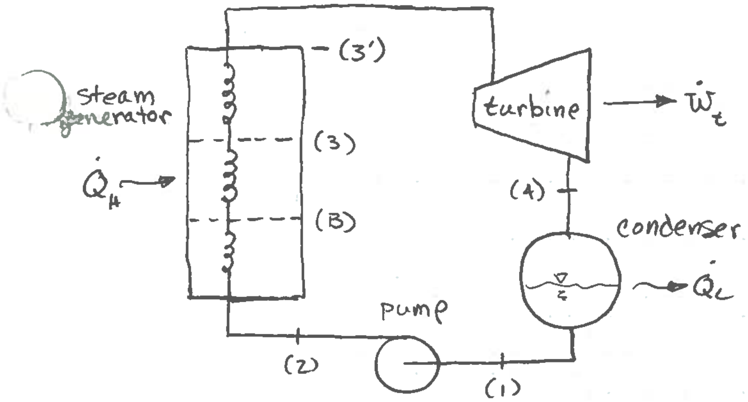
Ideal Rankine Cycle (all processes internally reversible)



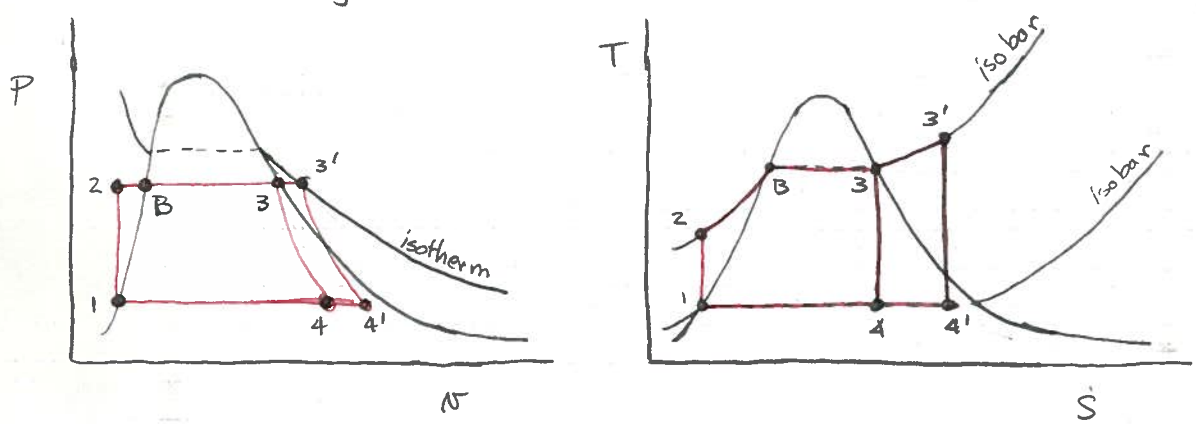
- heat exchangers used for heat addition & rejection (exception is steam locomotives)
- external heating/combustion
  - generally less polluting than internal
  - can use lower grade fuels
  - can use solar, nuclear & geothermal thermal energy sources
- common working fluids:
  - water (most common)
  - potassium
  - sodium
  - rubidium
  - ammonia
  - chain & aromatic hydrocarbons
  - fluorocarbons

1-2: Isentropic Compression (Pump)

2-3: Isobaric Heat Addition (Steam Generator)

3-4: Isentropic Expansion (Turbine)

4-1: Isobaric Heat Rejection (Condenser)



Saturated Rankine Cycle: 1-2-B-3-4-1

Superheated Rankine Cycle: 1-2-B-3-3'-4'-1

Steam Generator:

- State (B) is saturated liquid

  2-B Economizer → brings subcooled liquid up to saturation temperature

  B-3 Boiler/Evaporator → saturated liquid to saturated vapor

  3-3' Superheater → saturated vapor to superheated vapor

Analyzing Rankine Cycle requires determination of enthalpies:

1st Law: $q - w = \Delta e = \left[\underbrace{(u+Pv)}_{h} + \frac{1}{2}v^2 + gz\right]\Big|_{out} - \left[(u+Pv) + \frac{1}{2}v^2 + gz\right]\Big|_{in}$

- generally can neglect kinetic & potential energy changes

$$\boxed{q - w = \Delta h}$$

Processes:

1-2 isentropic compression (pump): $-w_p = h_2 - h_1$

2-3,3' isobaric heat addition (steam generator): $q_H = h_{3,3'} - h_2$

3,3'-4,4' isentropic expansion (turbine): $-w_t = h_{4,4'} - h_{3,3'}$

4,4'-1 isobaric heat rejection (condenser): $q_L = h_1 - h_{4,4'}$ ← not present on steam locomotives

net work, $w_{net} = w_t - |w_p| = (h_{3,3'} - h_{4,4'}) - (h_2 - h_1)$

thermal efficiency, $\eta_{th} = \dfrac{w_{net}}{q_H} = \dfrac{(h_{3,3'} - h_{4,4'}) - (h_2 - h_1)}{(h_{3,3'} - h_2)}$

work ratio $\equiv \dfrac{\text{net work}}{\text{gross work}} = \dfrac{w_{net}}{w_t}$

<u>pump work (1-2)</u>

- For small units, where P_2 is not too large compared to P_1, $h_2 \approx h_3$
  $\Rightarrow$ pump work is negligible compared to turbine work

- For modern power plants, $P_2 \geq 1000$ psia (70 bar) & $P_1 \sim 1$ psia (0.07 bar)

  h_1 is the saturated liquid enthalpy at P_1 (h_{1f})

  h_2 is found from subcooled liquid tables at T_2 & P_2

  - generally, $T_2 \approx T_1$ and the latter is used in lieu of T_2 which is difficult to obtain

A good approximation for the pump work can be obtained from the change in "flow work"

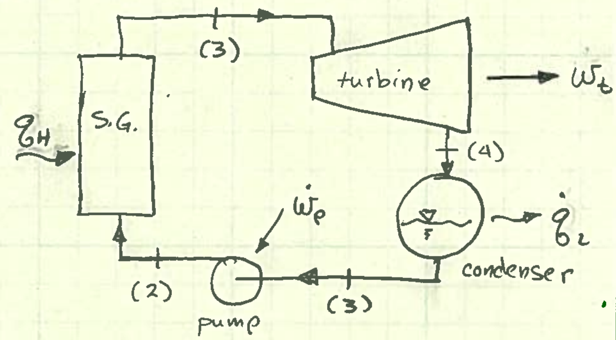
$$|w_p| = v_1(P_2 - P_1)$$

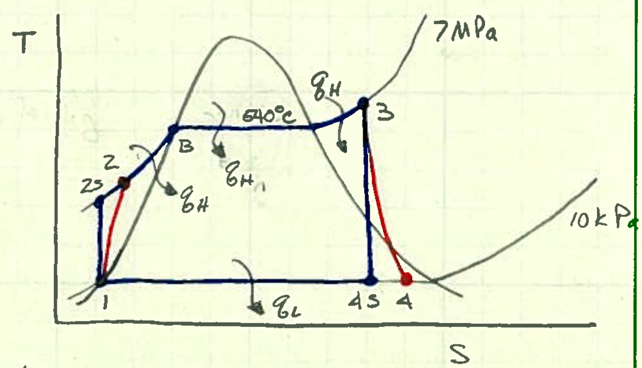
← $v_2 = v_1$ for incompressible fluid

$$h_2 - h_1 = \underbrace{(u_2 - u_1)}_{\approx 0 \text{ since } T_2 \approx T_1} + \underbrace{(P_2 v_2 - P_1 v_1)}_{v_1(P_2 - P_1)}$$

Simple Rankine Cycle

Find the thermal efficiency and the specific work of a simple Rankine cycle if the maximum temperature and pressure are 540°C and 7.0 MPa and the minimum pressure is 10 kPa. The turbine and pump efficiencies are both 85%.





State (1) - saturated liquid

$$h_1 = h_f(P_1 = 10\text{ kPa}) = 191.53\text{ kJ/kg}$$
$$v_1 = v_f(P_1 = 10\text{ kPa}) = 0.0010102\text{ m}^3/\text{kg}$$

State (2) - subcooled (compressed) liquid

$$h_2 = h_1 + |w_p|$$
$$|w_p| \approx v_1(P_2 - P_1)\cdot\left(\frac{1}{0.85}\frac{W_e}{W_m}\right)$$
$$h_2 = 191.53\text{ kJ/kg} + 8.31\text{ kJ/kg} = 199.84\text{ kJ/kg}$$

State (3) - superheat vapor

$$h_3 = h(T = 540°C, P = 7\text{ MPa}) = 3506.9\text{ kJ/kg}$$
$$s_3 = 6.9193\text{ kJ/kg K}$$

State (4) - saturated liquid & vapor $(0 < x < 1)$

$$\eta_t = \frac{h_3 - h_4}{h_3 - h_{4s}} \longrightarrow h_4 = h_{4s} + (1 - \eta_t)(h_3 - h_{4s})$$

$$s_{4s} = s_3$$

$$M_{4s} = \frac{s_{4g}(P = 10\text{ kPa}) - s_{4s}}{s_{4g}(P = 10\text{ kPa}) - s_{4f}(P = 10\text{ kPa})} = \frac{8.1502\text{ kJ/kg K} - 6.9193\text{ kJ/kg K}}{7.5009\text{ kJ/kg K}} = 0.1641 \Rightarrow x_{4s} = 1 - M_{4s}$$

$$h_{4s} = h_{4g} - x_{4s}(h_{4g} - h_{4f}) = 2584.7\text{ kJ/kg} - 0.1641(2392.8\text{ kJ/kg}) = 2192.0\text{ kJ/kg}$$

$$h_4 = 2192.0\text{ kJ/kg} + (0.15)(3506.9\text{ kJ/kg} - 2192.0\text{ kJ/kg}) = 2389.2\text{ kJ/kg}$$

- actual moisture content at turbine

$$M_4 = \frac{h_{4g}(P = 10\text{ kPa}) - h_4}{h_{4g}(P = 10\text{ kPa}) - h_{4f}(P = 10\text{ kPa})} = \frac{2584.7\text{ kJ/kg} - 2389.2\text{ kJ/kg}}{2392.8\text{ kJ/kg}} = 0.0817$$

- net specific work

$$w_{net} = \underbrace{(h_3 - h_4)}_{\text{turbine}} - \underbrace{(h_2 - h_1)}_{\text{pump}} = \left[3506.9\frac{\text{kJ}}{\text{kg}} - 2389.2\frac{\text{kJ}}{\text{kg}}\right] - \left[199.84\frac{\text{kJ}}{\text{kg}} - 191.8\frac{\text{kJ}}{\text{kg}}\right]$$

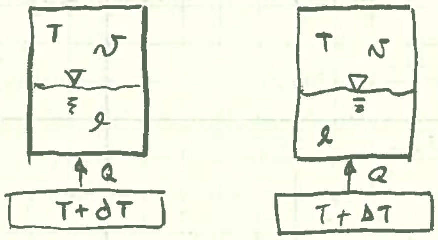
$$\boxed{w_{net} = 1109.7\text{ kJ/kg}}$$

- thermal efficiency

$$\eta_{th} = 1 - \frac{q_L}{q_H} = 1 - \frac{|(h_1 - h_4)|}{(h_3 - h_2)} = 1 - \left[\frac{\left|191.8\frac{\text{kJ}}{\text{kg}} - 2389.2\frac{\text{kJ}}{\text{kg}}\right|}{3506.9\text{ kJ/kg} - 199.84\text{ kJ/kg}}\right]$$

$$\boxed{\eta_{th} = 33.55\%}$$

Rankine Cycle - External Irreversibilities



evaporation/condensation is a reversible process

external irreversibilities primarily due to finite temperature difference, ΔT

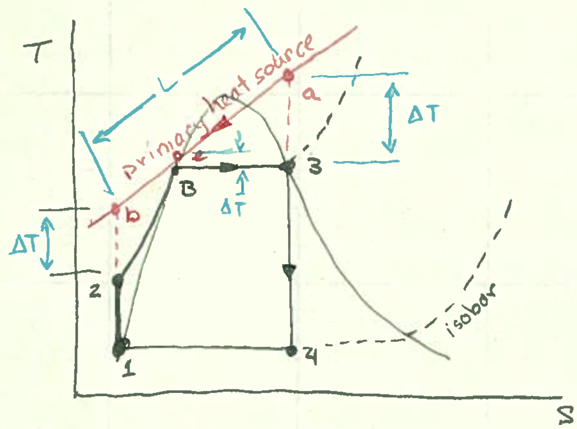
reversible heat transfer, external & internal

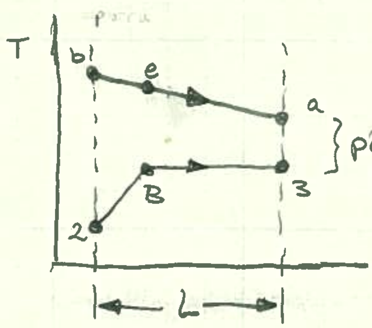
internally reversible heat transfer, but not externally

For Steam Generator, external irreversibilities primarily due to ΔT between $\underbrace{\text{primary heat sources}}$ & $\underbrace{\text{working fluid}}$.

