Homework #1 - due Tuesday, 1/23 by 4:00 pm
bonus problems - due Thursday, 1/25 by 4:00 pm

Readings for this homework assignment and upcoming lectures

1. Read lecture notes:
   - Part 1. Introduction to Energy
   - Part 2. Energy Perspectives
   - Part 3. Growth Rate and Hubbert’s Peak

2. Watch the sequence of videos on Prof. Bartlett’s lecture on Arithmetic, Population, and Energy; the link to the videos is on the course website and on Canvas.

   [link to Prof. Bartlett’s Lectures]

Homework Submission

For this first assignment, the homework is to be worked and submitted individually either in class or by dropping off at my office (MEEM 905).

Cite your sources of information for each problem.
Homework #1 – Tuesday, 1/23 by 4:00 pm

1. How many pounds of coal is equivalent to a million BTUs? [cite your sources]

**Solution:**

\[
1 \text{ ton coal } \equiv 20 \text{ MBtu}; \text{ estimated value for U.S. coal produced from 2010 to 2017 per EIA}
\]

\[
1 \text{ MBtu} \left( \frac{1 \text{ ton}}{20 \text{ MBtu}} \right) \left( \frac{2000 \text{ lbm}}{\text{ ton}} \right) = 100 \text{ lbm}
\]

2. How many gallons of gasoline is equivalent to a million BTUs? [cite your sources]

**Solution:**

\[
1 \text{ bbl gasoline } \equiv 5.222 \text{ MBtu}; \text{ estimated value for U.S. motor gasolines produced in 2016 per EIA}
\]

\[
1 \text{ MBtu} \left( \frac{1 \text{ bbl}}{5.222 \text{ MBtu}} \right) \left( \frac{42 \text{ gal}}{\text{ bbl}} \right) = 8.04 \text{ gal}
\]

3. How many therms of natural gas is equivalent to a million BTUs? [cite your sources]

**Solution:**

\[
1 \text{ therm } = 100,000 \text{ Btu}
\]

\[
1 \text{ ft}^3 \text{ natural gas } \equiv 1025 \text{ Btu for U.S. natural gas produced in 2016 per EIA}
\]

\[
1 \text{ MBtu} \left( \frac{1 \text{ therm}}{100,000 \text{ Btu}} \right) = 10 \text{ therms}
\]
4. A power plant delivers 100 units of work at 30% thermal efficiency. How many heat units are supplied to operate the plant? How many units of heat are rejected to the surroundings?

**Solution:** For a power cycle, conservation of energy is:

\[ \sum \dot{Q} - \sum \dot{W} = \dot{m}e|_{\text{out}} - \dot{m}e|_{\text{in}} \]

The energy in the fluid returns to the original state during the cycle, so the net change in stored energy is zero. Thus, conservation of energy becomes:

\[ \sum \dot{Q} - \sum \dot{W} = 0, \]

which can be rewritten as:

\[ \dot{Q}_{\text{out}} + \dot{Q}_{\text{in}} - \dot{W}_{\text{net}} = 0 \]

Efficiency of the cycle is the net power out per unit of heat input:

\[ \eta_{th} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{out}}} \]

For 100 units of net work at 30% efficiency, the thermal power required and rejected are:

\[ \dot{Q}_{\text{in}} = \frac{\dot{W}_{\text{net}}}{\eta_{th}} = 333.33 \text{ units} \]

\[ \dot{Q}_{\text{out}} = \dot{W}_{\text{net}} - \dot{Q}_{\text{in}} = -233.33 \text{ units} \]

5. Thomas Newcomen used the fact that the specific volume of saturated liquid is much smaller than the specific volume of saturated steam at the same pressure in his “atmospheric engine.” Calculate the work done on the piston by the atmosphere if steam is condensed at an average pressure of 6 psia by cooling in a tightly fitted piston-cylinder enclosure if the piston area is 1 ft\(^2\) and the piston stroke is 1 ft. If the process takes place 10 times a minute, what is the power delivered? Discuss what can be done to increase the power of the engine. Describe the characteristics of a Newcomen engine that would theoretically deliver 20 horsepower.

