



CM4650 Polymer Rheology *Just-in-time*




What exactly do we observe when we subject non-Newtonian fluids to deformation?



Professor Faith A. Morrison
Department of Chemical Engineering
Michigan Technological University

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Part II-A. Continuum versus molecular modeling 


What exactly do we observe when we subject non-Newtonian fluids to deformation?

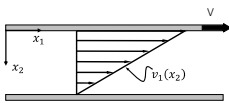
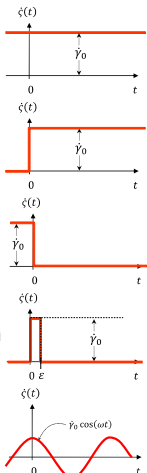
Rheology uses a sort of an ASTM or ISO-like technical standards approach to organize observations:

1. Choose a standard flow (shear or elongation)
2. Choose a set of flow kinematics (the speed and time-profile of the specific test)
3. Measure specified quantities (related to stress or deformation)
4. Report a standardized function (material function)


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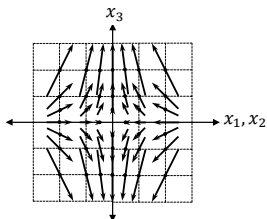
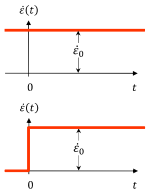
What exactly do we observe when we subject non-Newtonian fluids to deformation?



Standard flow	$\zeta(t) =$	Standard kinematics	What do we observe?
<div style="background-color: yellow; padding: 5px; color: red; font-weight: bold; margin-bottom: 10px;">Shear</div> $\underline{v} \equiv \begin{pmatrix} \zeta(t)x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$ 	<p>a) Steady</p> <p>b) Start-up</p> <p>c) Cessation</p> <p>d) Step-strain</p> <p>e) SAOS*</p>		<div style="border: 1px solid blue; border-radius: 50%; width: 100px; height: 150px; background-color: #e6e6fa; display: flex; align-items: center; justify-content: center; margin: 0 auto;"> <p style="text-align: center;">Shear Material Functions</p> </div>
3/5/2018	(*SAOS=small-amplitude oscillatory shear)		© Faith A. Morrison, Michigan Tech U. ³

What exactly do we observe when we subject non-Newtonian fluids to deformation?



Standard flow	$\epsilon(t) =$	Standard kinematics	What do we observe?
<div style="background-color: yellow; padding: 5px; color: red; font-weight: bold; margin-bottom: 10px;">Elongation</div> $\underline{v} \equiv \begin{pmatrix} -\frac{1}{2}\dot{\epsilon}(t)x_1 \\ -\frac{1}{2}\dot{\epsilon}(t)x_2 \\ \dot{\epsilon}(t)x_3 \end{pmatrix}_{123}$ 	<p>a) Steady</p> <p>b) Start-up</p> <p>(etc.)</p>		<div style="border: 1px solid blue; border-radius: 50%; width: 100px; height: 100px; background-color: #e6e6fa; display: flex; align-items: center; justify-content: center; margin: 0 auto;"> <p style="text-align: center;">Elongation Material Functions</p> </div>
3/5/2018	Note: the Society of Rheology convention has flow in the 1-direction rather than the 3-direction.		© Faith A. Morrison, Michigan Tech U. ⁴

What exactly do we observe when we subject non-Newtonian fluids to deformation?

Material Functions

Summarized on "recipe cards" ➔

- Vocabulary (framework) of material comparison
- Used to characterize a material as Newtonian vs. non-Newtonian

Newtonian

$\eta = \mu = \text{constant}$

$\Psi_1 = \Psi_2 = 0$

$G' = 0$

etc.

Non-Newtonian

$\eta = \eta(\dot{\gamma})$

$\Psi_1 \neq 0; \Psi_2 \neq 0$

$G', G'' \neq 0$

etc.