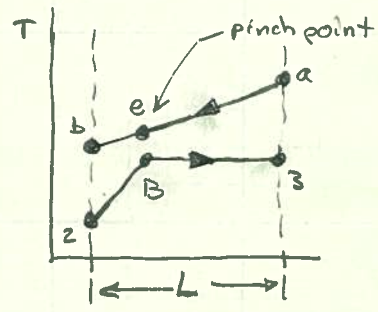
- combustion gases
- primary coolant from nuclear reactor
- solar concentrator

- steam
- ammonia
- refrigerants





parallel flow (co-flow) steam generator



counter flow steam generator

- The net ΔT is greater for parallel flow than for counterflow.
- Heat transfer process is also more efficient with counterflow.

— Use Counter Flow —

Pinch-Point Temperature Difference, $T_e - T_B$:

- small pinch-point → • lower overall temperature difference between steam lines & primary heat source lines; results in lower external irreversibilities
  • requires large, costly steam generators

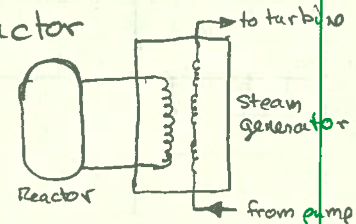
- large pinch-point → • small, inexpensive steam generators
  • higher overall temperature difference & external irreversibilities; reduction in plant efficiency

- most economical pinch-point ΔT is obtained by considering both capital costs and recurring costs.
  - capital costs ← price of steam generator
  - recurring costs ← operating & fuel costs

Types of Heat Source Fluids for Steam Generator:

gas → • combustion gases
• primary coolant from a gas-cooled reactor, CO_2 or He

liquid → • water; pressurized-water reactor (PWR)
• liquid metal; liquid-metal fast-breeder reactor



Heat Transfer: $\dot{Q} - \cancel{\dot{W}}^{\,0} = \underbrace{\dot{m} c_p}\,\Delta T$

← Heat Source Fluids have a variety of c_p & $\dot{m}$ required to achieve $\dot{Q}$

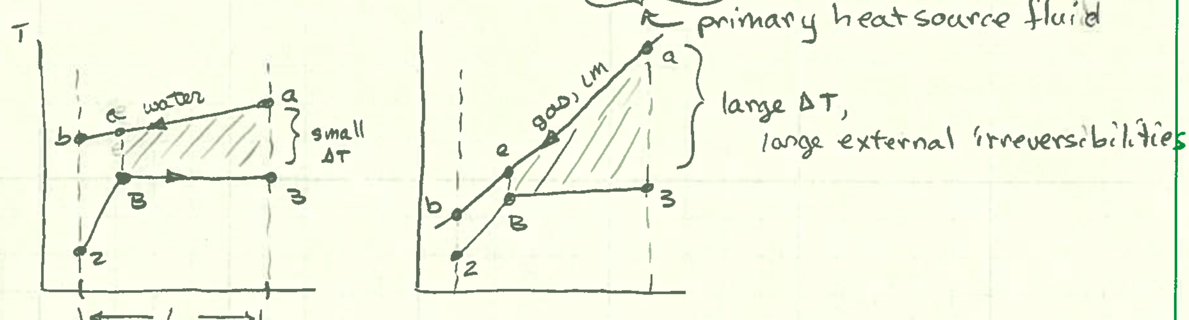
(PWR) water has a higher c_p than gas, but also has a much higher $\dot{m}$ in order to limit temperature rise in water flowing through the reactor; need to maintain uniform moderation of neutrons

$\therefore$ $\dot{m} c_p$ much greater for water than for gases

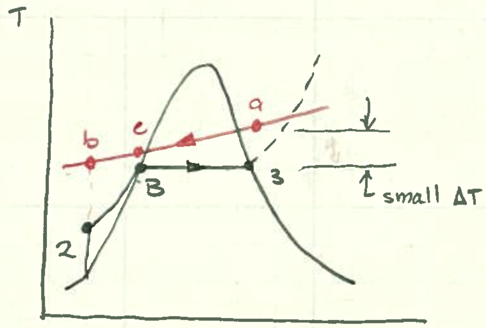
$$\frac{dQ}{dL} = \dot{m} c_p \frac{dT}{dL} \longrightarrow$$ slope of line a-b is proportional to $\frac{1}{\dot{m} c_p}$

or $\underbrace{\frac{dT}{dL} \sim \frac{1}{\dot{m} c_p}}$ for a fixed $\dot{Q}$

← primary heat source fluid

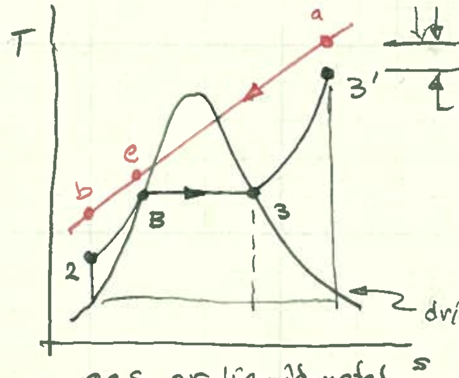


To Superheat or Not To Superheat (& Reheat)?



water as primary heat source fluid

→ PWR, BWR



ΔT reduced by using superheat; therefore, external irreversibilities also reduced

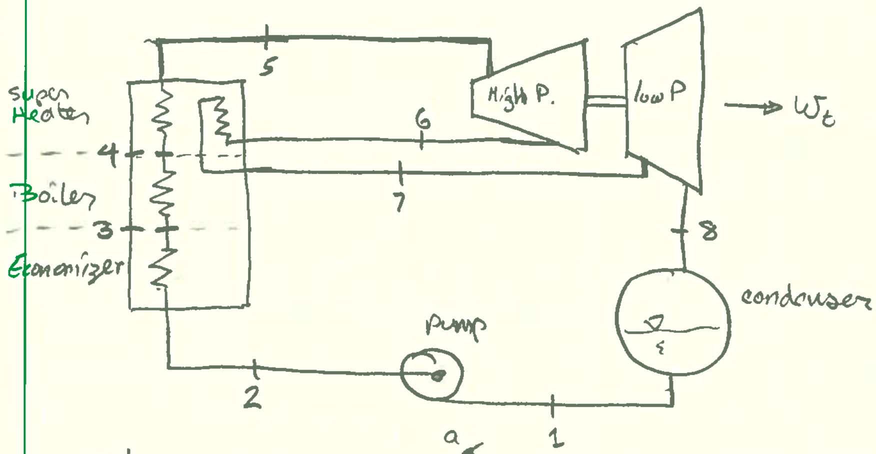
drier steam

↑ more efficient & less prone to blade damage

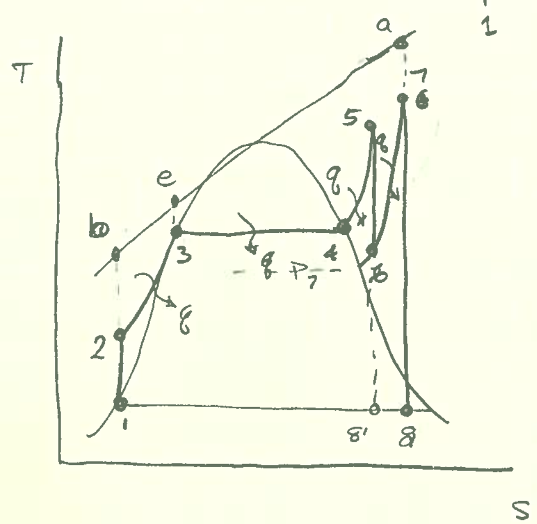
gas or liquid metal as primary heat source fluid

→ fossil fuel
gas-cooled reactors
liquid-metal-cooled reactors

REHEAT



- Improved efficiency
- drier steam at exhaust
- all modern power plants use steam heat & at least 1 stage of reheat



$$-w_t = (h_6 - h_5) + (h_8 - h_7)$$

$$-w_p = (h_2 - h_1)$$

$$w_{net} = [h_5 - h_6 + h_8 - h_7] - [h_2 - h_1]$$

$$q_h = (h_5 - h_2) + (h_7 - h_6)$$

$$\eta_{th} = \frac{w_{net}}{q_h} = \frac{h_5 - h_6 + h_8 - h_7 - h_2 + h_1}{h_5 - h_2 + h_7 - h_6}$$

Superheat Reheat Power Plant often designated as $\underbrace{P_5/T_5/T_6}_{\text{turbine inlet}}$

- The pressure at which the steam is reheated is critical. A reheat pressure ($P_6 = P_7$) too close to the initial pressure ($P_2 = P_3 = P_4 = P_5$) results in little increase to the cycle efficiency. The efficiency increase due to reheat improves as the reheat pressure ($P_6 = P_7$) is lowered; reaching a peak at a pressure ratio P_6/P_5 of 0.2 to 0.25

- Too low of a pressure ratio could result in superheated exhaust steam, unfavorable for condenser.