**Solution:**

The power of the engine is:

\[ W = \frac{\{\text{force}\} \{\text{distance}\}}{\{\text{time}\}} = \frac{P_{\text{net}} \Delta \dot{V}}{\{\text{time}\}} = \frac{P_{\text{net}} \{\text{Area}\} \{\text{Stroke}\}}{\{\text{time}\}} \]

For the engine described:

\[ W = \frac{(8.7 \text{ psid}) \left(144 \text{ in}^2/\text{ft}^2\right) (1 \text{ ft}^3)}{(0.10 \text{ min}) (60 \text{ s/min})} = 208.8 \text{ ft-lbf/s} \times \frac{1 \text{ hp}}{550 \text{ ft-lbf}} = 0.34 \text{ hp} \]

In order to increase the power, the stroke may be longer and/or faster and the area may be increased. Lowering the internal saturation pressure will also increase the power.
6. Explain why is the magnitude of $g_c$ equal to 1 for all of the common engineering unit systems except the English Engineering system? What is another set of common units for which $g_c$ does not have a magnitude equal to 1.

**Solution:** The English Engineering unit system defines mass, $m$ (lbm), and force, $F$ (lbf), independently such that the magnitude of the two dimensions are the same. Since these two dimensions are related to one another through Newton’s second law of motion $F = ma$ and the acceleration, $a$, is not restricted to a magnitude of 1, a conversion factor is necessary, $g_c$. Hence, Newton’s second law of motion is more properly expressed as $g_c F = ma$. In the British Gravitational unit system, force is independently defined and mass is a secondary dimension defined in terms of force so that $g_c = \text{1 slug} \cdot \text{ft}/\text{lbf} \cdot \text{s}^2$. In the SI unit system, mass is independently defined and force is a secondary dimension defined in terms of mass so that again the magnitude of $g_c$ is 1. Another set of units for which $g_c$ does not have a magnitude of one are Gravitational Metric units.
7. Determine the energy and power equivalent of gasoline: [cite your sources of information]
   
   (a) Calculate the energy in 25 gallons of gasoline in terms of kJ, Btu’s, and tons of TNT using the average API gravity in your calculation. Gasoline has an average API (American Petroleum Institute) gravity of 70 °API.

   (b) If it takes 5 minutes to transfer 25 gallons of gasoline. What is the equivalent power during this transfer in kW and hp?

Solution:

File: energy_equivalency_gasoline_2.EES

"energy_equivalency_gasoline_2.ees"

"(a) The higher heating value (HHV) of gasoline is 47,590 kJ/kg [Appendix G, Culp (1991)]. The American Petroleum Institute (API) specific gravity for gasoline is 70 degrees API [Culp (1991)]."

\[
\text{tankvolume} = 21 \text{ [gallons]}
\]
\[
\text{filltime} = 6 \text{ [minutes]}
\]

"The specific gravity as defined in fluid mechanics can be found from the API value:"

\[
SG = \frac{141.5}{131.5 + 70}
\]

"Therefore, the density may be taken to be SG*1000 kg/m^3 and the mass of gasoline found is:"

\[
\rho = SG*1000
\]
\[
m = \rho*\text{volume}
\]
\[
\text{HHV} = 47590 \text{ [kJ/kg]} \quad \text{"Culp (1991)"}
\]

"total energy in this volume"

\[
E = \text{HHV}^\text{m}
\]
\[
E_{\text{BTU}} = E^*\text{convert(kJ,Btu)}
\]
\[
c = 2.381\times10^{-10} \quad \text{"ton TNT/J"}
\]
\[
E_{\text{tnt}} = E^*c*1000
\]

"equivalent power in energy transfer"

\[
E_{\dot{}} = E/(\text{filltime}\times60)
\]
\[
E_{\dot{}}_{\text{hp}} = E_{\dot{}}^*\text{convert(kW,HP)}
\]

"eof"

\[
\text{SOLUTION}
\]