Steady Shear Flow Material Functions (Michigan Tech)

Imposed Kinematics: $\dot{\gamma}(t) = \dot{\gamma}_0 = \text{constant}$

Material Stress Response: $\tau_{21}(t)$, $N_{11}(t)$, $N_{22}(t)$

Material Functions: Viscosity $\eta(\dot{\gamma}_0) = \frac{\tau_{21}}{\dot{\gamma}_0} = -\frac{\tau_{12}}{\dot{\gamma}_0}$

First normal-stress coefficient $\Psi_1(\dot{\gamma}_0) \equiv \frac{\tau_{11} - \tau_{22}}{\dot{\gamma}_0^2} = -\frac{(\tau_{11} - \tau_{22})}{\dot{\gamma}_0^2}$

Second normal-stress coefficient $\Psi_2(\dot{\gamma}_0) \equiv \frac{\tau_{22} - \tau_{33}}{\dot{\gamma}_0^2} = -\frac{(\tau_{22} - \tau_{33})}{\dot{\gamma}_0^2}$

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What exactly do we observe when we subject non-Newtonian fluids to deformation?

Material Functions

Summarized on "recipe cards" ➔

- Vocabulary (framework) of material comparison
- Used to characterize a material as Newtonian vs. non-Newtonian
- Within non-Newtonian fluids, used to further categorize materials

Cross-linked rubber: $G' = G_0 = \text{constant}$
 Entangled melt: characteristic $G'(\omega), G''(\omega)$ shapes
 Shear thinning, shear thickening melt: characteristic $\eta(\dot{\gamma})$
 Branched polymer: characteristic $G'(\omega), G''(\omega)$ shapes

Steady Shear Flow Material Functions (Michigan Tech)

Imposed Kinematics: $\dot{\gamma}(t) = \dot{\gamma}_0 = \text{constant}$

Material Stress Response: $\tau_{21}(t)$, $N_{11}(t)$, $N_{22}(t)$

Material Functions: Viscosity $\eta(\dot{\gamma}_0) = \frac{\tau_{21}}{\dot{\gamma}_0} = -\frac{\tau_{12}}{\dot{\gamma}_0}$


First normal-stress coefficient $\Psi_1(\dot{\gamma}_0) \equiv \frac{\tau_{11} - \tau_{22}}{\dot{\gamma}_0^2} = -\frac{(\tau_{11} - \tau_{22})}{\dot{\gamma}_0^2}$

Second normal-stress coefficient $\Psi_2(\dot{\gamma}_0) \equiv \frac{\tau_{22} - \tau_{33}}{\dot{\gamma}_0^2} = -\frac{(\tau_{22} - \tau_{33})}{\dot{\gamma}_0^2}$

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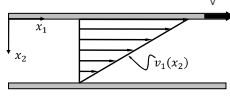
Part II-A. Continuum versus molecular modeling



Michigan Tech

Shear Material Functions

$$\underline{v} \equiv \begin{pmatrix} \dot{\zeta}(t)x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$$



"Recipe cards"

a) Steady

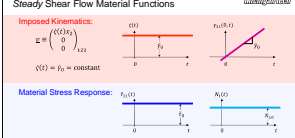
Steady Shear Flow Material Functions

Imposed Kinematics:

$$\underline{\Sigma} = \begin{pmatrix} \dot{\zeta}(t)x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$$

(t) = $\dot{\zeta}_0$ = constant

Material Stress Response:



Material Functions:

First normal stress coefficient: $\Psi_1^*(\dot{\zeta}_0) \equiv \frac{\tau_{11}(\dot{\zeta}_0)}{\dot{\zeta}_0} = \frac{\tau_{11}(\dot{\zeta}_0)}{\dot{\zeta}_0}$

Second normal stress coefficient: $\Psi_2^*(\dot{\zeta}_0) \equiv \frac{\tau_{22}(\dot{\zeta}_0)}{\dot{\zeta}_0} = \frac{\tau_{22}(\dot{\zeta}_0)}{\dot{\zeta}_0}$