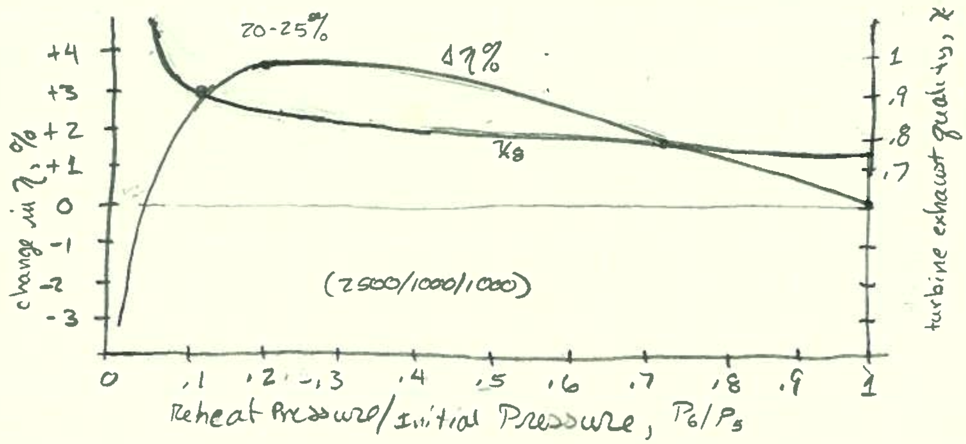
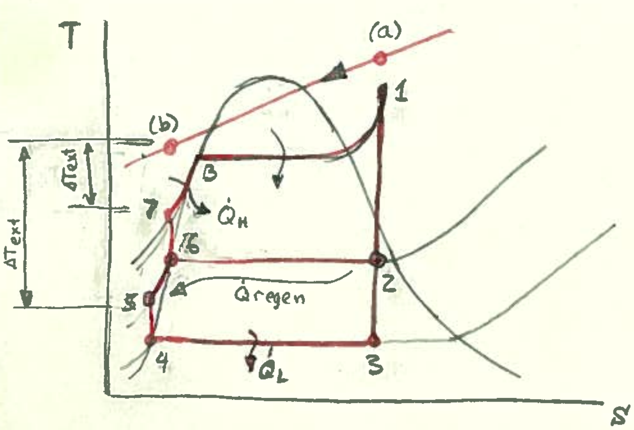
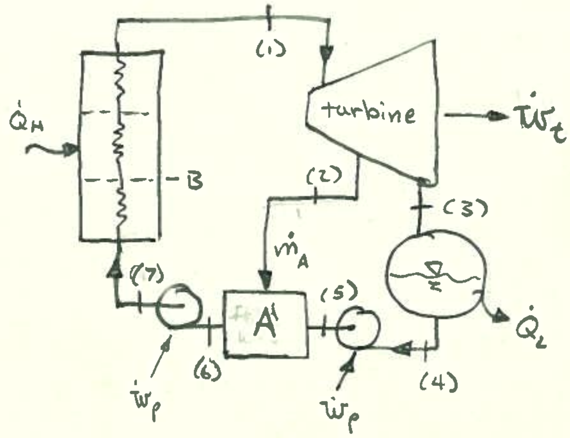


Fig. 2-9

Regeneration — Feedwater Heating

- Superheat addresses external irreversibilities in the boiler.
- Regeneration would address external irreversibilities in the economizer;
  - transfer heat from exhaust side of cycle to compressed liquid prior to heat addition process. (+)
  - For the Rankine cycle, heat rejection at the condenser is isothermal and below the temperature of the compressed liquid; can not transfer thermal energy from a cold fluid to a warm fluid. (#)
- Regeneration is therefore accomplished by "bleeding" a small amount of steam from the turbine and heating the "feedwater" (compressed liquid) prior to the feedwater entering the steam generator.

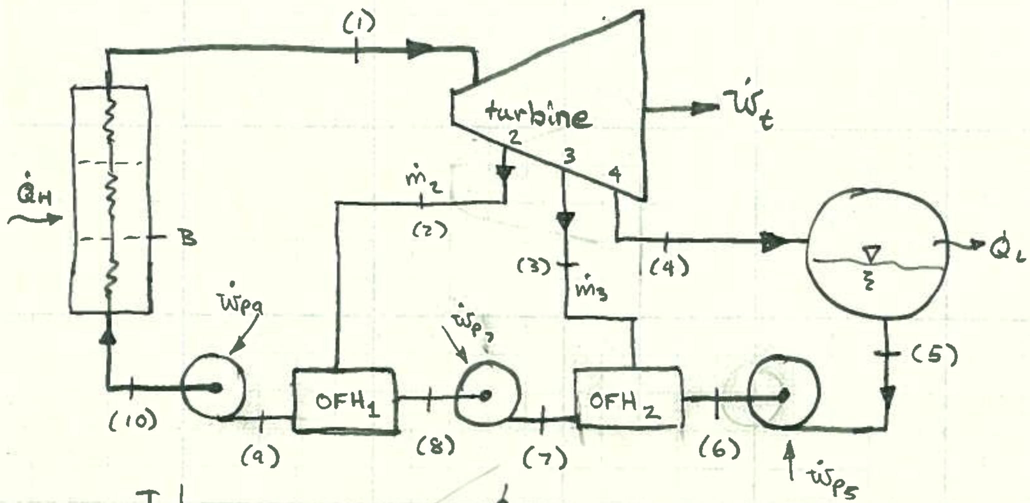


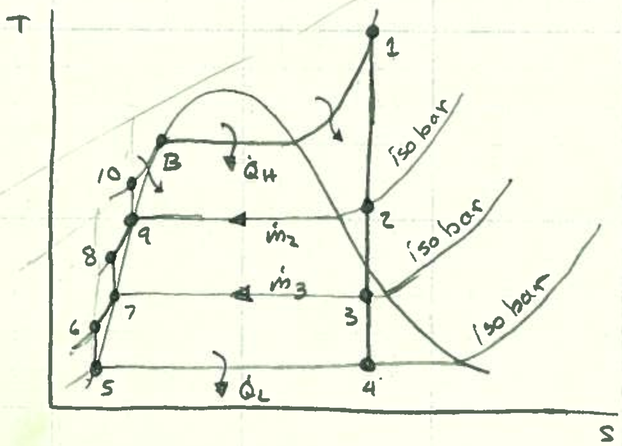


- Feedwater is the compressed liquid flowing towards the steam generator
- compressed liquid at (5) is heated with small amount of steam (2) to bring the feedwater back to saturated liquid state (6)
- heat transfer takes place in a heat exchanger → "feedwater heater"
- dates back to the 1920's, same time period that steam temperatures reached 725°F
- modern power plants use between 5 and 8 feedwater heaters
- still need an economizer (7-B), but a much smaller one than without feedwater heating; compare the temperature difference from (b) to (7) and from (b) to (5).
- three types of feedwater heaters:
  1. open (direct contact) → steam is mixed with feedwater
  2. closed with drains cascading backwards
  3. closed with drains cascading forwards

  } steam is run through a tube-and-shell heat exchanger.

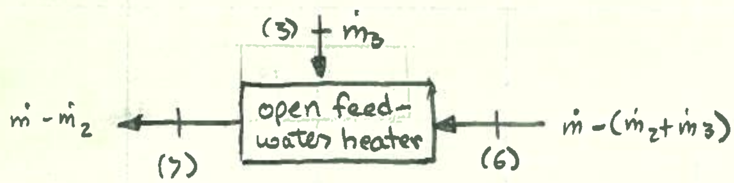
Open Feedwater Heater





- Large amount of energy released from extracted steam as it condenses; small amount of energy required to bring feedwater back to saturated liquid
- Therefore, $\dot{m}_2$ and $\dot{m}_3$ are small fractions of the total $\dot{m}$ through the turbine.

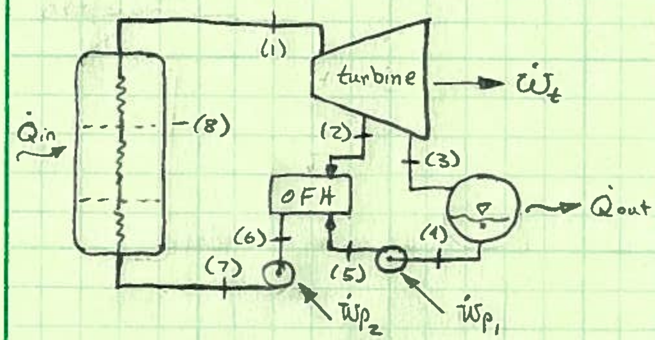
- Steam is "bled off" at (2) and (3) and mixed with subcoold water at (6) and (8) to produce saturated water at the temperature of the bled steam.

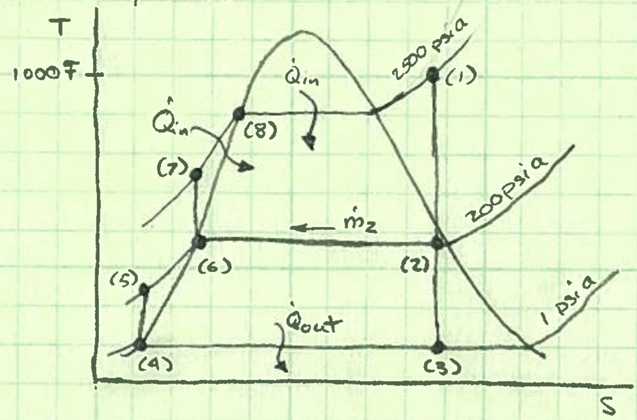


- state (3) & state (6) mix to become state (7)

- The amount of steam bled, $\dot{m}_3$, is that which will saturate the subcooled liquid, $\dot{m} - (\dot{m}_2 + \dot{m}_3)$
- $P_6 = P_7 \leq P_3$ (reverse flow into turbine would occur if $P_7 > P_3$)
- requires a pump after the feedwater heater & each pump has to handle nearly all of the mass flow in cycle
- open feedwater heaters also serve as deaerators to remove non-condensible gases such as O_2, H_2, and CO_2.

An ideal Rankine cycle operates between 2500 psia and 1000°F at throttle and 1 psia in the condenser. One open feed water heater is placed at 200 psia. Calculate the net work and heat input in Btu/lbm.





(1) $T_1 = 1000\,°F$, $P_1 = 2500$ psia } superheated steam

$h_1 = 1457.5$ Btu/lbm

$s_1 = 1.15269$ Btu/lbm°R

(2) $P_2 = 200$ psia

$s_2 = s_1 = 1.15269 \frac{Btu}{lbm°R} < s_g(500\,psia)$ } saturated steam

$s_{2f} = 0.5438$ Btu/lbm°R, $s_{2g} = 1.5454$ Btu/lbm°R } $x_2 = 0.9815$

$h_{2f} = 355.5$ Btu/lbm, $h_{2g} = 1198.3$ Btu/lbm } $h_2 = 1182.7 \frac{Btu}{lbm}$

– $\dot{m}_2$ is drawn off at this point –

(3) $\dot{m}_3 = \dot{m} - \dot{m}_2$ saturated steam

$P_3 = 1$ psia

$s_3 = s_1 = 1.1526\cancel{9}$ Btu/lbm°R $< s_g(1\,psia)$

$s_{3f} = 0.1326$ Btu/lbm°R, $s_{3g} = 1.9781$ Btu/lbm°R } $x_3 = 0.7555$

$h_{3f} = 69.73$ Btu/lbm, $h_{3g} = 1105.83$ Btu/lbm } $h_3 = 852.2 \frac{Btu}{lbm}$

(4) saturated liquid

$h_4 = h_{3f} = 69.73$ Btu/lbm

$v_4 = 0.016136$ ft³/lbm

$s_4 = s_{3f} = 0.1326$ Btu/lbm°R

$P_4 = 1$ psia

(5) Subcooled Liquid

$P_5 = 200$ psia

$v_5 \approx v_4 = 0.016136\ ft^3/lbm$

$s_5 = s_4$

$w_{p_1} = |{-_4w_5}| = h_5 - h_4 = v_4 (P_5 - P_4) = 0.59 \frac{Btu}{lbm}$

$h_5 = h_4 + w_{p_1} = 70.32\ Btu/lbm$

(6) Saturated Liquid

$P_6 = 200$ psia

$h_6 = h_{2f} = 355.5\ Btu/lbm$

$v_6 = 0.01839\ ft^3/lbm$

(7) Subcooled Liquid

$P_7 = 2500$ psia

$w_{p_2} = |{-_6w_7}| = h_7 - h_6 = v_6 (P_7 - P_6) = 7.83\ Btu/lbm$

$h_7 = h_6 + w_p = 363.33\ Btu/lbm$

(8) Saturated Liquid

$P_8 = 2500$ psia

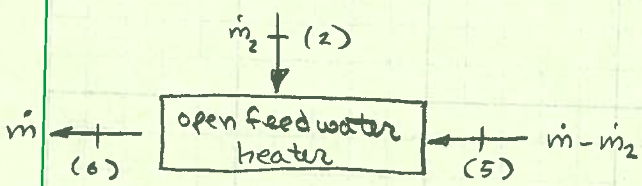
$h_8 = \cancel{1699.9}2\ Btu/lbm$

731.7

Summary of States

	h [Btu/lbm]
(1) Superheated Vapor	1457.5
(2) $x = 0.9815$	1182.7
(3) $x = 0.7555$	852.2
(4) Saturated Liquid	69.73
(5) Subcooled Liquid	70.32
(6) Saturated Liquid	355.5
(7) Subcooled Liquid	363.33
(8) Saturated Liquid	1699.92

what is the mass fraction at (2)?



