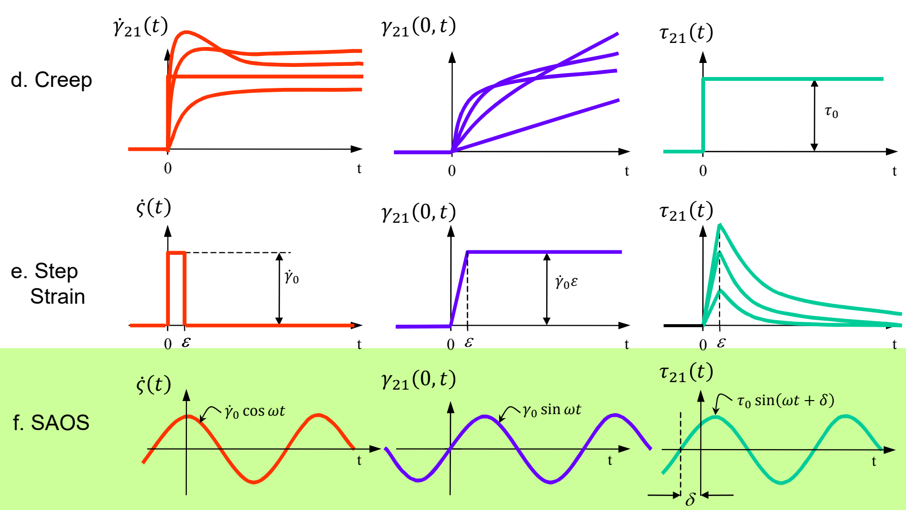
1) Choose a material function – Strain based

Summary of shear rate kinematics *(part 2; strain-based)*



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Small-Amplitude Oscillatory Shear Material Functions

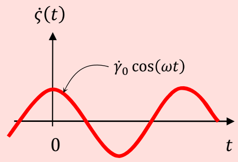
Michigan Tech

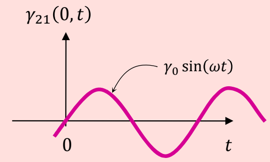
Imposed Kinematics:

$$\underline{v} \equiv \begin{pmatrix} \dot{\varsigma}(t) x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$$

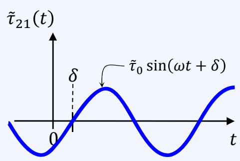
$$\dot{\varsigma}(t) = \dot{\gamma}_0 \cos(\omega t)$$

$$\gamma_0 = \frac{\dot{\gamma}_0}{\omega}$$





Material Stress Response:



δ = phase difference between stress and strain waves

$N_1(t) = N_2(t) = 0$
(linear viscoelastic regime)

Material Functions:

SAOS stress

$$\frac{\tilde{\tau}_{21}(t,\gamma_0)}{\gamma_0} = \frac{-\tau_{21}(t,\gamma_0)}{\gamma_0} = \tilde{\tau}_0 \sin(\omega t + \delta) = G' \sin(\omega t) + G'' \cos(\omega t)$$

Storage modulus $G'(\omega) \equiv \frac{\tilde{\tau}_0}{\gamma_0} \cos(\delta)$ Loss modulus $G''(\omega) \equiv \frac{\tilde{\tau}_0}{\gamma_0} \sin(\delta)$

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1) Choose a material function – Strain based

What is the strain in small-amplitude oscillatory shear?