Unit Settings: SI C kPa kJ mass deg
\[
c = 2.381E-10 \text{ [ton TNT/J]}
\]
\[
E_{\text{BTU}} = 2.518E+06 \text{ [Btu]}
\]
\[
E_{\text{tnt}} = 9896 \text{ [hp]}
\]
\[
\text{filltime} = 6 \text{ [minutes]}
\]
\[
m = 55.82 \text{ [kg]}
\]
\[
SG = 0.7022 \text{ [-]}
\]
\[
\text{volume} = 0.07949 \text{ [m^3]}
\]
\[
\text{HHV} = 47590 \text{ [kJ/kg]}
\]
\[
\rho = 702.2 \text{ [kg/m^3]}
\]
\[
\text{tankvolume} = 21 \text{ [gallons]}
\]
8. A 2400-MW\textsubscript{e} power plant has the following power demand for a given day:

<table>
<thead>
<tr>
<th>Time</th>
<th>Power Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>12-5 a.m.</td>
<td>850 MW</td>
</tr>
<tr>
<td>9-12 a.m.</td>
<td>2150 MW</td>
</tr>
<tr>
<td>5-6 p.m.</td>
<td>2430 MW</td>
</tr>
<tr>
<td>5-7 a.m.</td>
<td>1250 MW</td>
</tr>
<tr>
<td>1-4 p.m.</td>
<td>2500 MW</td>
</tr>
<tr>
<td>8-10 p.m.</td>
<td>1500 MW</td>
</tr>
<tr>
<td>8-9 a.m.</td>
<td>1960 MW</td>
</tr>
<tr>
<td>4-5 p.m.</td>
<td>2450 MW</td>
</tr>
<tr>
<td>6-8 p.m.</td>
<td>1850 MW</td>
</tr>
<tr>
<td>7-8 a.m.</td>
<td>1840 MW</td>
</tr>
<tr>
<td>1-4 p.m.</td>
<td>2500 MW</td>
</tr>
<tr>
<td>8-10 p.m.</td>
<td>1150 MW</td>
</tr>
</tbody>
</table>

Find the total power output in:
(a) MW\textsubscript{e}·days/day, (b) kW\textsubscript{e}·h/day, (c) MeV\textsubscript{e}/day, (d) J\textsubscript{e}/day, and (e) Btu\textsubscript{e}/day.

Assume that the plant burns coal with a heating value (energy content) of 26,400 kJ/kg with an overall efficiency of 32%. Determine (f) the total mass of coal, in short tons consumed during the day’s operation, and (g) the maximum design coal rate, in short tons per hour, required for proper operation of the unit. Also evaluate (h) the heat rate of the unit in Btu/kWh, (i) the capacity factor of the unit for one day’s operation, and (j) the load factor for the same period of operation.

**Solution:** The solution shown is for slightly different values for power demand. The electric power output for this day is the power multiplied by the fraction of time per day for each average power level:

\[ P_e = \left( \frac{5 \text{ hr}}{\text{day}} \right) (850 \text{ MW}) + \left( \frac{2 \text{ hr}}{\text{day}} \right) (1250 \text{ MW}) + \ldots + \left( \frac{1 \text{ hr}}{\text{day}} \right) (23650 \text{ MW}) + \ldots \]

\[ P_e = 39,690 \text{ MW}_e \cdot \text{hr/day} \]

Find the total power output in:
(a) MW\textsubscript{e}·days/day
\[ P_e = \left( \frac{39,690 \text{ MW}_e \cdot \text{hr}}{\text{day}} \right) \left( \frac{1 \text{ day}}{24 \text{ hr}} \right) = 1653.75 \text{ MW}_e \cdot \text{day/day} \]

(b) kW\textsubscript{e}·h/day
\[ P_e = \left( \frac{39,690 \text{ MW}_e \cdot \text{hr}}{\text{day}} \right) \left( \frac{1000 \text{ kW}}{\text{MW}} \right) = 39.69 \cdot 10^6 \text{ kW}_e \cdot \text{hr/day} \]

(c) MeV\textsubscript{e}/day
\[ P_e = \left( \frac{39,690 \text{ MW}_e \cdot \text{hr}}{\text{day}} \right) \left( \frac{3600 \text{ s}}{\text{hr}} \right) \left( \frac{10^6 J/s}{\text{MW}} \right) \left( \frac{6.242 \cdot 10^{12} \text{ MeV}}{J} \right) = 891.88 \cdot 10^{24} \text{ MeV}_e \cdot \text{day/day} \]