Applying Cons. of Energy: $\cancel{\dot{Q}}^{0} - \cancel{\dot{W}}^{0} = \dot{m}_6 h_6 - \dot{m}_5 h_5 - \dot{m}_2 h_2$

- steady
- uniform
- incompressible

$\dot{m}_6 = \dot{m}$

$\dot{m}_5 = \dot{m} - \dot{m}_2$

$$\dot{m} h_6 = (\dot{m} - \dot{m}_2) h_5 + \dot{m}_2 h_2$$
$$= \dot{m} h_5 + \dot{m}_2 (h_2 - h_5)$$
$$\dot{m}_2 = \dot{m}\left(\frac{h_6 - h_5}{h_2 - h_5}\right) = \dot{m}\left[\frac{355.5 - 70.32 \text{ Btu/lbm}}{1182.7 - 70.32 \text{ Btu/lbm}}\right] = \underline{0.2564\, \dot{m}}$$

Turbine Work:

$$\dot{W}_t = \dot{m}\left\{(h_1 - h_2) + \left(1 - \frac{\dot{m}_2}{\dot{m}}\right)(h_2 - h_3)\right\}$$

$$w_t = \left(1457.5 \frac{\text{Btu}}{\text{lbm}} - 1182.7 \frac{\text{Btu}}{\text{lbm}}\right) + (1 - 0.2564)\left(1182.7 \frac{\text{Btu}}{\text{lbm}} - 852.2 \frac{\text{Btu}}{\text{lbm}}\right)$$
$$= 274.80 \text{ Btu/lbm} + 245.76 \text{ Btu/lbm}$$
$$= \underline{520.56 \text{ Btu/lbm}}$$

Pump Work:

$$\dot{W}_p = (\dot{m} - \dot{m}_2)\, {}_4w_5 + \dot{m}_6\, {}_6w_7 = \dot{m}\left\{\left(1 - \frac{\dot{m}_2}{\dot{m}}\right) {}_4w_5 + {}_6w_7\right\}$$

$$w_p = (1 - 0.2564)(0.59 \text{ Btu/lbm}) + (7.83 \text{ Btu/lbm}) =$$
$$= 0.44 \text{ Btu/lbm} + 7.83 \text{ Btu/lbm}$$
$$= \underline{8.27 \text{ Btu/lbm}}$$

Net Work: $w_{net} = w_t - w_p = \underline{512.29 \text{ Btu/lbm}}$

Heat Input: $q_H = h_1 - h_7 = \underline{1094.2 \text{ Btu/lbm}}$

$\left.\right\} \underline{\eta_{th} = 46.8\%}$

Closed Feedwater Heater - Cascading Drain Backwards

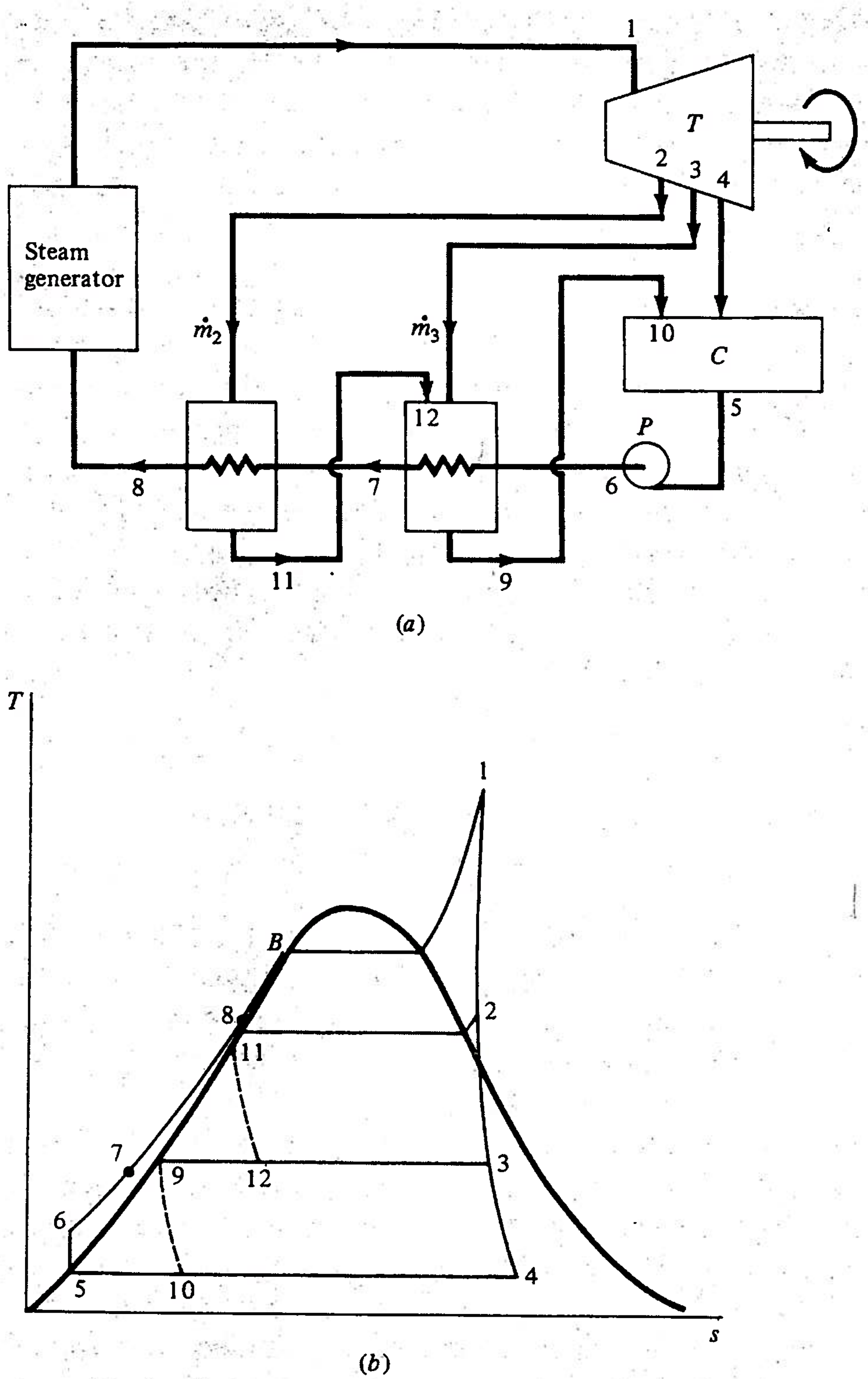
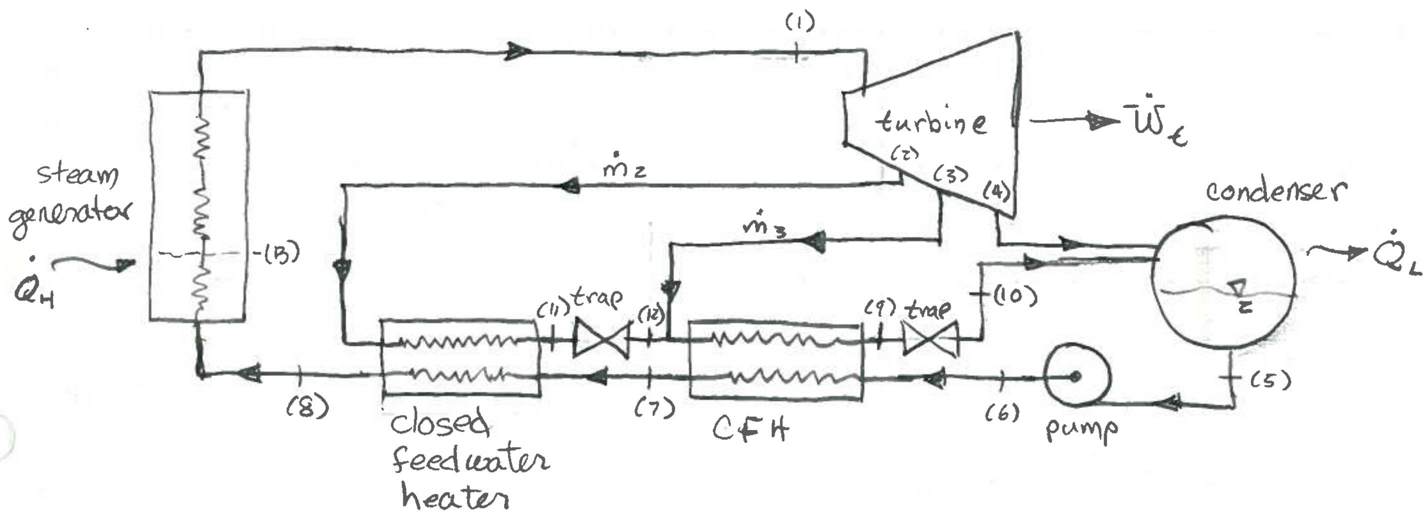


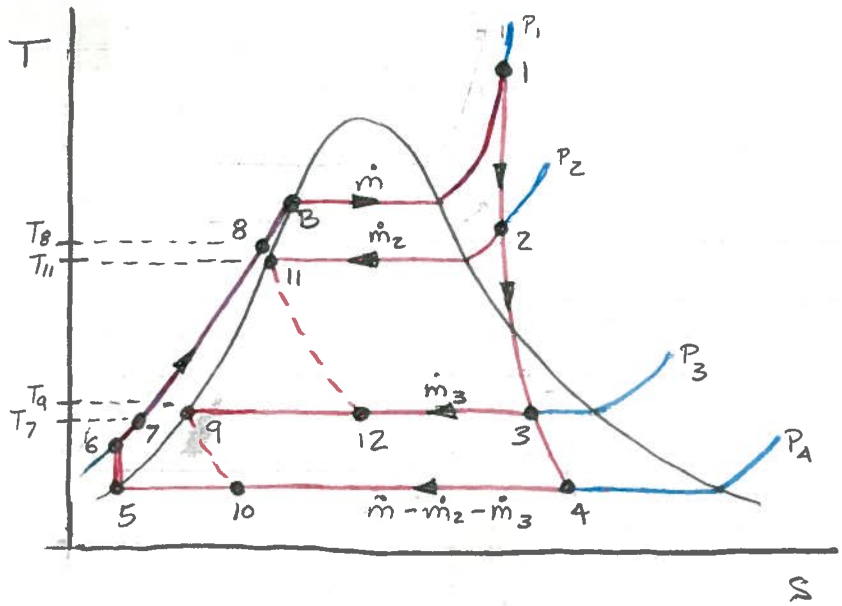
Figure 1: Closed feedwater heaters cascading backwards. From Figure 2-15, page 52, El-Wakil.