(let $t_{ref} = 0$)

Answer: $\gamma_{21}(0,t) = \frac{\dot{\gamma}_0}{\omega} \sin \omega t$

1) Choose a material function – Strain based

In SAOS the strain amplitude is small, and a sinusoidal imposed strain induces a sinusoidal measured stress.

$$-\tau_{21}(t) = \tau_0 \sin(\omega t + \delta)$$

$$\begin{aligned}
-\tau_{21}(t) &= \tau_0 \sin(\omega t + \delta) \\
&= \tau_0 \sin \omega t \cos \delta + \tau_0 \cos \omega t \sin \delta \\
&= \underbrace{[\tau_0 \cos \delta]}_{\text{portion in-phase with strain}} \sin \omega t + \underbrace{[\tau_0 \sin \delta]}_{\text{portion in-phase with strain-rate}} \cos \omega t
\end{aligned}$$

portion in-phase with strain *portion in-phase with strain-rate*

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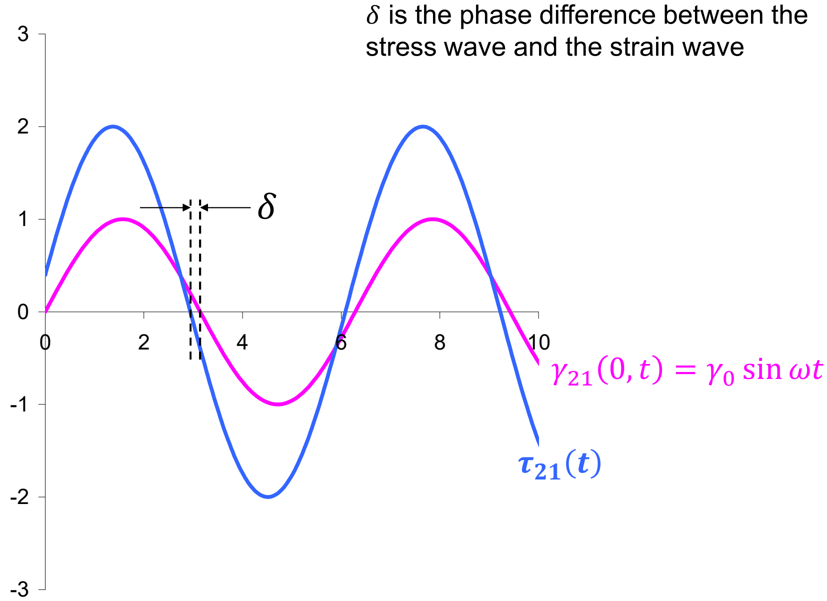
1) Choose a material function – Strain based

δ is the phase difference between the stress wave and the strain wave



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1) Choose a material function – Strain based

SAOS Material Functions

$$\frac{-\tau_{21}(t)}{\gamma_0} = \left[\frac{\tau_0 \cos\delta}{\gamma_0}\right]\sin\omega t + \left[\frac{\tau_0 \sin\delta}{\gamma_0}\right]\cos\omega t$$

portion in-phase with strain — G'

portion in-phase with strain-rate — G''

For Newtonian fluids, stress is proportional to strain rate: $\tau_{21} = -\mu\dot{\gamma}_{21}$

G" is thus known as the viscous loss modulus. It characterizes the viscous contribution to the stress response.

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1) Choose a material function – Strain based

SAOS Material Functions

$$\frac{-\tau_{21}(t)}{\gamma_0} = \left[\frac{\tau_0 \cos\delta}{\gamma_0}\right]\sin\omega t + \left[\frac{\tau_0 \sin\delta}{\gamma_0}\right]\cos\omega t$$

portion in-phase with strain — G'

portion in-phase with strain-rate — G''

$\tau_{21} = -G\gamma_{21}$

For Hookean solids, stress is proportional to strain :

G' is thus known as the elastic storage modulus. It characterizes the elastic contribution to the stress response.

(note: SAOS material functions may also be expressed in complex notation. See pp. 156-159 of Morrison, 2001)

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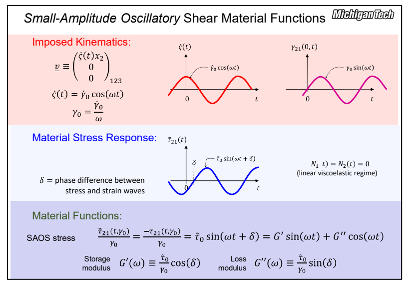
2) Predict what Newtonian fluids would do.