(d) J\textsubscript{e}/day
\[ P_e = \left( \frac{39,690 \text{ MW}_e \cdot \text{hr}}{\text{day}} \right) \left( \frac{3600 \text{ s}}{\text{hr}} \right) \left( \frac{10^6 J/s}{\text{MW}} \right) = 142.88 \cdot 10^{12} \text{ J}_e/\text{day} \]

(e) Btu\textsubscript{e}/day
\[ P_e = \left( \frac{39,690 \text{ MW}_e \cdot \text{hr}}{\text{day}} \right) \left( \frac{10^6 J/s}{\text{MW}} \right) \left( \frac{3.412 \text{ Btu/hr}}{\text{W}} \right) = 135.42 \cdot 10^9 \text{ Btu}_e/\text{day} \]

(f) the total mass of coal, in short tons consumed during the day’s operation:
Assume that the plant burns coal with a heating value (energy content) of 26,400 kJ/kg with an overall efficiency of 38%. The energy required to produce 39,690 MW\textsubscript{e}·hr/day is:

\[ \left( \frac{39,690 \text{ MW}_e \cdot \text{hr}}{\text{day}} \right) \left( \frac{1 \text{ MW}_{th}}{0.38 \text{ MW}_e} \right) \left( \frac{1000 \text{ kW}}{\text{MW}} \right) \left( \frac{3600 \text{ kJ}}{\text{kWh}} \right) \left( \frac{\text{kg}}{26,400 \text{kJ}_{th}} \right) \left( \frac{0.001102 \text{ short tons}}{\text{kg}} \right) \]

\[ \text{Daily Coal Rate} = 15,696 \text{ tons of coal/day} \]
(g) the maximum design coal rate is based on rated power:

\[
\text{Rated Power} = \left(2400 \cdot 10^6 \text{ W}_e\right) \left(\frac{1 \text{ MW}}{0.38 \text{ MW}_e}\right) \left(\frac{1 \text{ J/s}}{\text{ W}}\right) \left(\frac{\text{ kg}}{26,400 \cdot 10^3 \text{ J}}\right) \left(\frac{3600 \text{ s}}{\text{hr}}\right)
\]

\[
= (0.861 \cdot 10^6 \text{ kg/hr})(0.001102 \text{ ton/kg})
\]

= 949 tons of coal per hour

(h) the Heat Rate is the number of thermal Btu’s required to produce 1 kW·hr of electricity:

\[
\text{Heat Rate} = \frac{3412}{\eta_{th}} \left[\frac{\text{Btu}_t}{\text{ kW}_e \cdot \text{hr}}\right]
\]

\[
\text{HR} = \left(\frac{3412 \text{ Btu}_t}{\text{ kW}_e \cdot \text{hr}}\right) \left(\frac{1}{0.38 \text{ Btu}_t/Btu_\text{e}}\right) = 8979 \frac{\text{ Btu}_t}{\text{ kW}_e \cdot \text{hr}}
\]

(i) the capacity factor is the fraction of average power to rated power:

\[
\text{Capacity Factor} = \frac{P_{e,\text{avg}}}{P_{e,\text{rated}}} = \left(\frac{1653.75 \text{ MW}_e/\text{day}}{2400 \text{ MW}_e/\text{day}}\right) = 0.6891
\]

(j) the load factor is the fraction of average power to maximum power; for this day, the maximum power was 2650 MW\text{e}:

\[
\text{Load Factor} = \frac{P_{e,\text{avg}}}{P_{e,\text{max}}} = \left(\frac{1653.75 \text{ MW}_e/\text{day}}{2350 \text{ MW}_e/\text{day}}\right) = 0.7037
\]
For the specific values in the problem statement:

\[ \text{avgpower} = 1684 \ \text{[MW]} \]
\[ \text{CF} = 0.7017 \ \text{[\ - \]} \]
\[ \text{coalrate} = 18981 \ \text{[tons/day]} \]
\[ \eta_{\text{th}} = 0.32 \]
\[ \text{HeatRate} = 10663 \ \text{[Btu/hr/kW]} \]
\[ \text{HV} = 26400 \ \text{[kJ/kg]} \]
\[ \text{LF} = 0.6737 \ \text{[\ - \]} \]
\[ \text{maxpower} = 2500 \ \text{[MW]} \]
\[ \text{Pe}_1 = 4250 \ \text{[MW yr]} \]
\[ \text{Pe}_{10} = 3700 \ \text{[MW yr]} \]
\[ \text{Pe}_{11} = 3000 \ \text{[MW yr]} \]
\[ \text{Pe}_{12} = 2300 \ \text{[MW yr]} \]
\[ \text{Pe}_3 = 1840 \ \text{[MW yr]} \]
\[ \text{Pe}_5 = 1960 \ \text{[MW yr]} \]
\[ \text{Pe}_6 = 6450 \ \text{[MW yr]} \]
\[ \text{Pe}_7 = 2040 \ \text{[MW yr]} \]
\[ \text{Pe}_8 = 7500 \ \text{[MW yr]} \]
\[ \text{Pe}_9 = 2450 \ \text{[MW yr]} \]
\[ \text{Pe}_{10} = 2430 \ \text{[MW yr]} \]
\[ \text{power} = 40420 \ \text{[MW yr]} \]
\[ \text{powerA} = 1684 \ \text{[MW yr]} \]
\[ \text{powerB} = 4.042E+07 \ \text{[kW yr]} \]
\[ \text{powerC} = 9.083E+26 \ \text{[MeV yr]} \]
\[ \text{powerD} = 1.455E+14 \ \text{[J yr]} \]
\[ \text{powerE} = 1.379E+11 \ \text{[Btu yr]} \]
\[ \text{ratedpower} = 2400 \ \text{[MW]} \]

27 potential unit problems were detected.

**KEY VARIABLES**

\[ \text{power} = 40420 \ \text{[MW yr]} \] \hspace{1cm} \text{total power output}  
\[ \text{powerA} = 1684 \ \text{[MW yr]} \] \hspace{1cm} (a) MW yr/day  
\[ \text{powerB} = 4.042E+07 \ \text{[kW yr]} \] \hspace{1cm} (b) kWe yr/day  
\[ \text{powerC} = 9.083E+26 \ \text{[MeV yr]} \] \hspace{1cm} (c) MeV yr/day  
\[ \text{powerD} = 1.455E+14 \ \text{[J yr]} \] \hspace{1cm} (d) J yr/day  
\[ \text{powerE} = 1.379E+11 \ \text{[Btu yr]} \] \hspace{1cm} (e) Btu yr/day  
\[ \text{coalrate} = 18981 \ \text{[tons/day]} \] \hspace{1cm} (f) short tons of coal per day  
\[ \text{designcoalrate} = 1127 \ \text{[tons/hr]} \] \hspace{1cm} (g) tons/hr  
\[ \text{HeatRate} = 10663 \ \text{[Btu yr/kW yr]} \] \hspace{1cm} (h) heat rate  
\[ \text{avgpower} = 1684 \ \text{[MW]} \] \hspace{1cm} average power production  
\[ \text{maxpower} = 2500 \ \text{[MW]} \] \hspace{1cm} maximum power production  
\[ \text{CF} = 0.7017 \ \text{[\ - \]} \] \hspace{1cm} (i) capacity factor  
\[ \text{LF} = 0.6737 \ \text{[\ - \]} \] \hspace{1cm} load factor
9. Determine the area of solar cells required to drive a commuter electric car if the overall conversion efficiency of the propulsion system, including the electromagnetic-electric-mechanical conversion is 13 percent. Assume that the car requires 24 hp and that the average gross solar input is 650 W/m². If the system can store energy while sitting in the parking lot and is storing energy at a rate of 4 hours for each hour of operation, find the required area of the solar-cell array. Assume the storage efficiency of the batteries is 60 percent.