Closed Feedwater Heaters - reduce size and number of feedwater pumps

• two types – Drains Cascading Backward
  – Pump Forward

▪ Drains Cascading Backwards (Figure 2-15, p. 52)
- counterflow heat exchanger (shell & tube); no mixing as in open feedwater heaters
- no additional pumps required
- need to throttle ("trap") high pressure bleed back to low pressure bleed



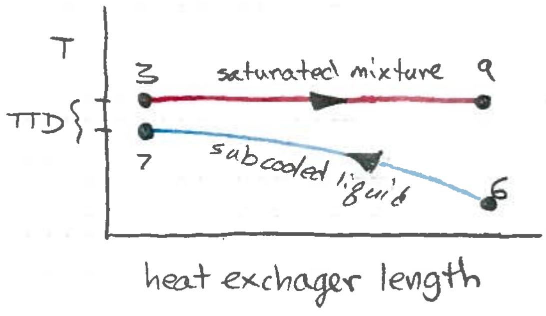


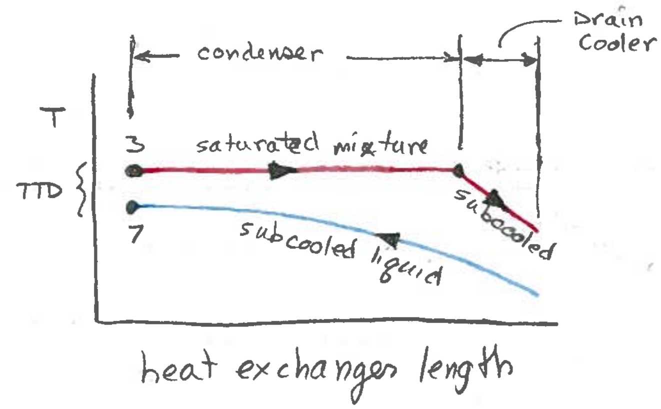
Terminal Temperature Difference (TTD or TD)

$$TTD = \left\{\begin{array}{l}\text{saturation temperature}\\ \text{of bled steam}\end{array}\right\} - \left\{\begin{array}{l}\text{exit temperature}\\ \text{of feedwater}\end{array}\right\}$$

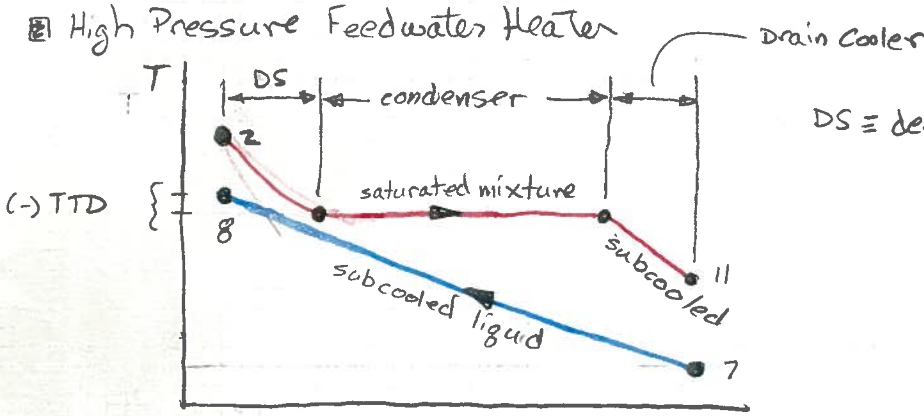
- measure of feedwater heater effectiveness

■ Low Pressure Feedwater Heater





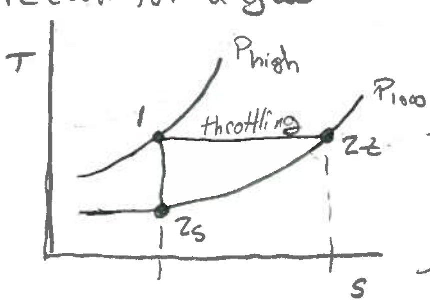
■ High Pressure Feedwater Heater



DS ≡ desuperheater

■ The drains must be throttled ("trapped") to lower the pressure back to the pressure of the earlier stage. This throttling results in a loss of availability

- recall for a gas



loss of $\underbrace{\text{potential to extract work}}_{\text{availability}}$

- also a loss of availability due to finite temperature difference during heat transfer & external irreversibilities

Closed Feedwater Heater - Pumped Forward

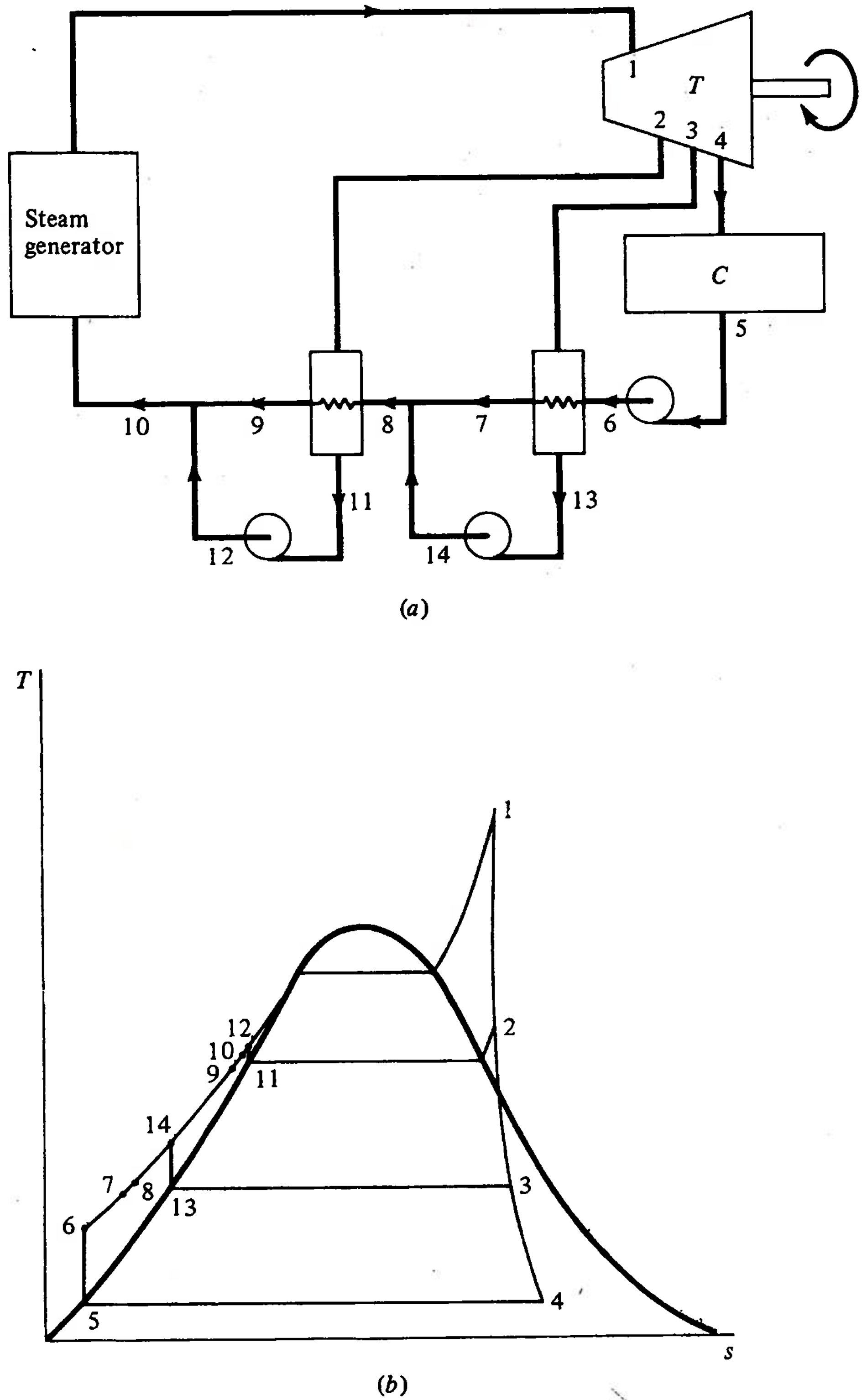
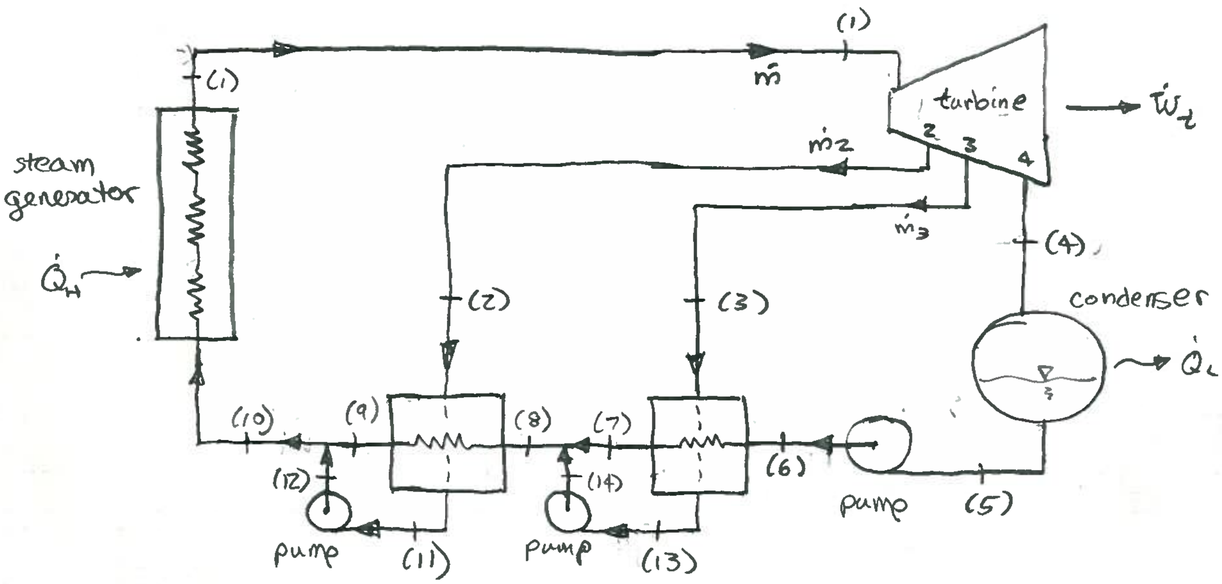
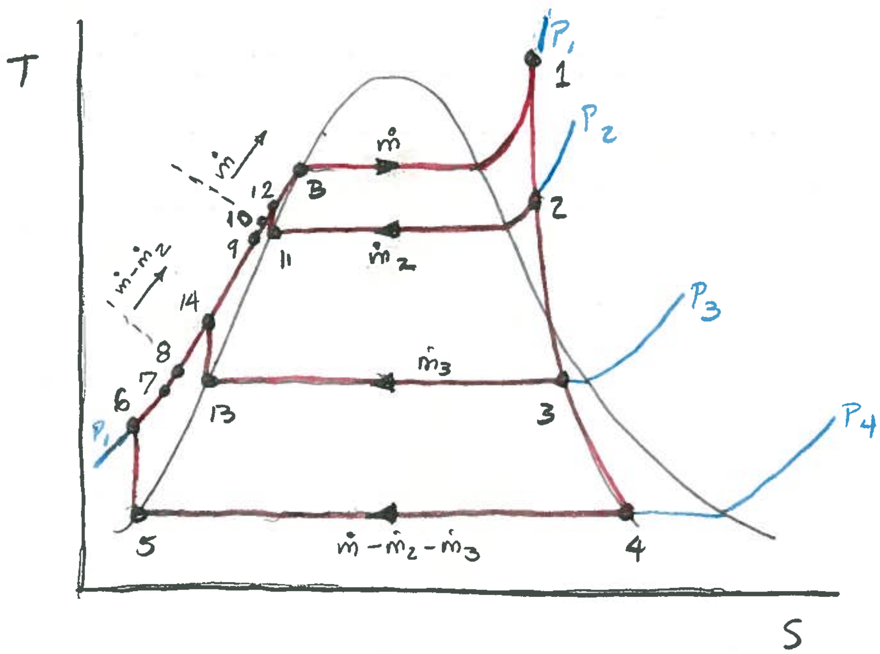


Figure 2: Closed feedwater heaters pumped forward. From Figure 2-18, page 58, El-Wakil.