Viscosity: $\eta(\dot{\zeta}_0) \equiv \frac{\tau_{12}(\dot{\zeta}_0)}{\dot{\zeta}_0} = \frac{\tau_{12}(\dot{\zeta}_0)}{\dot{\zeta}_0}$

b) Start-up

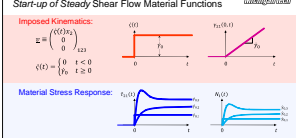
Start-up of Steady Shear Flow Material Functions

Imposed Kinematics:

$$\underline{\Sigma} = \begin{pmatrix} \dot{\zeta}(t)x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$$

(t) = $\dot{\zeta}_0$ for $t < 0$
 $\dot{\zeta}_0$ for $t \geq 0$

Material Stress Response:



Material Functions:

First normal stress growth coefficient: $\Psi_1^*(t, \dot{\zeta}_0) \equiv \frac{\tau_{11}(t, \dot{\zeta}_0)}{\dot{\zeta}_0}$

Second normal stress growth coefficient: $\Psi_2^*(t, \dot{\zeta}_0) \equiv \frac{\tau_{22}(t, \dot{\zeta}_0)}{\dot{\zeta}_0}$

Shear stress growth function: $\eta^*(t, \dot{\zeta}_0) \equiv \frac{\tau_{12}(t, \dot{\zeta}_0)}{\dot{\zeta}_0}$

c) Cessation

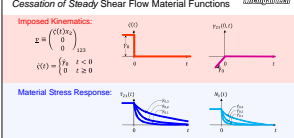
Cessation of Steady Shear Flow Material Functions

Imposed Kinematics:

$$\underline{\Sigma} = \begin{pmatrix} \dot{\zeta}(t)x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$$

(t) = $\dot{\zeta}_0$ for $t < 0$
0 for $t \geq 0$

Material Stress Response:



Material Functions:

First normal stress decay coefficient: $\Psi_1^*(t, \dot{\zeta}_0) \equiv \frac{\tau_{11}(t, \dot{\zeta}_0)}{\dot{\zeta}_0}$

Second normal stress decay coefficient: $\Psi_2^*(t, \dot{\zeta}_0) \equiv \frac{\tau_{22}(t, \dot{\zeta}_0)}{\dot{\zeta}_0}$

Shear stress decay function: $\eta^*(t, \dot{\zeta}_0) \equiv \frac{\tau_{12}(t, \dot{\zeta}_0)}{\dot{\zeta}_0}$

d) Step strain

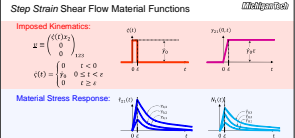
Step Strain Shear Flow Material Functions

Imposed Kinematics:

$$\underline{\Sigma} = \begin{pmatrix} \dot{\zeta}(t)x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$$

(t) = 0 for $t < 0$
 $\dot{\zeta}_0$ for $0 \leq t < \infty$

Material Stress Response:



Material Functions:

First normal stress relaxation modulus: $G_{11}(t, \dot{\zeta}_0) \equiv \frac{\tau_{11}(t, \dot{\zeta}_0)}{\dot{\zeta}_0}$

Second normal stress relaxation modulus: $G_{22}(t, \dot{\zeta}_0) \equiv \frac{\tau_{22}(t, \dot{\zeta}_0)}{\dot{\zeta}_0}$

Relaxation modulus: $G(t, \dot{\zeta}_0) \equiv \frac{\tau_{12}(t, \dot{\zeta}_0)}{\dot{\zeta}_0}$

e) SAOS

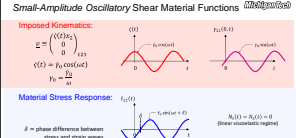
Small-Amplitude Oscillatory Shear Material Functions