1. Choose a material function
2. Predict what Newtonian fluids would do
3. See what non-Newtonian fluids **do**
4. Hypothesize a $\underline{\underline{\tau}}(\underline{v})$
5. Predict the material function
6. Compare with what non-Newtonian fluids **do**
7. Reflect, **learn**, revise model, repeat.

2) Predict what Newtonian fluids would do.

?



Answer: $\delta = \frac{\pi}{2}; G'' = \mu\omega$

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2) Predict what Newtonian fluids would do.

1. Choose a material function
2. Predict what Newtonian fluids would do
3. See what non-Newtonian fluids **do**
4. Hypothesize a $\underline{\underline{\tau}}(\underline{v})$
5. Predict the material function
6. Compare with what non-Newtonian fluids **do**
7. Reflect, **learn**, revise model, repeat.

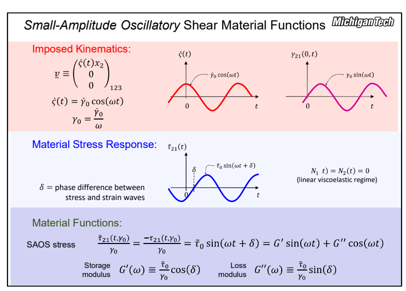
Hookean solids

2) Predict what ~~Newtonian fluids~~ would do.

Hookean Solid Constitutive Equation

$$\underline{\underline{\tau}} = -G\underline{\underline{\gamma}}(0,t)$$

?



In shear: $\tau_{21}(t) = -G\gamma_{21}(0,t)$

Answer: $\delta = 0; G' = G$

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3) See what non-Newtonian fluids do

1. Choose a material function
2. Predict what Newtonian fluids would do
3. See what non-Newtonian fluids **do**
4. Hypothesize a $\underline{\underline{\tau}}(\underline{v})$
5. Predict the material function
6. Compare with what non-Newtonian fluids **do**
7. Reflect, **learn**, revise model, repeat.

3) See what non-Newtonian fluids **do**

?

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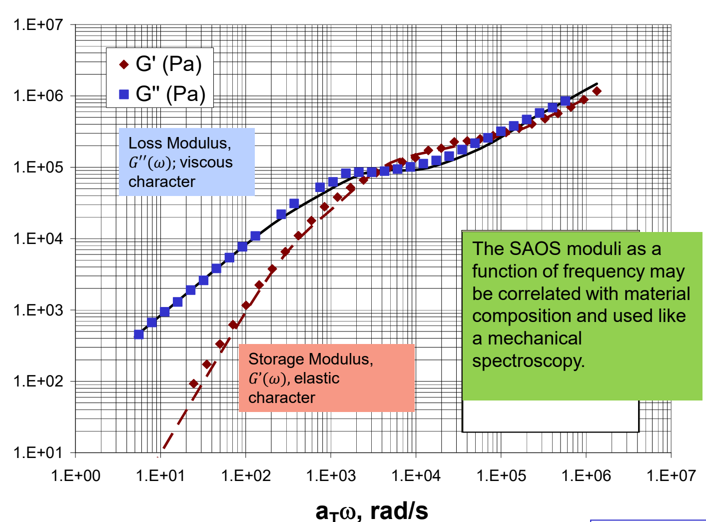
3) See what non-Newtonian fluids do

SAOS Moduli of a Polymer Melt



The SAOS moduli as a function of frequency may be correlated with material composition and used like a mechanical spectroscopy.

Figure 8.8, p. 284 data from Vinogradov, PS melt

3) See what non-Newtonian fluids do

SAOS Moduli of a Polymer Melt

We'll talk more about this frequency dependence in Chapter 6.