**Solution:**

The effective solar energy flux with energy storage on a per hour basis is:

\[
\frac{650 \text{ W}_\text{em}}{\text{m}^2} \left(1 \text{ h} + \frac{(0.60 \text{ W}_{\text{e, out}}/\text{W}_{\text{e, in}}) \times 4 \text{ h}}{1 \text{ h}}\right) = 2210 \frac{\text{W}_\text{em}}{\text{m}^2}
\]

The area of the solar panels required to power a 24 hp motor with a system efficiency of 13% is:

\[
(24 \text{ hp}_\text{m}) \left(\frac{\text{hp}_\text{em}}{0.13 \text{ hp}_\text{m}}\right) \left(\frac{\text{m}^2}{2210 \text{ W}_\text{em}}\right) \left(\frac{\text{W}_\text{m}}{0.001244 \text{ hp}_\text{em}}\right) = 67.15 \text{ m}^2
\]

**Homework #1 – 5290 only**

10. Two units of work are required to transfer 10 units of heat from a refrigerator to the environment. What is the COP of the refrigerator? Suppose that the same amount of heat transfer instead is by a heat pump into a house. What is the heat pump COP?

**Solution:**

From the first law: \( W_{\text{in}} = Q_{\text{out}} - Q_{\text{in}} \), therefore \( Q_{\text{in}} = 8 \text{ units} \)

- for refrigeration: \( \text{COP} = \frac{Q_{\text{out}}}{W_{\text{in}}} = \frac{10}{2} = 5 \)
- for heat pump: \( \text{COP} = \frac{Q_{\text{in}}}{W_{\text{in}}} = \frac{8}{2} = 4 \)

11. A 2007 Associated Press article states that Ann Arbor will replace street lights with Light-Emitting Diodes (LED’s). The article states:

(a) LED technology uses half the energy of traditional bulbs,

(b) the change could save Ann Arbor $100,000 per year,

(c) lighting consumes 22% of the electricity in the United States,

(d) Ann Arbor’s lighting conversion will reduce the city’s production of carbon dioxide and gases that contribute to global warming in an amount equal to taking 400 cars of the road.

Making reasonable engineering assumptions, evaluate if these are legitimate claims or a gross exaggerations. [cite your sources of information]
10. The world’s nuclear arsenal has been estimated to be 13,000 megatons of TNT. Determine the minutes of equivalent sunshine that would yield the same amount of energy to the earth. Also evaluate the length of time this stockpile could supply one hundred 1000-MWe power reactors with a thermal efficiency of 34 percent and a capacity factor of 70 percent.

**Solution:**

"energy of the nuclear arsenal"

\[ E_{\text{nuclear}} = (13000*10^6)/c_1 \]

"from Culp 2nd ed, Appendix B"

\[ c_1 = 1/(2.38*10^{-10}) \text{ [J/ton TNT]} \]

"solar energy impinging on earth; using projected area"

\[ D_{\text{earth}} = 12756000 \text{ [m]} \]

\[ A_{\text{proj}} = \pi D_{\text{earth}}^2/4 \]

\[ EMflux = 1350 \text{ [W/m}^2]\]

\[ c_2 = 3600 \text{ [J/W hr]} \]

\[ c_3 = 8766 \text{ [hr/y]} \]

\[ W_{\text{dot em}} = A_{\text{proj}} * EMflux * c_2 * c_3 \]

"equivalent time"

\[ W_{\text{dot em}} = E_{\text{nuclear}}/t_y \]

"much less than a year, try hours"

\[ t_h = t_y * c_3 \]

"still very small, try minutes"

\[ t_m = t_h * 60 \]

"way too large, try hours"

\[ t_h = t_y * c_3 \]

"still very small, try minutes"

\[ t_m = t_h * 60 \]

"too large, try days"

\[ t_d = t_y / 365.25 \]

"way too large, try hours"

\[ t_h = t_d * 24 \]

"still very small, try minutes"

\[ t_m = t_h * 60 \]

"too large, try years"

\[ t_y = t_d / 365.25 \]

"eot"
13. Use Equation (1.8) to derive a relation for the entropy change as a function of temperature for an isobaric process in a calorically perfect gas.

**Solution:** equation 1.8: \( T \, ds = dh - v \, dP \)

For an ideal gas, \( dh = c_p \, dT \) and \( v = RT/P \), therefore

\[
ds = c_p \frac{dT}{T} - R \frac{dP}{P}
\]

Integrating,

\[
s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}
\]

Since \( P_2 = P_1 \)

\[
s_2 - s_1 = c_p \ln \frac{T_2}{T_1}
\]