Imposed Kinematics:

$$\underline{\Sigma} = \begin{pmatrix} \dot{\zeta}(t)x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$$

(t) = $\dot{\zeta}_0 \cos(\omega t)$

Material Stress Response:



Material Functions:

SAOS stress: $\frac{\tau_{12}(t, \dot{\zeta}_0)}{\dot{\zeta}_0} = \tau_1 \sin(\omega t + \delta) = G' \sin(\omega t) + G'' \cos(\omega t)$

Storage modulus: $G'(\omega) \equiv \frac{\tau_1}{\dot{\zeta}_0} \cos(\delta)$

Loss modulus: $G''(\omega) \equiv \frac{\tau_1}{\dot{\zeta}_0} \sin(\delta)$

δ = phase difference between stress and strain waves

f) Creep

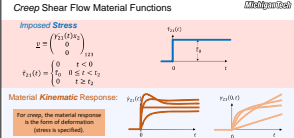
Creep Shear Flow Material Functions

Imposed Stress:

$$\underline{\Sigma} = \begin{pmatrix} \tau_{11}(t)x_1 \\ 0 \\ 0 \end{pmatrix}_{123}$$

(t) = 0 for $t < 0$
 τ_0 for $0 \leq t < \infty$

Material Kinematic Response:



Material Function:

Shear creep compliance: $J(t, \tau_0) \equiv \frac{\gamma(t, \tau_0)}{\tau_0}$


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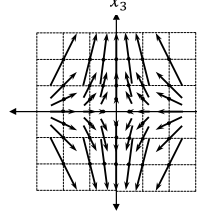
Part II-A. Continuum versus molecular modeling



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Elongation Material Functions

$$\underline{v} \equiv \begin{pmatrix} -\frac{1}{2}\dot{\epsilon}(t)x_1 \\ -\frac{1}{2}\dot{\epsilon}(t)x_2 \\ \dot{\epsilon}(t)x_3 \end{pmatrix}_{123}$$



"Recipe cards"

a) Steady

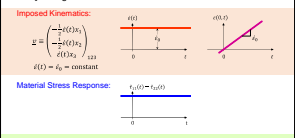
Steady Elongational Flow Material Functions

Imposed Kinematics:

$$\underline{\Sigma} = \begin{pmatrix} -\frac{1}{2}\dot{\epsilon}(t)x_1 \\ -\frac{1}{2}\dot{\epsilon}(t)x_2 \\ \dot{\epsilon}(t)x_3 \end{pmatrix}_{123}$$

(t) = $\dot{\epsilon}_0$ = constant

Material Stress Response:



Material Functions:

Elongational Viscosity: $\eta_e(\dot{\epsilon}_0) \equiv \frac{\tau_{11}(\dot{\epsilon}_0) - \tau_{22}(\dot{\epsilon}_0)}{\dot{\epsilon}_0} = \frac{\tau_{11}(\dot{\epsilon}_0) - \tau_{22}(\dot{\epsilon}_0)}{\dot{\epsilon}_0}$

Alternatively: $\eta_e(\dot{\epsilon}_0)$

b) Start-up

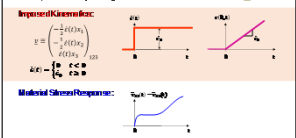
Start-up of Steady Elongation Material Functions

Imposed Kinematics:

$$\underline{\Sigma} = \begin{pmatrix} -\frac{1}{2}\dot{\epsilon}(t)x_1 \\ -\frac{1}{2}\dot{\epsilon}(t)x_2 \\ \dot{\epsilon}(t)x_3 \end{pmatrix}_{123}$$

(t) = $\dot{\epsilon}_0$ for $t < 0$
 $\dot{\epsilon}_0$ for $t \geq 0$

Material Stress Response:



Material Functions:

Elongational Viscosity Function: $\eta_e^*(t, \dot{\epsilon}_0) \equiv \frac{\tau_{11}(t, \dot{\epsilon}_0) - \tau_{22}(t, \dot{\epsilon}_0)}{\dot{\epsilon}_0}$