Closed Feedwater Heater - Drains Pumped Forward

- to reduce loss of availability
- adds some complexity with small pumps
- drains pumped forward into the main feedwater line
  - pumps only handle a fraction of the flow, not all of the flow as with the open feedwater heaters
- efficiency is better without drain coolers
- typically used at the lowest feedwater pressure point in order to prevent throttling of a large flow (combined cascaded drains) into the condenser





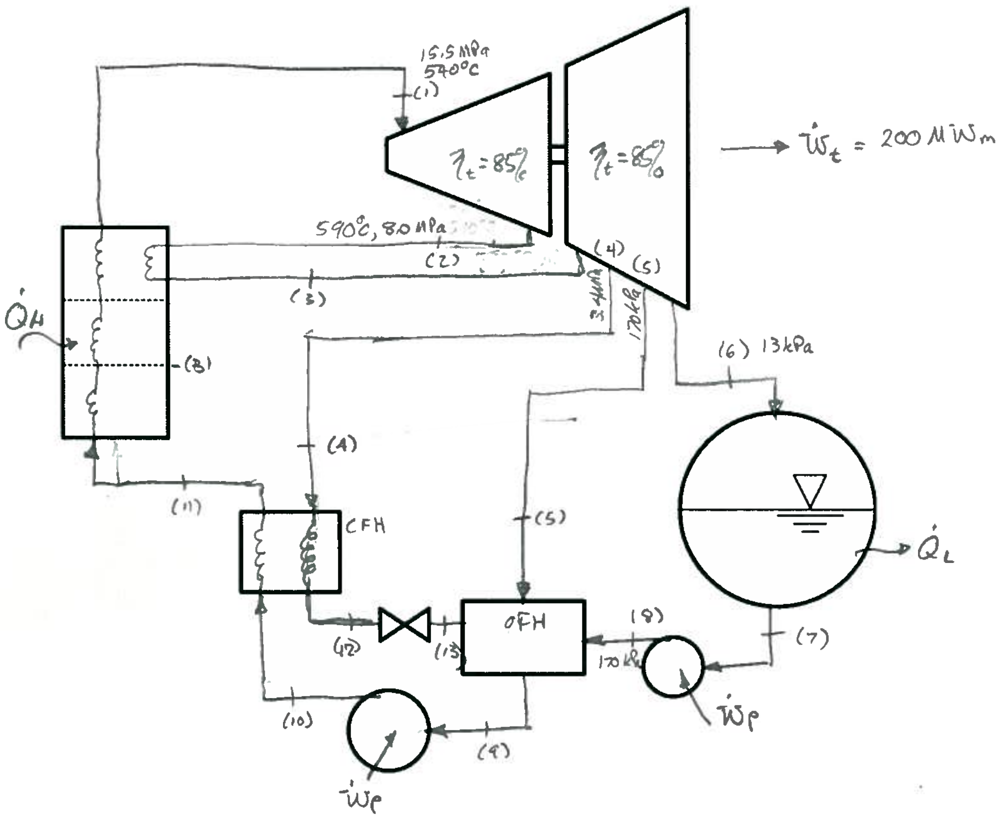
Determine the thermal efficiency, the required steam flow rate, and the moisture at the turbine exhaust for a reheat/regenerative cycle which is to produce 200 MW at the turbine coupling if the turbine throttle conditions are **15.5 MPa** and 540 °C; reheat is at 8.0 MPa and 590 °C; one closed feedwater heater is at 3.4 MPa; an open feedwater heater is at 170 kPa; and the condenser pressure is 13 kPa.

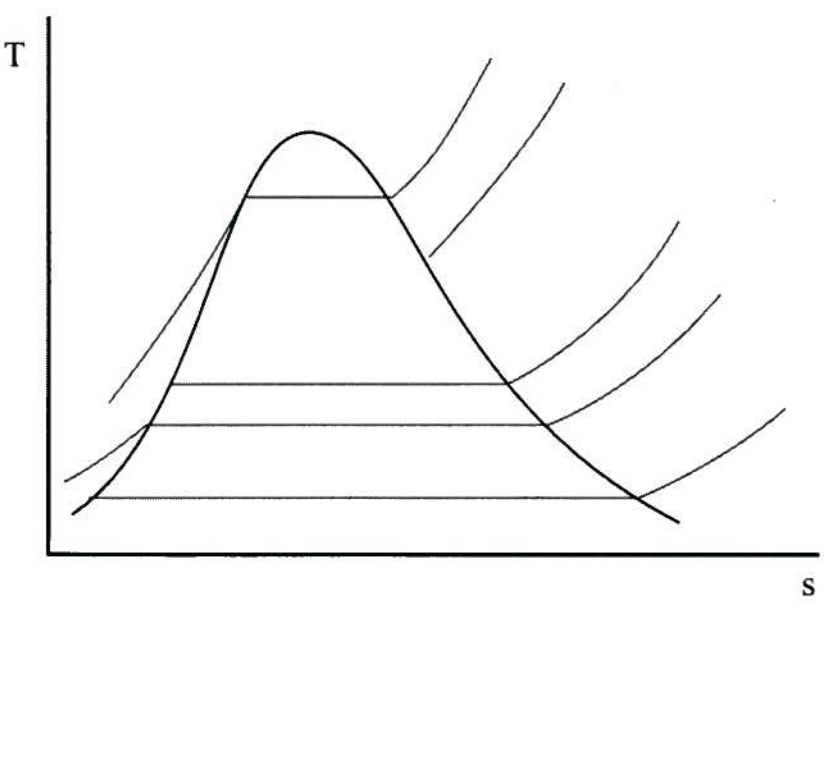
The adiabatic turbine and pump efficiencies are **85%**. The terminal temperature difference in the closed feedwater heater is 3°C and the drain for the closed feedwater heater is trapped to the open feedwater heater.

$\eta_{th} = ?$

$\dot{m} = ?$

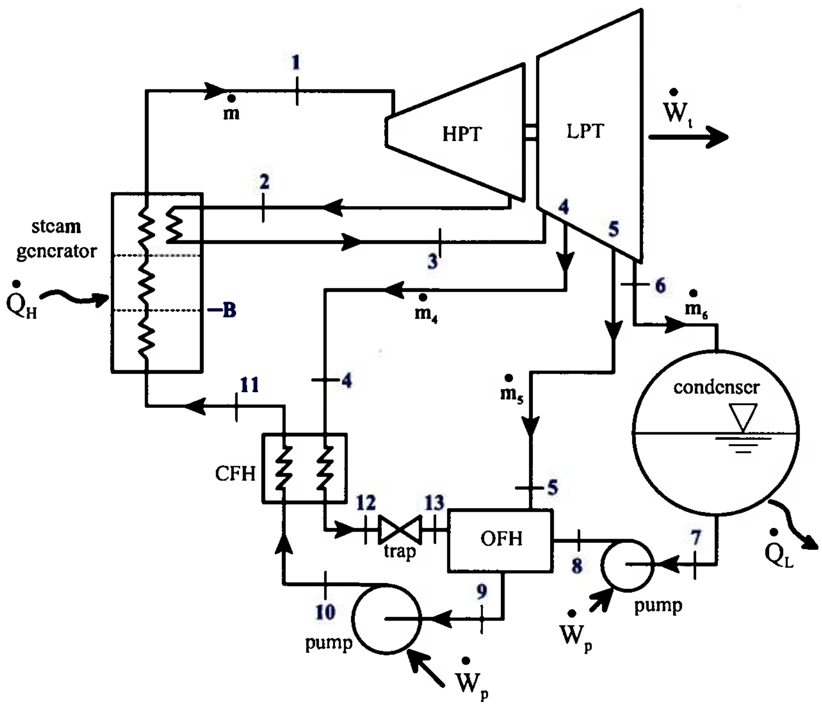
$1-x = ?$

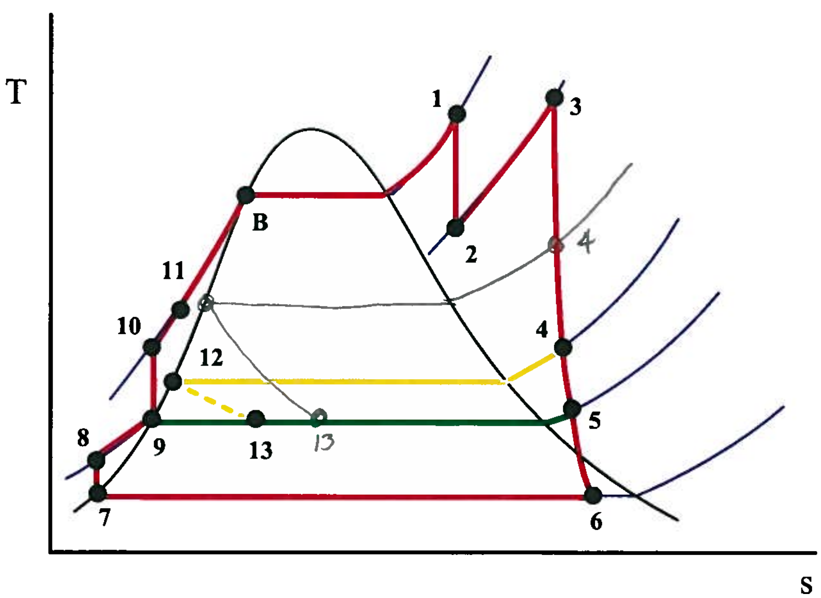




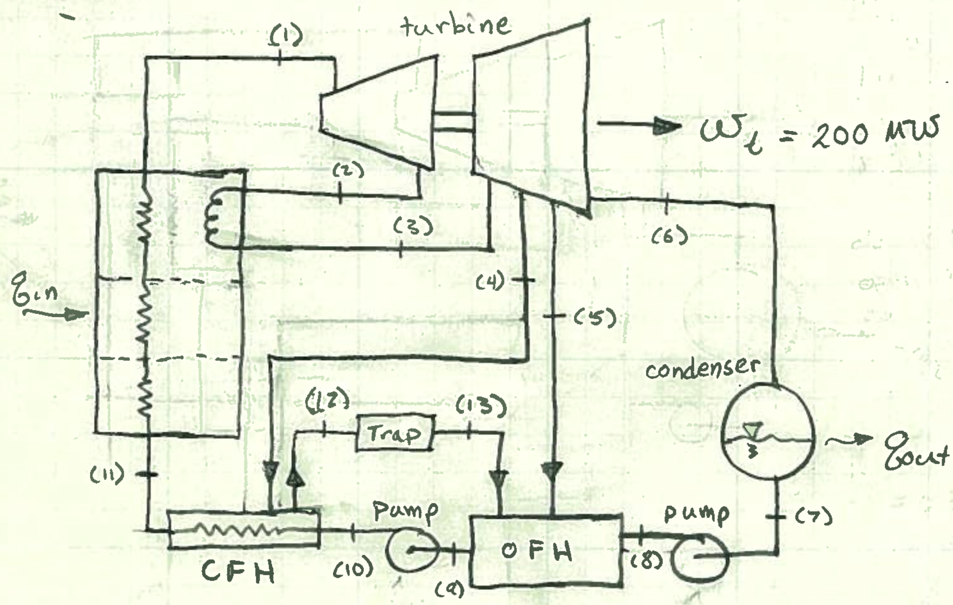
Determine the thermal efficiency, the required steam flow rate, and the moisture at the turbine exhaust for a reheat/regenerative cycle which is to produce 200 MW at the turbine coupling if the turbine throttle conditions are **15.5 MPa** and 540 °C; reheat is at 8.0 MPa and 590 °C; one closed feedwater heater is at 3.4 MPa; an open feedwater heater is at 170 kPa; and the condenser pressure is 13 kPa.