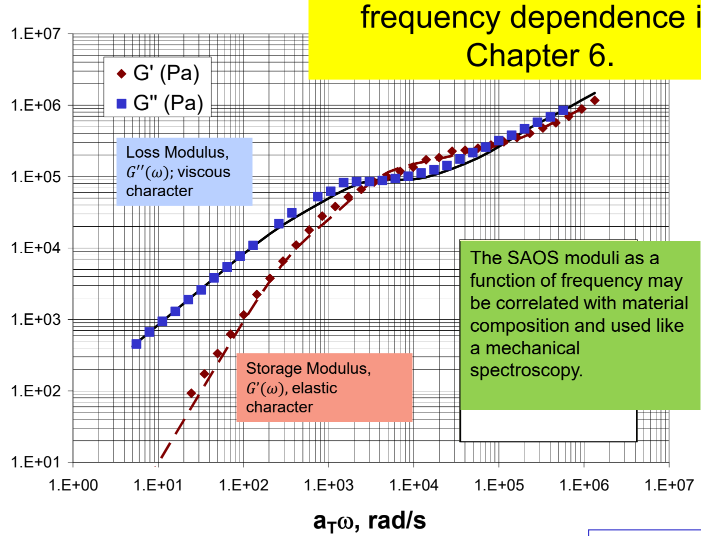
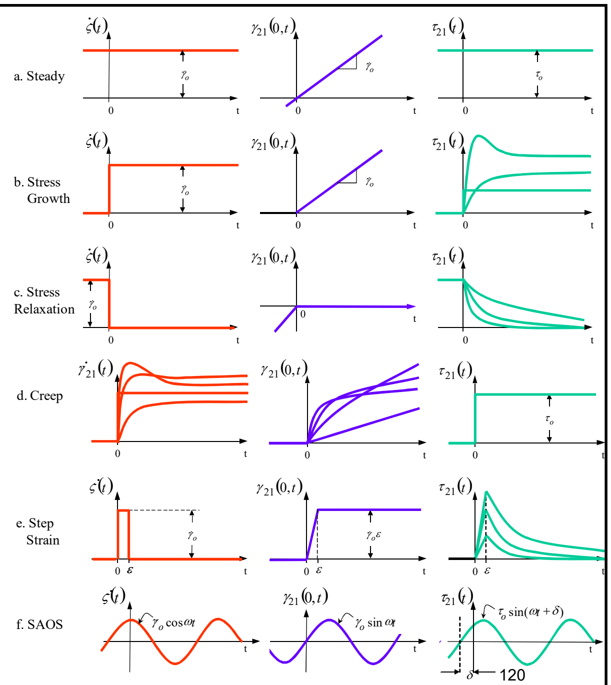


Figure 8.8, p. 284 data from Vinogradov, PS melt

We have discussed six shear material functions;

Now, the equivalent **elongational** material functions

$$\underline{v} \equiv \begin{pmatrix} -\frac{1}{2}\dot{\varepsilon}(t)x_1 \\ -\frac{1}{2}\dot{\varepsilon}(t)x_2 \\ \dot{\varepsilon}(t)x_3 \end{pmatrix}_{123}$$



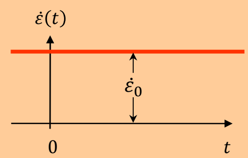
Steady Elongational Flow Material Functions

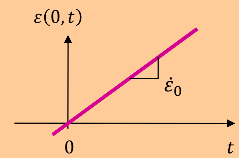
Michigan Tech

Imposed Kinematics:

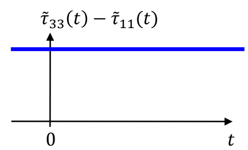
$$\underline{v} \equiv \begin{pmatrix} -\frac{1}{2}\dot{\varepsilon}(t)x_1 \\ -\frac{1}{2}\dot{\varepsilon}(t)x_2 \\ \dot{\varepsilon}(t)x_3 \end{pmatrix}_{123}$$

$$\dot{\varepsilon}(t) = \dot{\varepsilon}_0 = \text{constant}$$





Material Stress Response:



Material Functions:

Elongational Viscosity

$$\eta_e(\dot{\varepsilon}_0) \equiv \frac{\tilde{\tau}_{33} - \tilde{\tau}_{11}}{\dot{\varepsilon}_0} = \frac{-(\tau_{33} - \tau_{11})}{\dot{\varepsilon}_0}$$

Alternatively, $\bar{\eta}(\dot{\varepsilon}_0)$

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1) Choose a material function – elongational flow

1. Choose a material function
2. Predict what Newtonian fluids would do
3. See what non-Newtonian fluids **do**
4. Hypothesize a $\underline{\underline{\tau}}(\underline{v})$
5. Predict the material function
6. Compare with what non-Newtonian fluids **do**
7. Reflect, **learn**, revise model, repeat.

The other elongational experiments are analogous to shear experiments *(see text)*

- Elongational stress growth
- Elongational stress cessation *(nearly impossible)*
- Elongational creep
- Step elongational strain
- Small-amplitude Oscillatory Elongation (SAOE) *(Redundant with SAOS)*

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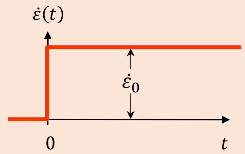
Start-up of Steady Elongation Material Functions

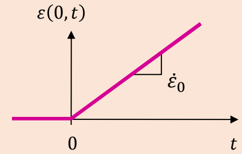
Michigan Tech

Imposed Kinematics:

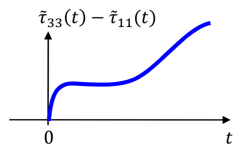
$$\underline{v} \equiv \begin{pmatrix} -\frac{1}{2}\dot{\varepsilon}(t)x_1 \\ -\frac{1}{2}\dot{\varepsilon}(t)x_2 \\ \dot{\varepsilon}(t)x_3 \end{pmatrix}_{123}$$

$$\dot{\varepsilon}(t) = \begin{cases} 0 & t < 0 \\ \dot{\varepsilon}_0 & t \geq 0 \end{cases}$$





Material Stress Response:



Material Functions:

Elongational Start-up Function

$$\eta_e^+(t,\dot{\varepsilon}_0) \equiv \frac{\tilde{\tau}_{33} - \tilde{\tau}_{11}}{\dot{\varepsilon}_0} = \frac{-(\tau_{33} - \tau_{11})}{\dot{\varepsilon}_0}$$

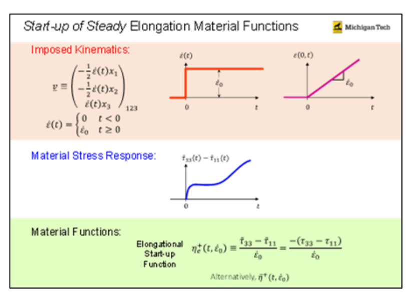
Alternatively, $\bar{\eta}^+(t,\dot{\varepsilon}_0)$

3) See what non-Newtonian fluids do

1. Choose a material function
2. Predict what Newtonian fluids would do
3. See what non-Newtonian fluids **do**
4. Hypothesize a $\underline{\underline{\tau}}(\underline{v})$
5. Predict the material function
6. Compare with what non-Newtonian fluids **do**
7. Reflect, **learn**, revise model, repeat.

3) See what non-Newtonian fluids **do**

?