Alternatively: $\eta_e^*(t, \dot{\epsilon}_0)$

c) Cessation

(currently unobservable)

d) Step strain

(exists, but less often discussed)

e) SAOE

(exists, but easily converted to SAOS so is redundant)

f) Creep

(exists)

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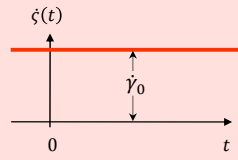
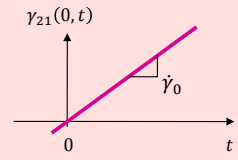
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Steady Shear Flow Material Functions

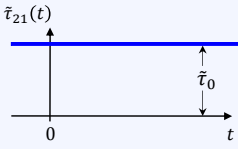
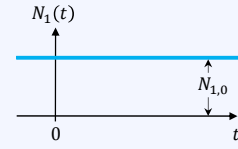
Imposed Kinematics:

$$\underline{v} \equiv \begin{pmatrix} \dot{\zeta}(t)x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$$

$$\dot{\zeta}(t) = \dot{\gamma}_0 = \text{constant}$$

Material Stress Response:

Material Functions:

Viscosity $\eta(\dot{\gamma}_0) \equiv \frac{\tilde{\tau}_{21}}{\dot{\gamma}_0} = \frac{-\tau_{21}}{\dot{\gamma}_0}$	First normal-stress coefficient $\Psi_1(\dot{\gamma}_0) \equiv \frac{\tilde{\tau}_{11} - \tilde{\tau}_{22}}{\dot{\gamma}_0^2} = \frac{-(\tau_{11} - \tau_{22})}{\dot{\gamma}_0^2}$	
	Second normal-stress coefficient $\Psi_2(\dot{\gamma}_0) \equiv \frac{\tilde{\tau}_{22} - \tilde{\tau}_{33}}{\dot{\gamma}_0^2} = \frac{-(\tau_{22} - \tau_{33})}{\dot{\gamma}_0^2}$	

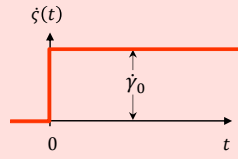
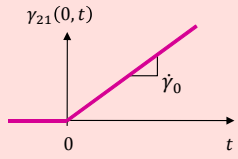
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Start-up of Steady Shear Flow Material Functions

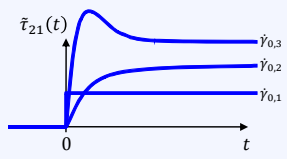
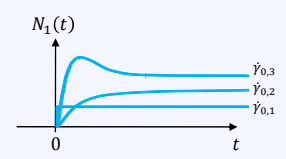
Imposed Kinematics:

$$\underline{v} \equiv \begin{pmatrix} \dot{\zeta}(t)x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$$

$$\dot{\zeta}(t) = \begin{cases} 0 & t < 0 \\ \dot{\gamma}_0 & t \geq 0 \end{cases}$$

Material Stress Response:

Material Functions:

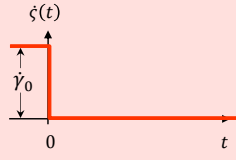
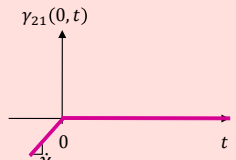
Shear stress growth function $\eta^+(t, \dot{\gamma}_0) \equiv \frac{\tilde{\tau}_{21}(t)}{\dot{\gamma}_0} = \frac{-\tau_{21}(t)}{\dot{\gamma}_0}$	First normal-stress growth coefficient $\Psi_1^+(t, \dot{\gamma}_0) \equiv \frac{\tilde{\tau}_{11} - \tilde{\tau}_{22}}{\dot{\gamma}_0^2}$	
	Second normal-stress growth coefficient $\Psi_2^+(t, \dot{\gamma}_0) \equiv \frac{\tilde{\tau}_{22} - \tilde{\tau}_{33}}{\dot{\gamma}_0^2}$	