The adiabatic turbine and pump efficiencies are **85%**. The terminal temperature difference in the closed feedwater heater is 3°C and the drain for the closed feedwater heater is trapped to the open feedwater heater.





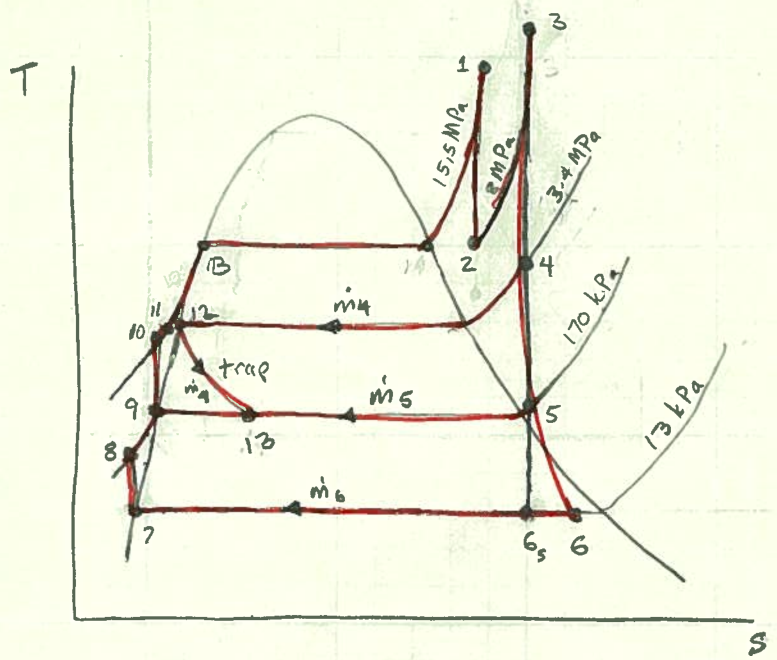
Determine the thermal efficiency, the required steam flow rate, and the moisture at the turbine exhaust for a reheat-regenerative cycle which is to produce 200 MW at the turbine coupling if the throttle conditions are 15.5 MPa and 540°C; reheat is at 8.0 MPa and 590°C; one closed feed water heater at 3.4 MPa; an open feed water heater is at 170 kPa; and the condenser pressure is 13 kPa. The turbine and pump efficiencies are 84%. The terminal temperature difference from both heaters is 3°C and the drain from the closed heater is trapped to the open heater. Also, sketch the cycle on a T-s diagram.



$\eta_{th} = ?$

$\dot{m} = ?$

moisture content at (6) = ?



turbine inlet : $P_1 = 15.5$ MPa, $T_1 = 540°C$

reheat: $P_2 = P_3 = 8$ MPa, $T_3 = 590°C$

closed FWH : $P_4 = 3.4$ MPa

open FWH : $P_5 = 170$ kPa

Condenser : $P_6 = 13$ kPa

$\eta_t = \eta_p = 84\%$

terminal temperature = 3°C

Property Data obtained from EES.

(1) $P_1 = 15.5$ MPa, $T_1 = 540\,°C$ } superheated vapor
$h_1 = 3446$ kJ/kg
$s_1 = 6.466$ kJ/kgK

(2) $P_2 = 8.0$ MPa
$s_{2s} = 6.466$ kJ/kgK, $h_{2s} = 3209$ kJ/kg } isentropic expansion

$h_2 = h_1 - \eta_t(h_1 - h_{2s}) = 3242$ kJ/kg
$s_2 = 6.513$ kJ/kgK
$T_2 = 711.4$ K
} actual expansion, superheated vapor

(3) $P_3 = 8.0$ MPa, $T_3 = 590\,°C$ } superheated vapor
$h_3 = 3617$ kJ/kg
$s_3 = 6.992$ kJ/kgK

(4) $P_4 = 3.4$ MPa, $T_4 = 462.1\,°C$ } superheated vapor
$h_{4s} = 3318$ kJ/kg for $s = s_3$

$h_4 = h_3 - \eta_t(h_3 - h_{4s}) = 3366$ kJ/kg
$s_4 = 7.058$ kJ/kgK

(5) $P_5 = 170$ kPa
$h_{5s} = 2625$ kJ/kg for $s = s_3$
$h_5 = h_3 - \eta_t(h_3 - h_{5s}) = 2784$ kJ/kg
} superheated vapor

$s_5 = 7.389$ kJ/kgK
$T_5 = 156.4\,°C$

(6) $P_6 = 13$ kPa
$h_{6s} = 2248$ kJ/kg for $s = s_3$
$h_6 = h_3 - \eta_t(h_3 - h_{6s}) = 2467$ kJ/kg
} $x < 1$

$x_6 = 0.947$
$T_6 = 51\,°C$
$s_6 = 7.668$ kJ/kgK

(7) $P_7 = 13$ kPa ← saturated liquid, $x = 0$
$h_7 = 213.6$ kJ/kg
$T_7 = 51\,°C$
$s_7 = 0.7169$ kJ/kgK
$v_7 = 0.001013$ m³/kg

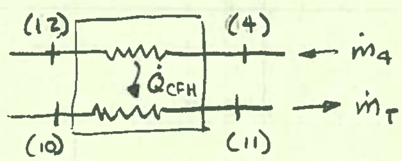
(8) $P_8 = 170$ kPa ← subcooled liquid
$v_8 \approx v_7$, incompressible fluid

$$|w_p|_{ideal} = h_{8s} - h_7 = v_7 (P_8 - P_7) = 0.159 \text{ kJ/kg}$$
$$|w_p|_{actual} = \frac{1}{\eta_p} |w_p|_{ideal} = 0.1893 \text{ kJ/kg} = h_8 - h_7$$
$$h_8 = 213.8 \text{ kJ/kg}$$

(9) $P_9 = P_8 = 170$ kPa ← saturated liquid
$T_9 = 115\,°C$
$h_9 = 483.2$ kJ/kg
$s_9 = 1.475$ kJ/kg K
$v_9 = 0.001056$ m³/kg

(10) $P_{10} = 15{,}500$ kPa ← subcooled liquid
$v_{10} \approx v_9$, incompressible fluid
$$|w_p|_{ideal} = h_{10s} - h_9 = v_9 (P_{10} - P_9) = 19.27 \text{ kJ/kg}$$
$$|w_p|_{actual} = \frac{1}{\eta_p} |w_p|_{ideal} = 19.27 \text{ kJ/kg} = h_{10} - h_9$$
$$h_{10} = 502.5 \text{ kJ/kg}$$

Closed Feedwater Heater



$$\dot{Q}_{CFH} = \dot{m}_4 (h_4 - h_{12}) = \dot{m}_T (h_{11} - h_{10})$$
$$m_4^* = \frac{\dot{m}_4}{\dot{m}_T} = \frac{h_{11} - h_{10}}{h_4 - h_{12}}$$

Terminal Temperature Difference: $T_{12} - T_{11} = 3\,°C$

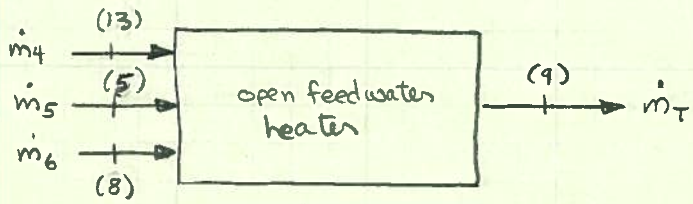
(11) $P_{11} = 15{,}500$ kPa ← subcooled liquid
$T_{11} = T_{12} - 3\,°C$
↑ $T_{sat,12} = 241\,°C$ } $T_{11} = 239\,°C$
$h_{11} = 1029$ kJ/kg

(12) $P_{12} = 3400$ kPa
$T_{12} = 241\,°C$ } ← saturated liquid
$h_{12} = 1042$ kJ/kg
$s_{12} = 2.71$ kJ/kg K

$$m_4^* = \frac{h_{11} - h_{10}}{h_4 - h_{12}} = 0.2267 \text{ kg/kg}$$

(13) Throttling Process $h_{13} = h_{12} = 1042$ kJ/kg
$P_{13} = 170$ kPa
$T_{13} = 115\,°C$
$x = 0.252$

Open Feedwater Heater



Conservation of Mass: $\dot{m}_6 = \dot{m}_T - (\dot{m}_4 + \dot{m}_5)$

$$m_6^* = 1 - (m_4^* + m_5^*)$$

Conservation of Energy:

$$\dot{m}_4 h_{13} + \dot{m}_5 h_5 + \dot{m}_6 h_8 = \dot{m}_T h_9$$

$$m_4^* h_{13} + m_5^* h_5 + (1 - m_4^* - m_5^*) h_8 = h_9$$

$$m_4^* (h_{13} - h_8) + m_5^* (h_5 - h_8) = (h_9 - h_8)$$

$$m_5^* = \frac{(h_9 - h_8) - m_4^* (h_{13} - h_8)}{(h_5 - h_8)} = 0.03181 \text{ kg/kg}$$

$$m_6^* = 0.7415 \text{ kg/kg}$$

Turbine Work:

$$\dot{W}_t = \dot{m}_T (h_1 - h_2) + \dot{m}_T (h_3 - h_4) + (\dot{m}_T - \dot{m}_4)(h_4 - h_5) + (\dot{m}_T - \dot{m}_4 - \dot{m}_5)(h_5 - h_6)$$

$$w_t = (h_1 - h_2) + (h_3 - h_4) + (1 - m_4^*)(h_4 - h_5) + (1 - m_4^* - m_5^*)(h_5 - h_6)$$

$$= (170 \text{ kJ/kg}) + (251 \text{ kJ/kg}) + 0.7733 (582 \text{ kJ/kg}) + 0.7415 (317 \text{ kJ/kg})$$

$$= 1106 \text{ kJ/kg}$$

Turbine Power: $\dot{W}_t = 200 \text{ MW} = \dot{m}_T w_t$

$$\boxed{\dot{m}_T = 180.8 \text{ kg/s}}$$

Pump Work: $|\dot{W}_p| = (\dot{m}_T - \dot{m}_4 - \dot{m}_5)(h_8 - h_7) + \dot{m}_T (h_{10} - h_9)$

$$|w_p| = (1 - m_4^* - m_5^*)(h_8 - h_7) + (h_{10} - h_9) = 19.45 \text{ kJ/kg}$$