3) See what non-Newtonian fluids do

Start-up of Steady Elongation

Strain-hardening

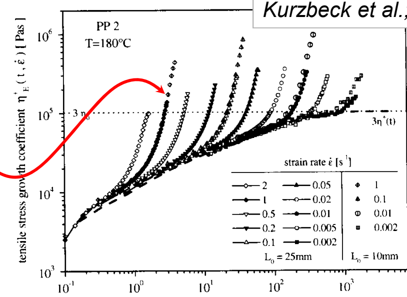


Figure 6.64, p. 218 Kurzbeck et al.; PP

Fit to an advanced constitutive equation (12 mode pom-pom model)

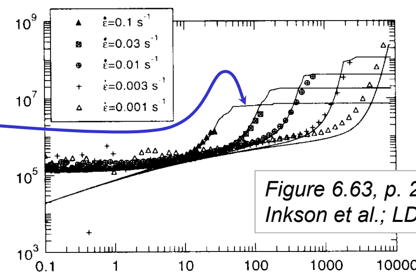


Figure 6.63, p. 217 Inkson et al.; LDPE

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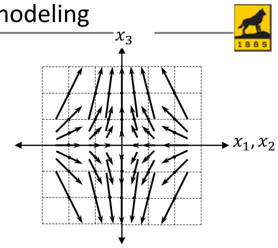
Part II-A. Continuum versus molecular modeling

Michigan Tech

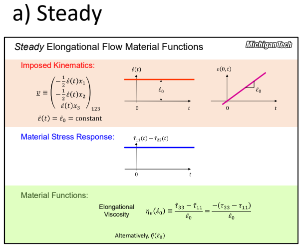
Elongation Material Functions

$$\underline{v} \equiv \begin{pmatrix} -\frac{1}{2}\dot{\varepsilon}(t)x_1 \\ -\frac{1}{2}\dot{\varepsilon}(t)x_2 \\ \dot{\varepsilon}(t)x_3 \end{pmatrix}_{123}$$

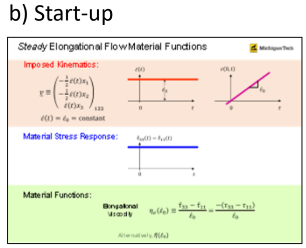


"Recipe cards"

a) Steady



b) Start-up



c) Cessation

(currently unobservable)

d) Step strain

(exists, but less often discussed)

e) SAOE

(exists, but easily converted to SAOS so is redundant)

f) Creep

(exists)

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What's next?

Make better constitutive equations

1. Add invariants (replace $\dot{\gamma}_0$, which is flow-specific).
2. Make constitutive equations that reference flow in the past (not purely instantaneous)
3. Investigate strain

Be inspired by material behavior

1. Become informed on more rheological behavior
2. Get more of a feel for what is observed, and when

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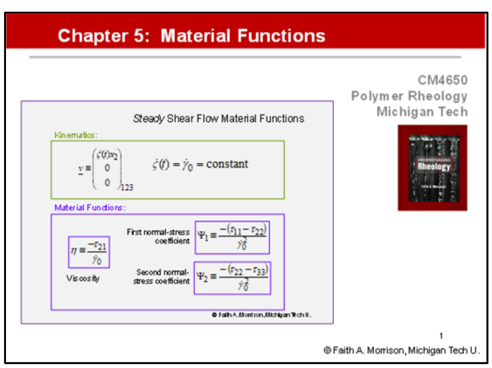
Done with Material Functions.



Let's move on to Experimental Data

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Chapter 6: Experimental Data

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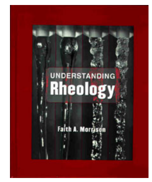

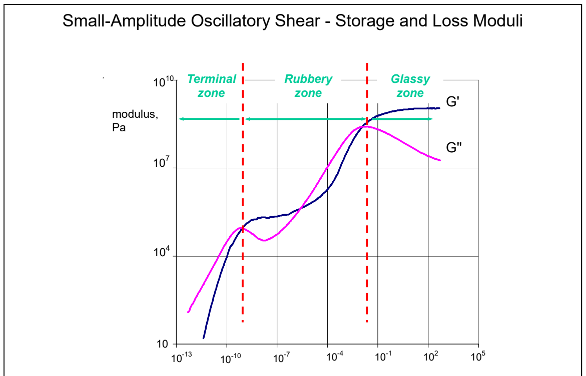

Figure 6.30, p. 192 Plazek and O'Rourke; PS

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