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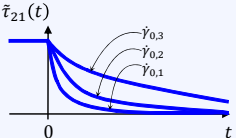
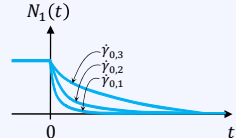
Cessation of Steady Shear Flow Material Functions

Imposed Kinematics:

$$\underline{v} \equiv \begin{pmatrix} \dot{\zeta}(t)x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$$

$$\dot{\zeta}(t) = \begin{cases} \dot{\gamma}_0 & t < 0 \\ 0 & t \geq 0 \end{cases}$$



Material Stress Response:

Material Functions:

Shear stress decay function $\eta^-(t, \dot{\gamma}_0) \equiv \frac{\bar{\tau}_{21}(t)}{\dot{\gamma}_0} = \frac{-\tau_{21}(t)}{\dot{\gamma}_0}$

First normal-stress decay coefficient $\Psi_1^-(t, \dot{\gamma}_0) \equiv \frac{\bar{\tau}_{11} - \bar{\tau}_{22}}{\dot{\gamma}_0^2}$

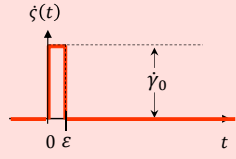
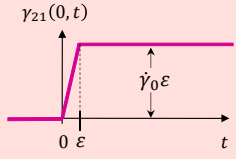
Second normal-stress decay coefficient $\Psi_2^-(t, \dot{\gamma}_0) \equiv \frac{\bar{\tau}_{22} - \bar{\tau}_{33}}{\dot{\gamma}_0^2}$

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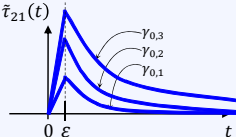
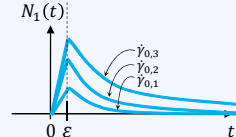
Step Strain Shear Flow Material Functions

Imposed Kinematics:

$$\underline{v} \equiv \begin{pmatrix} \dot{\zeta}(t)x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$$

$$\dot{\zeta}(t) = \begin{cases} 0 & t < 0 \\ \dot{\gamma}_0 & 0 \leq t < \varepsilon \\ 0 & t \geq \varepsilon \end{cases}$$



Material Stress Response:

Material Functions:

Relaxation modulus $G(t, \gamma_0) \equiv \frac{\bar{\tau}_{21}(t, \gamma_0)}{\gamma_0} = \frac{-\tau_{21}(t, \gamma_0)}{\gamma_0}$

First normal-stress relaxation modulus $G_{\Psi_1}(t, \gamma_0) \equiv \frac{\bar{\tau}_{11} - \bar{\tau}_{22}}{\gamma_0^2}$

Second normal-stress relaxation modulus $G_{\Psi_2}(t, \gamma_0) \equiv \frac{\bar{\tau}_{22} - \bar{\tau}_{33}}{\gamma_0^2}$

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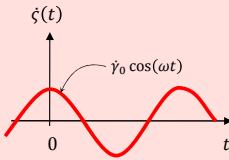
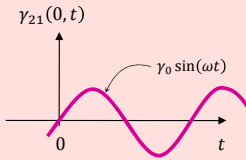
Small-Amplitude Oscillatory Shear Material Functions

Imposed Kinematics:

$$\underline{v} \equiv \begin{pmatrix} \dot{\zeta}(t)x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$$

$$\dot{\zeta}(t) = \dot{\gamma}_0 \cos(\omega t)$$

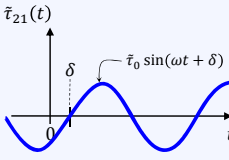
$$\gamma_0 = \frac{\dot{\gamma}_0}{\omega}$$