Net Work: $w_{net} = w_t - |w_p| = 1090.55 \text{ kJ/kg}$

Heat Input: $\dot{Q}_{in} = \dot{m}_T (h_1 - h_{11}) + \dot{m}_T (h_3 - h_2)$

$$q_{in} = (h_1 - h_{11}) + (h_3 - h_2) = 2762 \text{ kJ/kg}$$

Thermal Efficiency: $\boxed{\eta_{th} = \frac{w_{net}}{q_{in}} = 39.5\%}$

$\boxed{\text{Moisture Content at Turbine Exhaust} = 1 - x_6 = 5.3\%}$

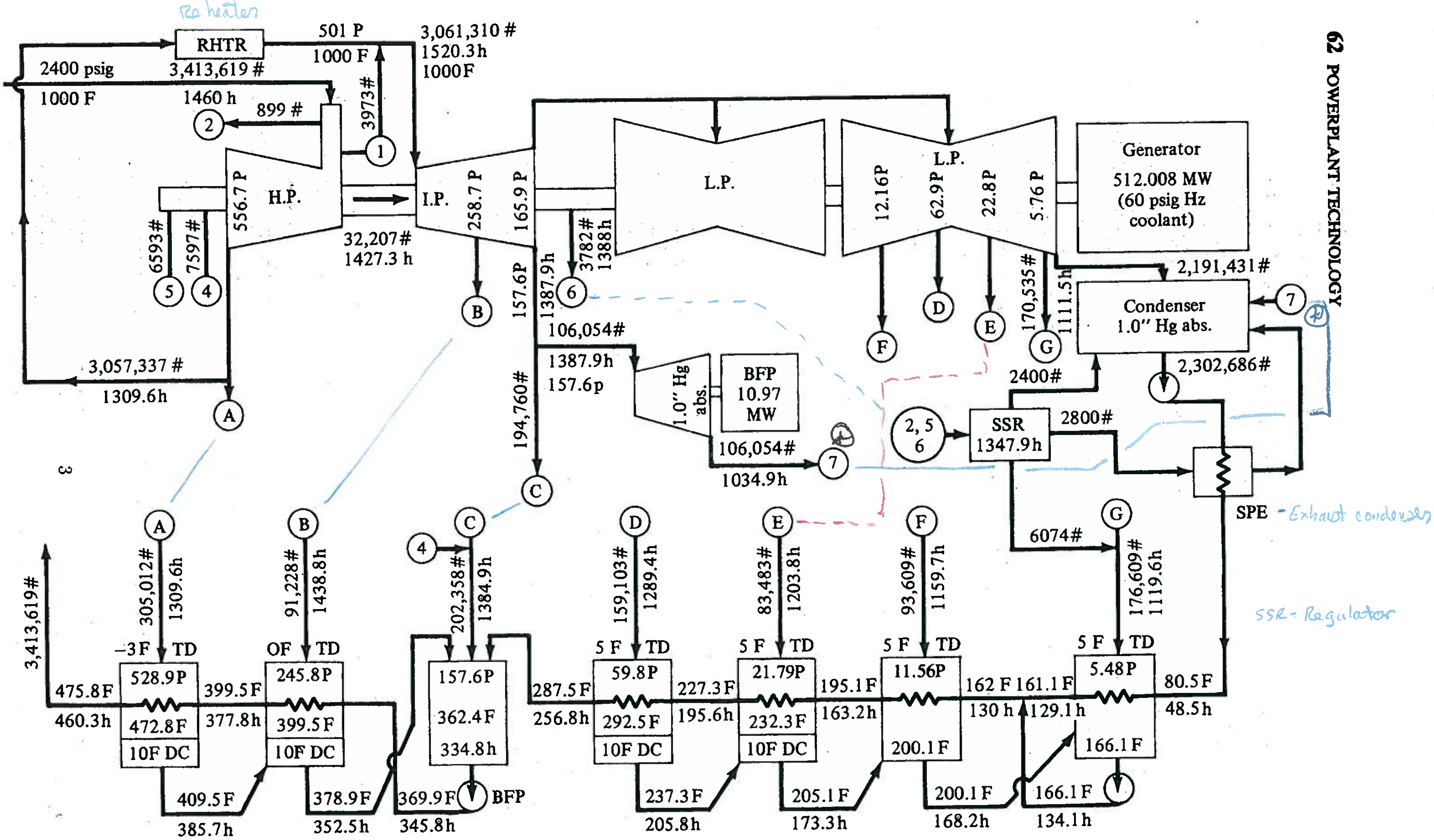
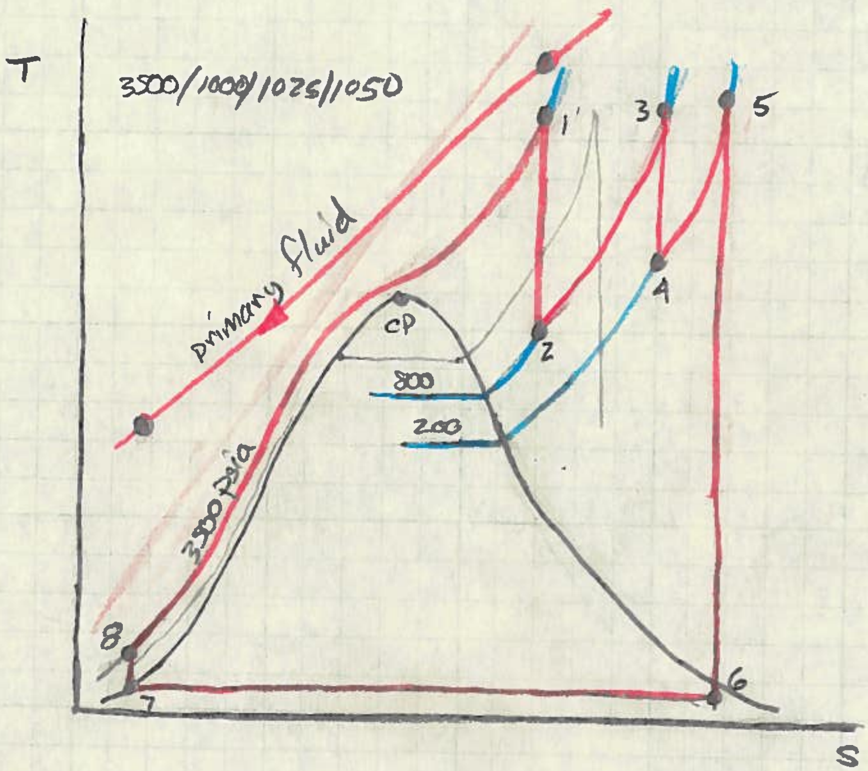


Figure 2-20 Flow diagram of an actual 512-MW 2400 psig/1000°F/1000°F reheat powerplant with seven feedwater heaters. *(Courtesy Wisconsin Power & Light Co.)*

supercritical Cycle



- critical point of steam is 3208 psia
- gradual change in temperature and density, but no phase change
- smaller external irreversibilities
  - no abrupt change in temperature in 2-phase region
- supercritical cycle receives more thermal energy at high temperatures than a subcritical cycle with the same turbine inlet temperature
- single stage expansion would result in very wet steam; often use reheat or two reheats
- The temperature can be increased in reheat sections (3500/1000/1025/1050) because the pressures are lower (steam tube materials limit).

Co-Generation

Simultaneous generation of electricity and process steam in a single plant. The process steam may be used as a source of superheated vapor or as a source of heat. Co-generation is commonly used by the chemical industry, paper mills, municipalities for district heating, and in cement kilns.

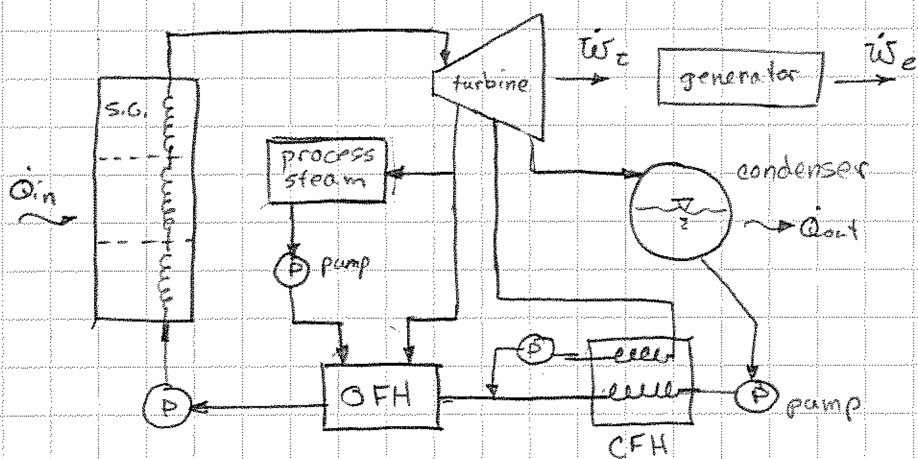


Fig. Co-Generation shown as a topping cycle with extraction and regeneration.

Topping Cycle: High pressure steam is used to generate electrical power, and low pressure steam (0.5 - 40.0 bar) is used for process. The process steam may be extracted from the turbine or from the turbine exhaust. The latter is known as a back pressure turbine since the turbine exhaust is at a higher pressure than the condenser.

Bottoming Cycle: High pressure steam is used for process, and low pressure steam is used to produce electrical power. An example of a bottoming cycle is certain types of cement kilns. Typically, co-generation with a bottoming cycle has very low overall efficiency.

Co-Generation Efficiency

Co-generation is only beneficial if less energy is used as compared to separate generation of electricity and process steam (heat).

Powerplant Efficiency, $\eta_{plant} = \dfrac{\dot{W}_e}{\Delta \dot{H}_{fuel}} = \underbrace{\eta_{SG}}_{\dot{Q}_{in}/\Delta \dot{H}_{fuel}} \underbrace{\eta_{th}}_{\dot{W}_m/\dot{Q}_{in}} \underbrace{\eta_e}_{\dot{W}_e/\dot{W}_m}$

$\left\{ \text{Heat Rate, } HR = \dfrac{\Delta \dot{H}_{fuel}}{\dot{W}_e} \right\}$

Co-Generation Plant Efficiency, $\eta_{cogen} = \dfrac{\dot{W}_e + \Delta \dot{H}_{process}}{\Delta \dot{H}_{fuel}}$

In order to assess whether or not co-generation is an improvement over separate generation, the fraction of fuel used to generate electrical power must be determined. Define the fraction of electrical power output to total power output as:

$$\chi = \frac{\dot{W}_e}{\dot{W}_e + \Delta \dot{H}_{process}}$$

and

$$1 - \chi = \frac{\Delta \dot{H}_{process}}{\dot{W}_e + \Delta \dot{H}_{process}}$$

The reciprical of the co-generation plant efficiency, which is similar to a Heat Rate, allows for an assessment of efficiency improvements.

$$\frac{1}{\eta_{cogen}} = \frac{\Delta \dot{H}_{fuel}}{\dot{W}_e + \Delta \dot{H}_{process}} = \underbrace{\frac{\Delta \dot{H}_{fuel}}{\dot{W}_e}}_{\frac{1}{\eta_{SG}\eta_t\eta_e}} \chi + \underbrace{\frac{\Delta \dot{H}_{fuel}}{\Delta \dot{H}_{process}}}_{\frac{1}{\eta_{SG}}} (1-\chi)$$

$$\eta_{cogen} = \frac{\dot{W}_e + \Delta \dot{H}_{process}}{\Delta \dot{H}_{fuel}} \qquad (1)$$

$$\eta_{sep} = \frac{1}{\dfrac{\chi}{\eta_{SG}\eta_{th}\eta_e} + \dfrac{1-\chi}{\eta_{SG}}} \qquad (2)$$

Co-generation may be beneficial if $\underset{(eq.\ 1)}{\eta_{cogen}} > \underset{(eq.\ 2)}{\eta_{sep}}$.