Material Stress Response:

$$\bar{\tau}_{21}(t) = \bar{\tau}_0 \sin(\omega t + \delta)$$

δ = phase difference between stress and strain waves



$N_1(t) = N_2(t) = 0$
(linear viscoelastic regime)

Material Functions:

SAOS stress $\frac{\bar{\tau}_{21}(t, \gamma_0)}{\gamma_0} = \frac{-\tau_{21}(t, \gamma_0)}{\gamma_0} = \bar{\tau}_0 \sin(\omega t + \delta) = G' \sin(\omega t) + G'' \cos(\omega t)$

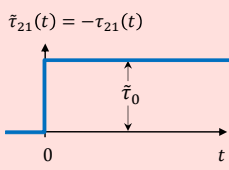
Storage modulus $G'(\omega) \equiv \frac{\bar{\tau}_0}{\gamma_0} \cos(\delta)$ Loss modulus $G''(\omega) \equiv \frac{\bar{\tau}_0}{\gamma_0} \sin(\delta)$

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Creep Shear Flow Material Functions

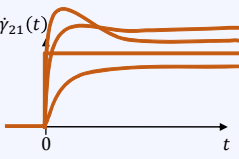
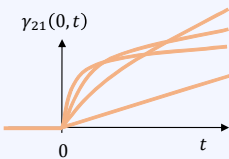
Imposed Stress

$$\underline{v} \equiv \begin{pmatrix} \dot{\gamma}_{21}(t)x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$$

$$\bar{\tau}_{21}(t) = -\tau_{21}(t) = \begin{cases} 0 & t < 0 \\ \bar{\tau}_0 & 0 \leq t < t_2 \\ 0 & t \geq t_2 \end{cases}$$


Material Kinematic Response:

For *creep*, the material response is the form of deformation (stress is specified).

Material Function:

Shear creep compliance $J(t, \bar{\tau}_0) \equiv \frac{\gamma_{21}(0, t; \bar{\tau}_0)}{\bar{\tau}_0}$

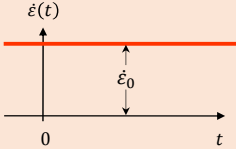
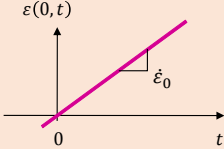
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Steady Elongational Flow Material Functions

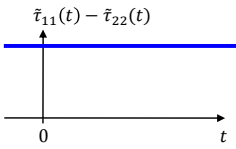
Imposed Kinematics:

$$\underline{v} \equiv \begin{pmatrix} -\frac{1}{2}\dot{\epsilon}(t)x_1 \\ -\frac{1}{2}\dot{\epsilon}(t)x_2 \\ \dot{\epsilon}(t)x_3 \end{pmatrix}_{123}$$

$$\dot{\epsilon}(t) = \dot{\epsilon}_0 = \text{constant}$$

Material Stress Response:



Material Functions:

Elongational Viscosity $\eta_e(\dot{\epsilon}_0) \equiv \frac{\bar{\tau}_{33} - \bar{\tau}_{11}}{\dot{\epsilon}_0} = \frac{-(\tau_{33} - \tau_{11})}{\dot{\epsilon}_0}$

Alternatively, $\bar{\eta}(\dot{\epsilon}_0)$

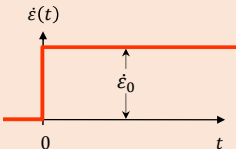
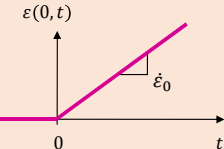
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Start-up of Steady Elongation Material Functions

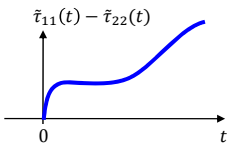
Imposed Kinematics:

$$\underline{v} \equiv \begin{pmatrix} -\frac{1}{2}\dot{\epsilon}(t)x_1 \\ -\frac{1}{2}\dot{\epsilon}(t)x_2 \\ \dot{\epsilon}(t)x_3 \end{pmatrix}_{123}$$

$$\dot{\epsilon}(t) = \begin{cases} 0 & t < 0 \\ \dot{\epsilon}_0 & t \geq 0 \end{cases}$$

Material Stress Response:




Material Functions:

Elongational Start-up Function $\eta_e^+(t, \dot{\epsilon}_0) \equiv \frac{\bar{\tau}_{33} - \bar{\tau}_{11}}{\dot{\epsilon}_0} = \frac{-(\tau_{33} - \tau_{11})}{\dot{\epsilon}_0}$

Alternatively, $\bar{\eta}^+(t, \dot{\epsilon}_0)$

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What exactly do we observe when we subject non-Newtonian fluids to deformation?



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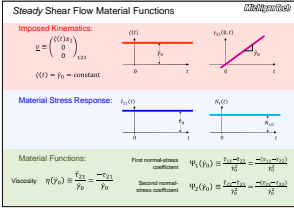
Summary

Rheological Material Functions

- Are the answer to the question *“What exactly do we observe when we subject non-Newtonian fluids to deformation?”*
- Are based on continuum view
- Provide a framework/vocabulary of comparison
- Help to categorize and organize observed material responses

Material Functions do not:

- Identify a material conclusively
- Tell us the form of $\underline{\tau}(\underline{\nu})$




Steady Shear Flow Material Functions (Morrison/2006)
Imposed Kinematics: $\underline{\Sigma} = \begin{pmatrix} \sigma_{11} & & \\ & \sigma_{22} & \\ & & \sigma_{33} \end{pmatrix}$, $\dot{\gamma} = \begin{pmatrix} \dot{\gamma}_1 & & \\ & \dot{\gamma}_2 & \\ & & \dot{\gamma}_3 \end{pmatrix}$, $\dot{\gamma} = \dot{\gamma}_0 = \text{constant}$
Material Stress Response: $\tau_{11}(\dot{\gamma}_1)$, $\tau_{22}(\dot{\gamma}_2)$, $\tau_{33}(\dot{\gamma}_3)$
Material Functions: First normal stress coefficient $\Psi_1(\dot{\gamma}_0) = \frac{\tau_{11}(\dot{\gamma}_0) - \tau_{22}(\dot{\gamma}_0)}{2\dot{\gamma}_0^2}$, $\Psi_2(\dot{\gamma}_0) = \frac{\tau_{22}(\dot{\gamma}_0) - \tau_{33}(\dot{\gamma}_0)}{2\dot{\gamma}_0^2}$
Viscosity: $\eta(\dot{\gamma}_0) = \frac{\tau_{11}(\dot{\gamma}_0) - \tau_{22}(\dot{\gamma}_0)}{2\dot{\gamma}_0}$, $\eta(\dot{\gamma}_0) = \frac{\tau_{22}(\dot{\gamma}_0) - \tau_{33}(\dot{\gamma}_0)}{2\dot{\gamma}_0}$

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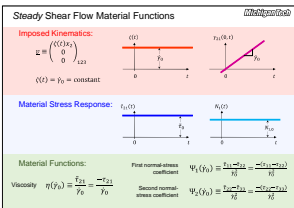
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Cannot conclusively identify, but can classify and can be used to assess proposed models

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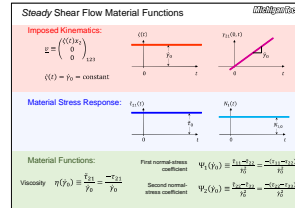
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Summary

Rheological Material Functions

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Material Functions do not:

- Identify a material conclusively
- Tell us the form of $\underline{\tau}(\underline{\nu})$

Cannot conclusively identify, but can be used to assess proposed models

Except for Newtonian fluids, the structure of the model for stress is not known $\underline{\tau}(\underline{\nu}) = ?$, and remains a mystery

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